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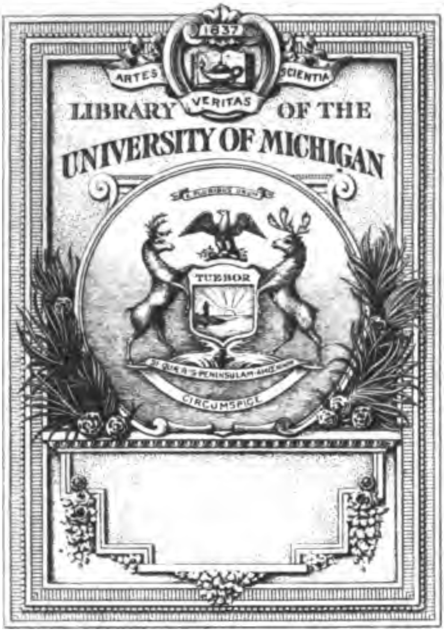
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THE
PREFACE.

THE Plan of the following Work was drawn up for the Uſe of a few private Friends; it is now enlarged by increaſing the Number of Examples, and making the Rules and Directions more copious; but whether it will prove as generally ſucceſsful, in conveying the Knowledge of this moſt uſeful Science, as it was beneficial to my Friends, Experience muſt declare.

It might be cenſured as Vanity, ſhould I draw a Parallel between this Work, in Regard to it's Uſefullneſs for Beginners, and what ſeveral learned Gentlemen have publiſhed on this Subject: But when it was determined it ſhould appear in Publick, I principally ſtudied, to make it as uſeful as poſſible to the Publick Schools, and at the ſame Time to provide that Perſons, by their own Application, might without further help, acquire a conſiderable Knowledge in the Elements of *Algebra*.

There are two Things abſolutely neceſſary, to make the Acquiſition of any Science as eaſy as it's Nature will admit. That is, the Diſpoſition of the Work, ſo that the Rules be clear and diſtinct; and then the illuſtration of theſe Rules by a ſufficient Number of proper and pertinent Examples. And tho' the excellent *Elements* of the judicious EUCLID are of a different Nature, yet in this I have induſtriouſly ſtudied to imitate him, that I propoſe no new Rule or Article in this *Art of Inveſtigation*, till it become neceſſary to carry the Learner to a further Degree of Knowledge.

Science may be compared to a highly finiſhed Pile of Building, all the Parts of which being diſpoſed in the moſt exact Symmetry, they muſt affect our Perception, and gratify our internal Senſation with a more exquiſite Pleaſure, than if viewed in a ſeparate State: For in ſuch a State, to all but the Learned, they would appear broken and inconnected Materials of a mighty Structure, which the Mind wanting Power to conceive, could enjoy no Satisfaction in the Contemplation of ſuch a Train of Imperfect and confuſed Ideas. But, when thus exhibited in their true Proportion, it will be eaſy, even for the youngeſt Scholar, to gain a perfect Notion of each, and, as he advances, a gradual Comprehenſion of the Beauty reſulting from their Connection, and how they mutually aſſiſt and ornament each other.

In teaching the abſtract Sciences, Examples have a ſtrong as well as natural Tendency to illuſtrate the Precepts concerning our abſtract Reaſonings, eſpecially in this Science of *Inveſtigation*: In the *Synthetic* Method, or Method by Demonſtration, one Example or Propoſition is ſufficient, for a Number of the ſame Propoſitions is only a Repetition of the ſame ſucceſſive Train of Ideas; whereas, in the *Analytic* Method, or Method by *Inveſtigation*, the ſame Concluſion is gained, tho' there may be a great Variety in the Connection of the Reaſoning, by the different Diſpoſition of the Ideas, notwithſtanding they are directed by the ſame general Rule.

And as the principal Difficulty in this Science, is acquiring the Knowledge of ſolving of Queſtions, I have given a great Variety of theſe in reſpect to Numbers and Geometry, and their Solutions I choſe to give in the moſt particular, diſtinct, and plain Manner; and for which the Reader will find full and explicit Directions. As it is prepoſterous, and abſurd, to expect a Perſon to be a *Critic* in any Language as ſoon as he has paſſed thro' his Grammar; ſo, I cannot help thinking it wrong to expect a Learner ſhould ſee the Reaſon of particular elegant Methods of Solution, before he has practiſed a general and univerſal Method; but after the general Rules are become eaſy and familiar the Learner may then apply himſelf to the particular Methods. And I know of no Work, that has illuſtrated and exemplified the general and univerſal Rules in ſo copious a Manner as will be found in the following Sheets.

In the Arithmetical Operations, the Decimal Fractions are continued to two Places only, theſe being ſufficient to ſhew the Reader, that if the Queſtion admits not of an exact Anſwer, he is yet near the Truth, and may proſecute the Anſwer to any required Degree of Exactneſs. I have avoided all tedious Numerical Calculations, as they have no Tendency to increaſe the Learner's Knowledge in *Algebra*.

The Rules of Vulgar Fractions in *Algebra* are omitted, being generally very perplexing to Learners, but as I have given ſufficient Directions how they are managed whenever they occur in the Solution of a Queſtion, the Reader will find no Difficulty in reducing an Equation with Fractional Quantities.

It is neceſſary my Reader ſhould underſtand Vulgar Fractions in common Arithmetick, and the Extraction of the Square Root, and then I know no Reaſon why a Perſon, who has a few vacant Hours, may not make himſelf a perfect Maſter of the following Work, excepting the Geometrical Queſtions, which he may omit, and proceed to thoſe which require no ſkill in Geometry, for through the whole, where it was neceſſary, I have given the ſame Directions, as if I was actually teaching a Scholar.

THE INTRODUCTION.

AS all Arts have their Beginning, rude and weak, and reach Perfection by Degrees, ſo that, which is the Subject of the following Sheets, has been cultivated by ſo many illuſtrious Men in our own, as well as in foreign Nations, that it cannot but appear a natural Introduction to this Treatiſe, if we digeſt the Hiſtory of it's Riſe and Progreſs into a ſuccinct Diſcourſe; the rather becauſe Books of this ſort are now become very numerous in ours, as well as in other Languages, and therefore, it is the more neceſſary to record the Names of ſuch as have eminently improved ſo uſeful a Branch of Knowledge.

The Word *Algebra* is certainly derived from the *Arabic*, but there have been ſome Miſtakes as to it's Meaning. When it was firſt introduced in *Europe*, it was underſtood to be the Invention of the famous Philoſopher *Geber*; and therefore *Michael Stifelius* calls it ſometimes *Regula Algebræ*, and ſometimes *Regula Gebri*, whence it is plain he underſtood by it no more than the Rule of *Geber*, or as we uſually expreſs it *Geber*'s Rule. But, when we became better acquainted with *Arabic* Learning, this Derivation appeared ill founded: In that Language, this Art is called *Al-gjâbr W'al-mokâbala*, which is literally, the Art of Reſolution and Equation. Hence it is plain, we had the Word Algebra from the *Arabic* Name of the Art, and not from the pretended Inventor. But it may not be amiſs to obſerve, that the *Arabic* Name contains a Definition, or is rather an emphatic Declaration of the Nature and End of this Science; for the *Arabic* Verb *jábara* ſignifies to reſet, and is properly uſed in reſpect to Diſlocations, and the Verb *bábala*, implies oppoſing, or comparing; and how applicable this is to what we call Algebra, the Reader, when he is thoroughly acquainted with this Book, will eaſily underſtand. As it became better known to the *Europeans*, it received different Names; the *Italians* ſtiled it *Ars magna*, in their own Language *l'Arte Magjore*, oppoſing to it common Arithmetick, as the leſſer or minor Art. It was alſo called *Regula Coſæ*, the Rule of *Coſs*, for an odd Reaſon: The *Italians* make Uſe of the Word *Coſa*, to ſignify what we call the Root, and from thence, this kind of Learning being derived to us from them, the Root, the Square, and the Cube, were called *Coſſick* Numbers, and this Science the Rule of *Coſs*. I ſhould not have dwelt ſo long on ſo dry a Subject, but that it is abſolutely neceſſary for the Underſtanding what follows.

It is a Point ſtill diſputed, whether the Invention of Algebra ought to be aſcribed to the *Oriental* Philoſophers, or to the *Greeks*; but it is a thing certain, that we received it from the *Moors*, who had it from the *Arabians*, who own themſelves indebted for it to the *Perſians* and *Indians*; and yet, which is ſtrange enough, the *Perſians* refer the Invention to the *Greeks*, and particularly to *Ariſtotle*. Yet, notwithſtanding this, it muſt be allowed that the Algebra taught us by the *Arabians* differs very much from that contained in the Works of *Diophantus*, the eldeſt *Greek* Author on this Art, which is now extant, and which was diſcovered and publiſhed long after the Algebra taught by the *Arabians* had been ſtudied and improved in the Weſt. But all theſe Difficulties, which have given ſome great Men ſo much

Trouble, may be eaſily ſurmounted, if we ſuppoſe that the Invention was originally taken from the *Greeks*, and new modelled by the *Arabians*, in the ſame Manner as we know that common Arithmetick was; for this, which is at leaſt extreamly probable, makes the whole plain and clear, and leaves us at liberty to purſue the Progreſs of this Art from the firſt printed Treatiſes about it.

Lucas Paciolus a Franciſcan Friar, commonly known by the Name of *Lucas de Burgo Sancti Sepulchri*, publiſhed at *Venice*, under the Title of, *A Compleat Treatiſe of Arithmetick and Geometry, Proportions and Equations*, the firſt Book at preſent extant on this Subject. It was printed ſo early as 1494, and is a very correct Treatiſe: He aſcribes the Invention of Algebra to the *Arabians*, uſes their Method, and treates very clearly of Quadratick Equations. After him, ſeveral Authors wrote on the ſame Subject in *Italy*, and in *Germany*; but ſtill the Art advanced little till the famous *Jerom Cardan* printed, at *Nuremberg* in 1545 in Folio, a Treatiſe with this Title, *Artis magnæ, ſive de Regulis Algebraicis Liber unus*; and ſoon after a ſmaller Piece, with the Title of *Sermo de Plus & Minus*, wherein were contained Rules for reſolving Cubick Equations, which have ſince been called *Cardan*'s Rules, though they were not invented by him, but, as himſelf owns, by *Scipio Ferreus* of *Bononia*, and *Tartalea*. The next celebrated Writer was a *French* Monk, whoſe Name was *Boteon*, better known to the learned by his *Latin* Appellation of *Buteo*; he publiſhed in 1559 his *Logiſtica*, in which there was a Treatiſe of Algebra which gained him great Reputation: Yet his Excellency lay in a clear and copious Manner of Writing, nor does it appear that he added any thing to what had been already diſcovered, except ſome Corrections as to *Tartalea*'s Method of managing Cubick Equations.

Hitherto nothing was known in *Europe* of the *Greek* Analyſis, but in 1575 *Xilander* publiſhed *Diophantus*, or at leaſt a part of his Works, which are ſtill remaining; and this quickly changed the Face of Things, for it preſently appeared that his was a neater and more eaſy Method, and withal opened a Path to much greater Diſcoveries, which was the Reaſon that ſucceeding Algebraiſts quitted the Terms made Uſe of by *Arabic* Writers, and followed his. The Time in which *Diophantus* flouriſhed is not thoroughly ſettled. *Voſſius* thinks he lived in the ſecond Century, but others place him in the fourth. His Works were known to the *Arabians*, and tranſlated by them; nay, it is ſaid, that they have ſtill thoſe ſeven Books of his Arithmetick which are loſt to us. The famous *Arabian* Hiſtorian *Abul-Pharaijus*, whoſe Works were publiſhed by the learned *Pocock*, not only mentions him, but aſcribes to him the Invention of Algebra; but in this he is to be underſtood, as writing according to the Lights he had; for tho' it be true, that *Diophantus Alexandrinus* is the oldeſt Author we have which treats expreſsly of the *Analytick Art*, yet the Footſteps thereof are viſible in much older Writers. *Theo*, who is thought to have explained the five firſt Propoſitions of the Thirteenth Book of *Euclid* in the Analytick Way, gives the Honour of this Invention to *Plato*; and indeed, it ſeems very agreeable to his Genius, and Method of Reaſoning on Mathematical Subjects. By the Junction of both Lights, and a proper Connection of the *Arabic* Method of Inveſtigation with the *Greek* Terms, which were ſhorter and eaſier, Algebra quickly became a much more uſeful, as well as conſiderable Science than it was before.

In our own Country, the firſt Writer upon Algebra that we know of was Dr *Robert Record* a Phyſician, who diſtinguiſhed himſelf in the Reign of Queen *Mary*, by his Skill in the Mathematicks. He firſt publiſhed a Treatiſe of Arithmetick, which continued the Standard in that Branch of Knowledge for many Years, and in 1557, he ſent abroad a ſecond Part, under the Title of *Cos Ingenii, or the Whetſtone of Wit*, which is a Treatiſe of Algebra; the Word *Cos* alluding to *Coſſick* Numbers, or the Rule of *Coſs*, by which Name, as we have before ſhewn, this Art was known abroad. This Treatiſe is really a great Curioſity, conſidering the Time in which it was publiſhed, and together with his other Works, muſt give us a high Idea of this Man's Induſtry and Application, whoſe Memory notwithſtanding is almoſt buried in Oblivion. But, notwithſtanding the early Publication of this Piece, and that ſome *Engliſh* Gentlemen had in their Travels acquired ſome Knowledge of this Kind, as appears by a *Spaniſh* Treatiſe of Algebra, publiſhed by *Pedro Nunnez* in 1567, yet it continued to be ſo little cultivated in *England*, that *John Dee*, in his Mathematical Preface prefixed to Sir *Henry Billingſley*'s Tranſlation of *Euclid*, printed at *London* in 1570, ſpeaks of it in very high Terms, and as a Myſtery ſcarce heard of by the ſtudious in the Mathematicks here. It is however, plain from ſome of his Annotations on *Euclid*, that he was tolerably verſed therein, and was even acquainted with the Manner of applying it to Geometry. In 1579 *Leonard Digges*, a great Mathematician for thoſe Times, printed a Treatiſe of Algebra in his *Stratioticos*, after which it came to be better known and more ſtudied, to which contributed not a little, the Improvements made by the Author I am next to mention.

Francis Viete, better known by his *Latin* Name of *Franciſcus Vieta*, was a Native of *Poitou*, in *France*, and Maſter of Requeſts to Queen *Margaret*, firſt Wife to King *Henry* IV; his Affection to the Mathematicks, and eſpecially to this Part of it, was ſo ſtrong, that he frequently paſſed three whole Days and Nights in his Study without eating, drinking, or ſleeping, except a Nod now and then upon his Elbow *. He, about the Year 1590, publiſhed a Treatiſe of Algebra in quite a new Method, and by a judicious Mixture of the *Greek* and *Arabian* Rules, with ſome Improvements of his own, introduced that mode of Calculation which is ſtill in Uſe, under the Title of *Specious Arithmetick*. Before his Time, only unknown Quantities were marked by Letters, but ſuch as were known were ſet down in Figures according to the uſual Notation. He made Uſe of Letters for both, only with this Diſtinction, that the known Quantities he repreſented by Conſonants, and the unknown, by Vowels.

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* *Thuan.* Hiſt. A. D. 1603.

By this Contrivance he greatly extended the Science, and which was more, ſhewed it's Capacity of being farther extended. For, whereas former Algebraiſts had confined their Inveſtigations to the particular Queſtions propoſed to them, he by this Means produced Theorems capable of reſolving all Demands of a like Nature, inſtead of particular Solutions. The learned Dr *Wallis* has accounted very clearly for the new Title which *Vieta* gave to his Algebra. The *Romans* had a Method of ſtating Law Queſtions under general Names, ſuch as *Titius* and *Sempronius*, *Cajus* and *Mevius*, whence we derive our Way of uſing A, B, C, D, on ſuch Occaſions, which Method of ſtating, the *Civilians* ſtile *Species*, in Oppoſition to the ſtating of *real Caſes* by *true Names*. *Vieta* having made a Change of the ſame Nature in Algebra, and being as we obſerved before, a Lawyer by Profeſſion, he borrowed from that Science this Title of his new Invention, which was received with univerſal Applauſe. We have likewiſe many of his Works, under the Name of *Apollonius Gallus*, which he aſſumed on Account of his firſt attempting to reſtore the Works of *Apollonius Pergæus*. His Genius was ſo extenſive, and his Penetration ſo great, that it enabled him to apply his Mathematical Knowledge to moſt Subjects; of which we have a particular Inſtance, in his decyphering the Letters which paſſed between the Court of *Spain*, and the Faction of the League in *France*, notwithſtanding above Five hundred different Characters were made Uſe of in them. About the ſame Time flouriſhed *Raphael Bombelli*, an *Italian*, who publiſhed at *Florence* a Treatiſe of Algebra, wherein he firſt taught how to reduce a biquadratick Equation to two Quadraticks, by the help of a Cubic.

Our own Countryman, Mr *William Oughtred*, was the next great Improver of Algebra. Building however on what *Vieta* had already performed. He introduced ſuch a Conciſeneſs, and withal ſo plain and perſpicuous a Method of inveſtigating Geometrical Problems, as acquired him immortal Reputation. His *Clavis Mathematicæ*, Or, *Key of the Mathematicks*, was firſt publiſhed in 1631, and is perhaps the cloſeſt and moſt compendious Syſtem hitherto extant. In this Work he contented himſelf with the Solution of quadratick Equations, reſerving thoſe of higher Powers for another Work, which was his *Exegeſis Numeroſa*, which in later Editions is join'd to his *Clavis*. In both Pieces there were abundance of Additions and Improvements, and the Doctrine of Proportions more fully and clearly ſtated than hitherto it had been; but the greateſt Excellency in Mr *Oughtred*'s Book, was his Application of the Analytick Method to Geometry, which he did in a Variety of Caſes, and enabled his Diſciples to proceed ſtill farther than himſelf had done. By Profeſſion he was a Clergyman, and Rector of *Albury* in *Surry*, where he gave himſelf up entirely to his Studies, and to the Converſation of a very few Friends, he lived to the Age of Fourſcore and ſeven, and died then of Joy, on *May* 1, 1660, at hearing the Houſe of Commons had voted the King's Return. Some have cenſur'd his *Clavis* as too ſhort and obſcure, and ſo indeed it might prove for ſuch as were altogether unacquainted with theſe Studies, for whoſe uſe it is plain enough he never deſign'd it; but where Perſons are acquainted with the Elements of Geometry and Algebra, and have that Sagacity and Attention which is neceſſary to make any conſiderable Progreſs in this ſort of Learning, Mr *Oughtred*'s Key will be ſtill found a very uſeful Book, and its Stile the moſt perfect in its kind that has ever been uſed.

Contemporary with him was Mr *Thomas Harriot*, an excellent Mathematician, and who made ſtill greater Improvements in this Science. He is placed after *Oughtred*, tho' he died long before him, becauſe his Book was not publiſhed till ſome time after the firſt Edition of *Oughtred*'s *Clavis*. It was then printed in a thin Folio by the Care of Mr *Walter Warner*, under the Title of *Artis Analyticæ Praxis ad Æquationes Algebraicas novâ, expeditâ, & generali Methodo, reſolvendas, Tractatus poſthumus, &c. i.e.* A Treatiſe of the Analytick Art, containing a new, expeditious, and general Method of reſolving Equations, a poſthumous a poſthumous Tract, by the late learned Mr *Thomas Harriot.* The Publiſher, Mr *Warner*, prefix'd a Preface of his own, containing a very judicious, tho' very conciſe, Repreſentation of the ſeveral Parts of Algebra, their Nature and Dependance on each other, the Extent and Uſefulneſs of this Art, and the Progreſs thereof to that Time. In Mr *Harriot*'s Book, Algebra takes a new Form, and from him alone it met with more Improvement than from all who had ſtudied, or at leaſt all who had written upon it, before him. He was indeed one of the greateſt Men this Nation ever produced, and great Pity it was, that this Work of his did not appear in his Life-time, or that his other Pieces, which were of infinite Value, ſhould be buried in Oblivion. The true Cauſe of the former ſeems to have been his Courſe of Life, he was a Dependant on the Earl of *Northumberland*, and Sir *Walter Ralegh*, and afterwards upon Sir *Thomas Aylesbury*, to whom, if I am rightly inform'd, he left many of his Writings, and, as I hinted, the Reaſon of his not publiſhing them in his Life-time, ſeems to have been his Deference for his Benefactors. Happy had it been, if the reſt of the Mathematical Works he left had been ſent abroad (as in his Preface he ſeemed to promiſe they ſhould) by the intelligent Editor of this excellent Work.

It is divided into two Parts; and the Author begins his Improvements by removing every thing that was uſeleſs, ſuperfluous, or inelegant in former Methods; thus inſtead of Capitals, he introduced ſmall Letters; inſtead of the Terms, Squares, Cubes, Surſolids, *&c.* and their Contractions, he brought in the Powers themſelves, which made the Operations much more eaſy, natural, and perſpicuous than they were before. Having thus eſtabliſhed a plain and accurate Notation, he proceeds to a Multitude of new Diſcoveries, of which, to the Number of twenty-three, the Reader may find a full, diſtinct, and very judicious Account, in the celebrated Treatiſe of Dr *Wallis.* From this admirable Piece of Mr *Harriot's*, *Des Cartes* took all the Improvements he pretended to make, as the Doctor juſtly obſerves, and of which I ſhall furniſh the Reader with ſome conciſe, and I think concluſive, Proofs. *Firſt*, It appears from all the Accounts we have of the Life of *Des Cartes*, that he was here in *England* when *Harriot*'s Book was publiſhed, which being written in *Latin*, in a Branch of Learning about which that great Man was then very ſedulous, it is eaſy to conceive that he was one of it's moſt early Peruſers; *Secondly*, It is certain that he did not publiſh any thing on this Subject before that Year; *Thirdly*, His Treatiſe of Geometry, wherein theſe new Improvements firſt appeared, was printed in *French* in 1637 without his Name, which in all Probability was to try what Opinion the World would have of them, and whether any of the *French* Mathematicians could diſcern whence they were taken; *Fourthly*, Though he ſuffered the two firſt Parts of his Book to be publiſhed in *Latin*, with his Name, in 1644; yet the third Part, relating to Geometry, did not appear till 1649, when it was publiſhed by *Francis Van Schooten*. Theſe are probable Reaſons only, but then, *Fifthly*, He follows *Harriot* diſtinctly in *Nineteen* ſeveral Diſcoveries; which that they ſhould be made in the ſame Method and Manner, (except a few Miſtakes) without conſulting Mr *Harriot*, is altogether incredible, and was ſo held to be even by his own Countrymen, when, through the Information of the Honourable Mr *Cavendiſh*, they were made acquainted with Mr *Harriot*'s Book; *Sixthly*, There are ſome little Changes, particularly in the Marks made Uſe of by *Des Cartes*, and which were never followed by any Body, that plainly intimate he only introduced them, in order to diſguiſe his Method; *Seventhly*, It appears that *Des Cartes* himſelf was acquainted with the Charge brought againſt him upon this Head, and yet he never thought fit to juſtify himſelf, nor did ever ſo much as declare that he had not ſeen the Book he was ſaid to have copied. On the whole therefore, there is all the Reaſon in the World to believe, that the Honour due to the great Improvement of this Science, which fitted it for all that it has received ſince, from

Foreigners or *Engliſhmen*, belongs to our Author *Harriot*, and not to *Des Cartes*, who only accommodated theſe Diſcoveries to Geometrical Subjects.

After him Dr *John Pell*, who was Reſident for the *Commonwealth* of *England* in *Switzerland*, publiſhed ſome new Diſcoveries. The Method he took of doing it was this, he recommended to Mr *Thomas Brancker* a Treatiſe of Algebra written in the *German* Language by *Rhonius*, which when he had tranſlated, the Doctor reviſed, alter'd and added to it. In this Piece there are a great many curious Things relating eſpecially to *Diophantine Algebra*, but delivered very obſcurely, inſomuch, that the learned Dr *Wallis* ſeems to be in doubt, whether himſelf had reached Dr *Pell*'s true Meaning. Yet, to this Gentleman, who wrote in ſo perplexed a Way, we ſtand indebted for the Invention of the Regiſter; a Method of great Uſe, eſpecially to Beginners, the Practice of which was what chiefly recommended *Kerſey*'s Algebra, and which is conſtantly and judiciouſly preſerved throughout the following Treatiſe. It is very likely, that the Darkneſs complained of in Dr *Pell*'s Writings might be owing to his Circumſtances as well as Temper, for he was a very bad Œconomiſt, not through any Vice or Extravagancy, but by a neglect of his private Affairs, and ſpending all his Time in Study.

As for the Rules of *John Van Hudde*, Mr *Merry*, *Eraſmus Bartholine*, Mr *Hugens*, and others, I do not take Notice of them, becauſe in reality they are no more than Improvements on, or Deductions from, *Harriot*. The ſame thing may be ſaid of what has been written by Meſſ. *Farmat*, *de Billy*, *Fernicle*, and other *French* Mathematicans, who only propoſed Problems for other People to reſolve, and reſerved their own Methods of Solution as impenetrable Secrets: A Practice, which however it might entitle them to the Admiration of the Age in which they lived, can give them no juſt Claim to the Praiſe of Poſterity; ſince if we reap any Benefit from their Diſcoveries, it is indirectly, and in a Manner againſt their Intentions.

Dr *Wallis* himſelf has alſo made ſome very conſiderable Improvements in this Science, eſpecially in reſpect to impoſſible Roots in ſuperior Equations; and what he left unperfected has been ſupplied by the ingenious Mr *Abraham De Moivre*, whoſe accurate Performance on that Subject has been lately publiſhed, in the Algebra of Doctor *Saunderſon*.

In 1655 Dr *Wallis* publiſhed his *Arithmetica Infinitorum*, in which he ſquared a Series of Curves, and ſhewed that if this Series could be interpolated in the middle Spaces, the Interpolation would give the Quadrature of the Circle. This Treatiſe fell into the Hands of the ingenious Sir *Iſaac* then Mr *Newton*, in the Year 1664, when that Gentleman was about Two and Twenty; and he by a Sagacity peculiar to himſelf, and which can never be enough admired, derived from this Hint his celebrated Method of Infinite or Converging Series. In 1665, he computed the Area of the Hyperbola by this Series to Fifty-two Figures, which having communicated to Dr *Barrow*, he prevented Mr *Nicholas Mercator*'s running away with the Reputation of this Diſcovery, who in 1668 publiſhed the Quadrature of the Hyperbola by an infinite Series. This was received with univerſal Applauſe, and yet Mr *Newton* far exceeded him; ſince, without ſtopping at the Hyperbola, he extended this Method by general Forms to all Sorts of Curves, even ſuch as are Mechanical, to their Quadratures, Rectifications, and Centers of Gravity, to the Solids formed by their Rotations, and to the Superficies of thoſe Solids; ſo that ſuppoſing their Determinations to be poſſible, this Series ſtopped at a certain Point, or at leaſt their Sums were given by ſtated Rules. But if the abſolute Determinations were impoſſible, they could yet be infinitely approximated as he likewiſe ſhewed, and which, as a *French* Writer juſtly obſerves, is the happieſt and moſt refined Contrivance for ſupplying the Defects of human Knowledge, that Man's Imagination could poſſibly invent. It is alſo certain, that he attained his Invention of Fluxions by that Time he was Four and twenty, but his Modeſty was ſo great, that he forbore to publiſh his Diſcovery, which was the ſole Reaſon that the Honour of it was ever diſputed with him.

In 1707, he firſt publiſhed a Syſtem of Algebra under the Title of *Univerſal Arithmetick*, and in 1722 gave another Edition of it, wherein are contained all his Improvements in that Art.

From the Rules by him laid down, ſtill farther Lights were ſtruck out by ſucceeding Mathematicians, ſuch as Dr *Edmund Halley*, who publiſhed in the *Philoſophical Tranſactions*, a Method of finding the Roots of Equations without any previous Reduction, and the Conſtruction of Equations of the 3d and 4th Power, by the help of a Circle and Parabola. Mr *J. Colſon*, who obliged the learned World with a univerſal Reſolution, Geometrical and Mechanical, of Cubic and Biquadratic Equations. Mr *Colin Mac Laurin*, in his Treatiſe of impoſſible Roots, and many others too long to be enumerated here.

But after all theſe Diſcoveries and Improvements, there has ſtill been a general Complaint, that hitherto we have had no Book of Algebra plain enough to inſtruct ſuch as are inclined to ſtudy this Science without farther Aſſiſtance, or who live in Places where it is not to he had. To obviate this Objection, the following Treatiſe was drawn up, which will be found to contain a clear and copious Syſtem of Algebra, delivered in ſo eaſy and natural a Method, and with ſuch Perſpicuity and Condeſcenſion to the Feebleneſs of the Underſtanding, when firſt applied to this kind of Study, that I felicitate myſelf on having prevailed upon it's Author to make it publick, as I am perſwaded it will be of general Uſe, in preventing young People from being diſcouraged at their firſt Entrance into Algebra, which has hitherto hindered Numbers from cultivating their Inclinations to the Mathematicks, as conceiving thoſe Difficulties in the Science, which, in Fact, are owing to the Teachers Inſufficiency, as a Coach injudiciouſly hung will jolt let the Road be ever ſo good.

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ALGEBRA.

ALGEBRA.

HAVING given the Reader an Hiſtorical Account of this Science in the Introduction, we are now to explain the Signs, and Characters uſed by Analytic Writers, and mention thoſe Axioms or Self-evident Principles of Truth and Certainty, which are the Foundations of this celebrated Art.

Signs	*Names.*	*Significations.*
$+$	*Plus or more.*	The Sign of *Addition*; as $8+4$, is 8 is to be added to 4, and $m+n$ ſignifies the Quantity repreſented by m, is to be added to the Quantity repreſented by n; again, $2+3+5$, ſignifies they are all to be added into one Sum, and $b+m+d$ ſignifies that the Numbers repreſented by b, m, and d, are to be added into one Sum.
$-$	*Minus or leſs.*	The Sign of *Subſtraction*; as $5-2$, is 5 leſs by 2, or 2 is to be ſubſtracted from 5, and $a-b$ is a leſs b, or the Number repreſented by b is to be ſubſtracted from the Number repreſented by a; and $9-2-3$, is that from 9 there is to be ſubſtracted 2, and from that remainder 3 is to be ſubſtracted.
$\times$	*Into or with*	The Sign of *Multiplication*; as 5×7, is 5 is to be multiplied by 7, and $a\times b$, is the Number repreſented by a, is to be multiplied by the Number repreſented by b; and $7\times3\times2$, is that 7, 3, and 2 are to be multiplied into each other, and the product is 42.

$\div$ } { *By.*

The Sign of *Diviſion*; as $8 \div 4$, that is 8 is to be divided by 4, and $x \div y$ that is, the Number repreſented by x, is to be divided by the Number repreſented by y; or ſometimes they are placed like Vulgar Fractions thus $\frac{8}{4}$, that is, 8 is to be divided by 4, and $\frac{x}{y}$, that is, the Number repreſented by x, is to be divided by the Number repreſented by y.

$=$ } { *Equal.*

The Sign of *Equality* or *Equation*; thus $9 = 9$, that is, 9 is equal to 9, and $2 + 3 = 5$ that is, 2 added to 3 is equal to 5; Again, $m = n + y$, that is, the Number repreſented by m is equal to the Number repreſented by n, added to the Number repreſented by y; and $y - x = a + b$, that is, the Number repreſented by y being leſſened by the Number repreſented by x, the Remainder is equal to the Number repreſented by a, added to the Number repreſented by b.

$::$ } { *So is.*

The Sign of *Proportion*, or what is commonly called the Rule of Three, and $::$ is placed between the two middle Numbers thus, $3 : 5 :: 6 : 10$, that is, as 3 is to 5, ſo is 6 to 10, and $a : b :: c : d$ that is, as the Number repreſented by a is to the Number repreſented by b, ſo is the Number repreſented by c to the Number repreſented by d.

⊙ } { *Involution.*

The Sign of *Involution*, or raiſing any Number or Quantity to the *Square*, *Cube*, or any other Power; and the heighth of the Involution is generally expreſſed by the Number after the Sign thus, 7 ⊙ 2, is 7 involved to the Square or ſecond Power; and 7 ⊙ 3, is 7 involved or raiſed to the Cube or third Power, and a ⊙ 2, is a involved to the Square or ſecond Power.

ɯ	*Evolution.*	The Sign of *Evolution*, or the extracting of Roots; and the Root that is taken is likewiſe expreſſed by the Figure that follows the Sign, thus 9 ɯ 2, is the Square Root of 9 is to be extracted, and 27 ɯ 3, is the Cube Root of 27 is to be extracted, and *a a* ɯ 2, is the Square Root of *a a* is to be extracted.
$\sqrt{}$	*Irrationality,* or *a Surd Root.*	The Sign of *Irrationality*, or of a Surd Root; that is, the Number or Quantity has not ſuch a Root as is required to be extracted, thus the Square Root of 2 would be expreſſed thus $\sqrt{2}$, and the Square Root of 5 thus $\sqrt{5}$, and the Cube Root of 4 thus $\sqrt[3]{4}$, the little Figure ſtanding over the Sign being 3 ſhews it to be the Cube Root; again, $\sqrt[3]{15}$ is the Cube Root of 15, and where there is no ſuch Figure over the Sign it ſignifies the Square Root only.

Now before we go farther, it will be neceſſary to inform the Reader, that where there is any Number joined to a Quantity it ſhews how many Times that Quantity is taken; thus, 4 *a* is *four times a*, or the Number repreſented by *a* is to be taken *four times*; and 7 *m* is *ſeven times m*, and if *y* was to be taken *ſeven times* it may be expreſſed thus 7 *y*.

Theſe Numbers are called *Co-efficients*, or *Fellow-Factors*, as they multipy the Quantity, and if any Quantity is without a Co-efficient, then it is always implied that *Unity* or 1 is the *Co-efficient* of that Quantity; thus *a* is the ſame as 1 *a*, and *y* the ſame as 1 *y*, for when the Co-efficient is only *Unity*, or 1, it is generally omitted.

Quantities that are expreſſed or repreſented by ſingle Letters, or ſeveral joined together like a word, as *a*, *b*, *a b*, *a n z*, 7 *y z*, are called ſimple or ſingle Quantities.

But when theſe are connected by the Signs + or — as $a + b$, $a m - d$, $d n + a z$, they are called compound Quantities.

And ſometimes Quantities are ſet down in the Manner of Vulgar Fractions, thus, $\frac{a}{b}$, $\frac{a+b}{n}$, $\frac{m}{x+y}$.

The Sign that connects the Quantities belongs to that which follows the Sign thus, $a+b$, where the Sign $+$ belongs to the Quantity b; again, $a-c+d$ the Sign $-$ belongs to the Quantity c, and the Sign $+$ to the Quantity d.

As to thoſe ſingle Quantities which have no Sign before them, it is always underſtood they have the Sign $+$; thus a is the ſame as $+a$, and m is the ſame as $+m$, and therefore in ſingle Quantities if they are to have the Sign $+$ it is uſually omitted, and they are uſually ſet down without any Sign, but the Sign $-$ is never omitted but always placed before the Quantity to which it belongs.

And in compound Quantities, if the firſt or leading Quantity has no Sign, then it is always underſtood to have the Sign $+$, thus, $a+b$ is the ſame as $+a+b$, and $a-b$ is the ſame as $+a-b$; therefore in Compound Quantities, if the firſt or leading Quantity is to have the Sign $+$ it is generally omitted, but in theſe Compound Quantities as well as in Simple Quantities the Sign $-$ is never omitted, but always placed before the Quantity to which it belongs.

But Letters ſet or joined together like a Word ſignifies the Product or Rectangle of theſe Letters, thus, ab is the Product of a multiplied by b, and dny is the Product of d, n, and y, multiplied together.

The Operations in *Algebra* are founded on theſe *Axioms.*

AXIOM 1.

If equal Quantities be added to equal Quantities, the Sum of theſe Quantities will be equal.

AXIOM 2.

If equal Quantities be taken or ſubſtracted from equal Quantities, the Quantities remaining will be equal.

AXIOM 3.

If equal Quantities be multiplied with equal Quantities, their Products will be equal.

AXIOM. 4.

If equal Quantities be divided by equal Quantities, their Quotients will be equal.

AXIOM 5.

If there are ſeveral Quantities that are equal to one and the ſame thing, thoſe Quantities are equal one to another.

The Reader having premiſed theſe Things, and underſtanding what the Signs are intended to expreſs, he may proceed to the Rules of the Science; and if at firſt he meets with ſome little Difficulties about the Signs and Co-efficients, I would recommend him to read the foregoing Pages again; and if that and another Eſſay or two does not remove the Difficulties of any particular Example, then to omit that and proceed to the next, in which perhaps he may ſucceed, and that may cauſe the Difficulty in the other to vaniſh.

ADDITION,

In which there are three Caſes.

(1.) *Caſe* 1. WHEN the Signs of the Quantities to be added are both affirmative, or both negative, add the Co-efficients or prefixt Numbers together, and to their Sum join the Quantities, prefixing to them the Sign they have in the *Example.*

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
To	$2a$	$5m$	$-4y$	$-2z$
Add	$3a$	$2m$	$-3y$	$-6z$
Sum	$5a$	$7m$	$-7y$	$-8z$

In *Exam.* 1. the Co-efficients are 2 and 3, which added together makes 5 to which joining a the Quantity it is $5a$, and no

Sign being prefixt to either $2a$ or $3a$, the *affirmative* Sign is underſtood as prefixt to both; hence I ſay $5a$ or $+5a$ is the Sum required.

Exam. 2. The Co-efficients are 5 and 2, which being added make 7, to which joining m, it is $7m$, the Sum required, for the Signs of $5m$ and $2m$ are both *affirmative* by what was ſaid of the laſt Example.

Exam. 3. The Co-efficients are 4 and 3, which being added make 7, to which joining y it becomes $7y$, but as $4y$ and $3y$ have both the Sign — before them, therefore prefix the Sign — to $7y$ and then $-7y$ is the Sum required.

Exam. 4. The Co-efficients are 2 and 6, which together make 8, to which joining z it becomes $8z$, and prefixing the Sign — for the Reaſon in the laſt Example, we have $-8z$ the Sum required.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
To	$15my$	$-14azx$	$4ady$	$-16ymd$
Add	$7my$	$-2azx$	$3ady$	$-12ymd$
Sum	$22my$	$-16azx$	$7ady$	$-28ymd$

Exam. 5. The Sum of the Co-efficients 15 and 7 are 22, to which joining my it is $22my$ the Sum required, for $15my$ and $7my$ have both the affirmative Sign, there being no Sign prefixt.

Exam. 6. The Sum of the Co-efficients 14 and 2 are 16, to which joining azx it is $16azx$, to which prefixing the Sign —, as both the Quantities to be added have that Sign, then is $-16azx$ the Sum required.

Exam. 7, The Sum of the Co-efficients 4 and 3 is 7, to which joining ady it is $7ady$, and both the Quantities having the *affirmative* Sign, therefore $7ady$ is the Sum required.

Exam. 8. The Sum of the Co-efficients 16 and 12 is 28, to which joining ymd it is $28ymd$, to which prefixing the Sign — as both the Quantities to be added have that Sign, then is $-28ymd$ the Sum required.

The four following Examples are added for the Learner's Exerciſe.

	Exam. 9.	*Exam.* 10.	*Exam.* 11.	*Exam.* 12.
To	$2my$	$1an$	$21dy$	$-da$
Add	$3my$	$2an$	dy	$-da$
Sum	$5my$	$3an$	$22dy$	$-2da$

Exam. 11. The Co-efficients are 21 and 1, for there being no Co-efficient prefixt to dy, *Unity* or 1 is always underſtood in ſuch Caſes to be the Co-efficient, hence the Sum is $22dy$.

Exam. 12. There being no Co-efficients prefixt to either of the Quantities, *Unity* or 1 is the Co-efficient to each, and 1 being added to 1 makes 2, to which joining da it is $2da$, to which prefixing the *negative* Sign, we have $-2da$ the Sum required.

(2.) If there are two or more Quantities connected by the Signs + or — to be added to two or more Quantities connected by the Signs + or — they are added as in the former Examples, only taking due Care that the Quantities which compoſe their Sum are connected with their proper Signs, according to the Rule as in the following Examples.

	Exam. 13.	*Exam.* 14.	*Exam.* 15.
To	$2a+7b$	$6ma+5y$	$21ma+2yd$
Add	$3a+2b$	$2ma+3y$	$3ma+3yd$
Sum	$5a+9b$	$8ma+8y$	$24ma+5yd$

Exam. 13. Is $2a+7b$ to be added to $3a+2b$. The Quantities being firſt diſpoſed as in the Example, it follows from former Examples that $2a$ being added to $3a$ makes $5a$, and $7b$ added to $2b$ makes $9b$, but as $7b$ and $2b$ have both the affirmative Sign, prefix that to the $9b$, hence $5a+9b$ is the Sum required.

Exam. 14. Is $6ma+5y$ to be added to $2ma+3y$. Now by the former Examples $6ma$ being added to $2ma$ is $8ma$, and $5y$ being added to $3y$ is $8y$, but as $5y$ and $3y$ have both the *affirmative* Sign, therefore prefix the Sign + to $8y$ ſo will $8ma+8y$ be the Sum required.

Exam. 15. Where $21ma+2yd$ is to be added to $3ma+3yd$. Now by the former Examples $21ma$ being added to $3ma$ the Sum is $24ma$, and $2yd$ being added to $3yd$ the Sum is $5yd$. But as $2yd$ and $3yd$ have both the affirmative Sign, therefore prefixing the Sign + to $5yd$ we have $24ma+5yd$ the Sum required.

	Exam. 16.	*Exam.* 17.	*Exam.* 18.
To	$-7da-15m$	$9ma-14nd$	$-2mn+15yd$
Add	$-2da-4m$	$3ma-3nd$	$-4mn+4yd$
Sum	$-9da-19m$	$12ma-17nd$	$-6mn+19yd$

Exam. 16. Is $-7da-15m$ is to be added to $-2da-4m$. Now $7da$ added to $2da$ is $9da$, but as both thoſe Quantities have the Sign $-$, I therefore prefix the *negative* Sign to $9da$, and then it is $-9da$. Again, $15m$ added to $4m$ is $19m$, and both thoſe Quantities having likewiſe the *negative* Sign, I accordingly prefix it to $19m$, whence the Sum required is $-9da-19m$.

Exam. 17. Where $9ma-14nd$ is to be added to $3ma-3nd$. Now $9ma$ added to $3ma$ is $12ma$, and both thoſe Quantities having the Sign $+$ place down $12ma$ as in the Example: Then $14nd$ added to $3nd$ is $17nd$, but both thoſe Quantities having the Sign $-$, therefore place the Sign $-$ before $17nd$, that the Sum required is $12ma-17nd$.

Exam. 18. Is $-2mn+15yd$ to be added to $-4mn+4yd$. Now $2mn$ added to $4mn$ is $6mn$, but both thoſe Quantities having the negative Sign, prefix the Sign $-$ and then it is $-6mn$. And $15yd$ added to $4yd$ is $19yd$, and both thoſe Quantities having the affirmative Sign, prefix the Sign $+$ to $19yd$, hence the Sum is $-6mn+19yd$.

	Exam. 19.	*Exam.* 20.	*Exam.* 21.
To	$9yd-7a$	$14yd+15a$	$-14y+d$
Add	$2yd-a$	$2yd+a$	$-y+d$
Sum	$11yd-8a$	$16yd+16a$	$-15y+2d$

Exam. 19. When you come to add $-7a$ to $-a$, there being no Co-efficient prefixt to a, Unity or 1 is always in ſuch Caſes the Co-efficient, and then by what has been already taught $-7a$ being added to $-a$ the Sum is $-8a$ as in the Example.

Exam. 20. Thus, when $15a$ is to be added to a the Sum is for the ſame Reaſon $16a$.

Exam. 21. And $-14y$ being added to $-y$ the Sum is $-15y$, and d being added to d, for the ſame Reaſon the Sum is $2d$ or $+2d$.

(3.) *Caſe* 2. When the Signs of the Quantities to be added are one *affirmative* and the other *negative*, ſubſtract the leſſer Co-efficient from the greater, to the Remainder join the Quantity, and prefix to it the Sign of the greateſt Co-efficient.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
To	$5a$	$16m$	$21ad$	$14mz$
Add	$-2a$	$-12m$	$-7ad$	$-5mz$
Sum	$3a$	$4m$	$14ad$	$9mz$

Exam. 1. The Co-efficient 2 being ſubſtracted from 5 leaves 3; to which joining a it is $3a$, but the Sign of 5 the greateſt Co-efficient is *affirmative*, therefore $3a$ or $+3a$ is the Sum required.

Exam. 2. The Co-efficient 12 being ſubſtracted from 16 leaves 4, to which joining m it is $4m$, but the Sign of 16 the greateſt Co-efficient is *affirmative*, therefore $4m$ or $+4m$ is the Sum required.

Exam. 3. The Co-efficient 7 being ſubſtracted from 21 leaves 14, to which joining ad it is $14ad$, but as the Sign of 21 the greateſt Co-efficient is *affirmative*, hence $14ad$ is the Sum required.

Exam. 4. The Co-efficient 5 being ſubſtracted from 14 leaves 9, to which joining mz it is $9mz$, but as the Sign of 14 the greateſt Co-efficient is *affirmative*, hence $9mz$ or $+9mz$ is the Sum required.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
To	$-14m$	$-9y$	$5z$	$9am$
Add	$7m$	$2y$	$-9z$	$-14am$
Sum	$-7m$	$-7y$	$-4z$	$-5am$

Exam. 5. The Co-efficient 7 being ſubſtracted from 14 leaves 7, to which joining m it is $7m$, but the Sign of 14 the greateſt Co-efficient being $-$, prefix that Sign to $7m$ then is $-7m$ the Sum required.

Exam. 6. The Co-efficient 2 being ſubſtracted from 9, there remains 7, to which joining y it is $7y$, but as the Sign of 9 the greateſt Co-efficient is $-$, prefix that Sign to $7y$, and we ſhall have $-7y$ for the Sum required.

Exam. 7. The Co-efficient 5 being ſubſtracted from 9 leaves 4, to which joining z it is $4z$, but the Sign of 9 the greateſt Co-efficient being negative, prefix the Sign $-$ to $4z$, and we have $-4z$ for the Sum required.

Exam. 8. The Co-efficient 9 being ſubſtracted from 14 leaves 5, to which joining am it is $5am$, but the Sign of 14 the greateſt Co-efficient being negative, prefix the Sign — to $5am$, and we have $-5am$ for the Sum required.

	Exam. 9.	*Exam.* 10.	*Exam.* 11.	*Exam.* 12.
To	$7am$	$-8ad$	$-14ym$	$-ay$
Add	$-am$	$9ad$	$16ym$	$7ay$
Sum	$6am$	ad	$2ym$	$6ay$

Exam. 9. The Co-efficient of $-am$ being Unity, or 1, which ſubſtracted from 7 leaves 6, to which joining am it is $6am$, prefixing to it the Sign of 7, the greateſt Co-efficient, we have $6am$ or $+6am$ the Sum required.

Exam. 10. The Co-efficient 8 being ſubſtracted from 9 leaves 1, to which joining ad we have $1ad$ or ad, which having already the Sign of 9, the greateſt Co-efficient, hence ad is the Sum required.

Exam. 12. The Co-efficient of $-ay$ being *Unity*, or 1, which ſubſtracted from 7 leaves 6, to which joining ay it is $6ay$, which having the ſame Sign with 7, the greateſt Co-efficient, $6ay$ is the Sum required.

4. And if there are ſeveral Quantities connected by the different Signs of + and —, to be added to ſeveral Quantities connected by the different Signs of + and —, they are added as in the ſecond Article, only taking due Care to prefix the Signs, according to the Directions in the third Article.

	Exam. 13.	*Exam.* 14.	*Exam.* 15.
To	$14a+7m$	$-15my-14az$	$17ay+8am$
Add	$-8a-3m$	$7my+12az$	$-3ay-5am$
Sum	$6a+4m$	$-8my-2az$	$14ay+3am$

Exam. 13. Is $14a+7m$ to be added to $-8a-3m$. Now by the Rule at Art. 3. the difference between the Co-efficients 14 and 8 is 6, to which joining a it is $6a$, but 14 the greateſt Co-efficient having the *affirmative* Sign, hence $6a$ is the Sum of $14a$ added to $-8a$. And the difference between 7 and 3 the Co-efficients of m being 4, to which joining m it is $4m$, but as 7 the greateſt Co-efficient has the *affirmative* Sign, therefore prefix the Sign + to $4m$, ſo is $6a+4m$ the Sum required.

Exam. 14. Where $-15my-14az$ is to be added to $7my+12az$. Now the Difference between 15 and 7 the two Co-efficients of my is 8, to which joining my it is $8my$, but as 15 the greateſt Co-efficient hath the *negative* Sign, therefore prefix the Sign $-$ to $8my$, and it is $-8my$: And the Difference between 14 and 12, the two Co-efficients of az being 2, to which joining az it is $2az$, but as 14 the greateſt Co-efficient has the *negative* Sign, therefore prefix the Sign $-$ to $2az$ ſo is $-8my-2az$, the Sum required.

Exam. 15. The Difference between 17 and 3 the two Co-efficients of ay is 14, to which joining ay it is $14ay$, but as 17 the greateſt Co-efficient has the *affirmative* Sign, therefore place down $14ay$ or $+14ay$. And the Difference between 8 and 5 the two Co-efficients of am is 3, to which joining am it is $3am$, but as 8 the greateſt Co-efficient has the affirmative Sign, therefore prefix the Sign $+$ to $3am$, ſo is $14ay+3am$ the Sum required.

	Exam. 16.	*Exam.* 17.	*Exam.* 18.
To	$-7a+16m$	$-15y+7p$	$7am-16y$
Add	$3a-4m$	$7y-11p$	$-11am+18y$
Sum	$-4a+12m$	$-8y-4p$	$-4am+2y$

Exam. 16. By Art. 3. the Difference between 7 and 3 the two Co-efficients of a is 4, to which joining a it is $4a$, but as 7 the greateſt Co-efficient has the *negative* Sign, therefore prefix the Sign $-$ to $4a$, and it is $-4a$. And the Difference between 16 and 4 the two Co-efficients of m is 12, to which joining m it is $12m$, but 16 the greateſt Co-efficient having the *affirmative* Sign, prefix the Sign $+$ to $12m$, ſo is $-4a+12m$, the Sum required.

Exam. 17. By Art. 3. the Difference between 15 and 7 is 8, to which joining y it is $8y$, but 15 the greateſt Co-efficient having the *negative* Sign, prefix the Sign $-$ to $8y$, and it is $-8y$. And the Difference between 7 and 11, the two Co-efficients of p is 4, to which joining p it is $4p$, but as 11 the greateſt Co-efficient has the *negative* Sign, therefore prefix the Sign $-$ to $4p$, and it is $-4p$, ſo is $-8y-4p$, the Sum required.

Exam. 18. By Art. 3. the Difference between 7 and 11 is 4, to which joining am it is $4am$, but as 11 the greateſt Co-efficient has the negative Sign, therefore prefix the Sign $-$ to $4am$, and it is $-4am$. And the Difference between 16 and

and 18 is 2, to which joining y it is $2y$, but as 18 the greateſt Co-efficient has the *affirmative* Sign, therefore prefix the Sign $+$ to $2y$, and it is $+2y$, ſo is $-4am+2y$, the Sum required.

	Exam. 19.	*Exam.* 20.	*Exam.* 21.
To	$14my - ma$	$-5yd + 15z$	$-14dy + 5mp$
Add	$-3my + 4ma$	$yd - 3z$	$dy - mp$
Sum	$11my + 3ma$	$-4yd + 12z$	$-13dy + 4mp$

Exam. 19. The Co-efficient of $-ma$ is 1, which being by Art. 3. ſubſtracted from 4 leaves 3, to which joining ma it is $3ma$, as in the Anſwer, and by the ſame Method in

Exam. 20. If $-5yd$ is added to yd or $1yd$, the Sum is $-4yd$, and likewiſe in

Exam. 21. If $-14dy$ is added to dy or $1dy$, the Sum is $-13dy$.

5. If the Co-efficients are equal, and the Signs are one *affirmative*, and the other *negative*, they being added together deſtroy each other, or the Sum of them is a *Cypher* or *nothing*.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
To	$7a$	$-5y$	$14m$	$5ya$
Add	$-7a$	$5y$	$-14m$	$-5ya$
Sum	0	0	0	0

Exam. 1. By Art. 3. the Signs being unlike the Co-efficients are to be ſubſtracted, but 7 taken from 7 leaves 0, and if to this we join a it is $0a$, or *no times a*, that is, the Quantity a is to be taken no times or not at all, which is the ſame as *nothing*: So in the fourth Example, if 5 is ſubſtracted from 5, there remains 0, or *nothing*, to which if we join ya, we then have *no times ya*, or *nothing*.

6. *Caſe* 3. When the Quantities are unlike, that is, the Letters are different, then ſet them down one after the other, with the ſame Co-efficients and Signs they have in the Example, and this is the Sum required.

And they may be ſet in any Order, that is, any Quantity may be ſet firſt, in the middle or laſt, it being not material how they are ranged, ſo as they are but connected with their proper Signs.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
To	$2a$	$3m$	$a+d$
Add	$3d$	$5a$	$2y$
Sum	$2a+3d$	$3m+5a$	$a+d+2y$

Exam. 1. The Quantities or Letters being unlike by Art. 6. I place down $2a$, and becauſe $3d$ has the Sign $+$, therefore after the $2a$ put $+3d$ ſo is $2a+3d$, the Sum required.

Exam. 2. Having put down the $3m$, after that put $+5a$ the other Quantity with its Sign, ſo is $3m+5a$, the Sum required.

Exam. 3. Having put down a, after that put $+d$, and after that $+2y$, ſo is $a+d+2y$, the Sum required.

	Exam. 4.	*Exam.* 5.
To	$2a-7m$	$2a+15$
Add	$3y+5z$	$z-7d$
Sum	$2a-7m+3y+5z$	$2a+15+z-7d$

Exam. 4. Begin and place down $2a$, after that $-7m$, after that $+3y$, and after that $+5z$, ſo is $2a-7m+3y+5z$ the Sum required.

Exam. 5. Begin and place down $2a$, after that $+15$, after that $+z$, and after that $-7d$, ſo is $2a+15+z-7d$, the Sum required.

To	$7m+15y$	$-15m+7a$
Add	$-4a+mn$	$8y-2b$
Sum	$7m+15y-4a+mn$	$-15m+7a+8y-2b$
To	$16+7m$	$-14m-15y$
Add	$-2a-8d$	$a-7$
Sum	$16+7m-2a-8d$	$-14m-15y+a-7$

Examples wherein all the foregoing Caſes are promiſcuouſly uſed.

	Exam. 1.	*Exam.* 2.
To	$7a-15d+m$	$-8a+7m-21x$
Add	$5a+18d$	$11a-12m+5y$
Sum	$12a+3d+m$	$3a-5m-21x+5y$

Exam. 1. $7a$ added to $5a$ makes $12a$, by Art. 1. and $-15d$ added to $18d$ makes $3d$, by Art. 3. and there being no Quantity like m, that muſt be placed by itſelf, by Art. 6. and connecting theſe Quantities with their proper Signs we have $12a+3d+m$, for the Sum required.

Exam. 2. $-8a$ added to $11a$ makes $3a$, by Art. 3. and $7m$ added to $-12m$ is $-5m$, by the ſame, but $21x$ and $5y$ being different, place them down one after another as at Art. 6. ſo is $3a-5m-21x+5y$, the Sum required.

Exam. 3.

$$\begin{array}{ll} \text{To} & -15a+14m-16 \\ \text{Add} & 7a-14m+y \\ \hline \text{Sum} & -8a-16+y \end{array}$$

Exam. 4.

$$\begin{array}{l} 11am-7yd+mn \\ -5am-2yd-7a \\ \hline 6am-9yd+mn-7a \end{array}$$

Exam. 3. $-15a$ added to $7a$ is $-8a$, by Art. 3. $14m$ added to $-14m$, is *nothing* or 0, by Art. 5. therefore take no Notice of thoſe Quantities in the Sum, and 16 and y being different Quantities ſet them down by Art. 6. ſo is $-8a-16+y$, the Sum required.

Exam. 4. $11am$ added to $-5am$ is $6am$, by Art. 3. and $-7yd$ added to $-2yd$ is $-9yd$, by Art. 1. But mn and $-7a$ being different Quantities ſet them down by Art. 6. and ſo ſhall $6am-9yd+mn-7a$, be the Sum required.

Exam. 5.

$$\begin{array}{ll} \text{To} & 4a-17y+15ap \\ \text{Add} & -2ap+3a-2y \\ \hline \text{Sum} & 13ap+7a-19y \end{array}$$

Exam. 6.

$$\begin{array}{l} -7m+15+4a \\ -4a-11+8m \\ \hline m+4 \end{array}$$

Exam. 5. $15ap$ added to $-2ap$ is $13ap$, by Art. 3. $4a$ added to $3a$ is $7a$, by Art. 1. and $-17y$ added to $-2y$ is $-19y$, by Art. 1. hence, $13ap+7a-19y$, is the Sum required.

Exam. 6. $-7m$ added to $8m$, the Sum is m, by Art. 3. 15 added to -11, the Sum is 4, by Art. 3. and $4a$ added to $-4a$, the Sum is 0, or *nothing*, by Art. 5. hence $m+4$, is the Sum required.

The more perfectly Addition is underſtood, the eaſier it will render the Work of Subſtraction.

SUBSTRACTION

7. IS performed by one general Rule; change all the Signs of thoſe Quantities which are to be ſubſtracted, or ſuppoſe them in the Mind to be changed, then add theſe Quantities to the others, according to the ſeveral Rules of Addition, which will be the Difference or Remainder required.

I would adviſe the Learner to take out the Examples, and put down thoſe Quantities which are to be ſubſtracted with contrary Signs, to thoſe they have in the Examples; that is, making thoſe *affirmative* which were *negative*, and thoſe *negative* which were *affirmative*, and then proceed as directed in the general Rule.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
From	$5a$	$7m$	$-5y$	$-8z$
Subſtract	$3a$	$2m$	$-2y$	$-4z$
Remains	$2a$	$5m$	$-3y$	$-4z$

Exam. 1. Here $3a$ the Quantity to be ſubſtracted has the Sign $+$, which being made or ſuppoſed to be made $-$, then by our general Rule $5a$ is to be added to $-3a$, the Sum of which is $2a$, by Art. 3. and this is the Remainder required.

Exam. 2. In the ſame Manner $2m$ having, or being ſuppoſed to have the Sign $-$ prefixed to it, then by the general Rule $7m$ is to be added to $-2m$, the Sum of which is $5m$, by Art. 3. and this is the Remainder required.

Exam. 3. And if we ſuppoſe $-2y$ to be $2y$, or $+2y$, then by the general Rule $-5y$ added to $+2y$, the Sum is $-3y$, by Art. 3. and this is the Remainder required.

Exam. 4. And if we ſuppoſe $-4z$ to be $4z$, or $+4z$, then by the general Rule if $-8z$ is added to $4z$, the Sum is $-4z$, by Art. 3. and this is the Remainder required.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
From	$14mn$	$-7yd$	$-5yx$	$4ay$
Subſtract	$-2mn$	$+5yd$	$+3yx$	$-3ay$
Remains	$16mn$	$-12yd$	$-8yx$	$7ay$

Exam. 5. The Sign of $2mn$ being $-$ if we ſuppoſe it $+$, then by the general Rule $14mn$ added to $2mn$, the Sum is $16mn$, by Art. 1. and this is the Remainder required.

Exam. 6. If we ſuppoſe $5yd$ to be $-5yd$, then by the general Rule $-7yd$ added to $-5yd$, the Sum is $-12yd$, by Art. 1. and this is the Remainder required.

Exam. 7. And ſuppoſing the $3yx$ to be $-3yx$, then by the general Rule $-5yx$ being added to $-3yx$, the Sum is $-8yx$, by Art. 1. the Remainder required.

Exam. 8. And if we ſuppoſe the $-3ay$ to be $3ay$, then by the general Rule $4ay$ being added to $3ay$, the Sum is $7ay$, by Art. 1. the Remainder required.

From	$5am$	$-ay$	$-7ad$	$5yd$
Subſtract	$-am$	$-5ay$	$+ad$	yd
Remains	$6am$	$4ay$	$-8ad$	$4yd$

The Truth of Subſtraction may be proved as in common Arithmetic, by adding the Remainder to the Quantity which is ſubſtracted, and if their Sum is the ſame as that from which the Quantity was ſubſtracted, the Work is true, otherwiſe it is erroneous.

Thus in the four laſt Examples $-am$ added to $6am$, the Sum is $5am$.

And $-5ay$ added to $4ay$, the Sum is $-ay$.

And $-8ad$ added to ad, the Sum is $-7ad$.

And $4yd$ added to yd, the Sum is $5yd$. And in the ſame Manner may any of the other Examples be proved.

8. And if there are two or more Quantities connected by the Signs $+$ or $-$, to be ſubſtracted from other Quantities connected by the Signs $+$ or $-$, it is done in the ſame Manner, only taking due Care to connect the remaining Quantities with their proper Signs, as was done in the Addition of compound Quantities.

	Exam. 9.	*Exam.* 10.	*Exam.* 11.
From	$12a+7b$	$7ma+5y$	$-5zy-2am$
Take	$3a+2b$	$6ma+4y$	$3zy+4am$
Remains	$9a+5b$	$ma+y$	$-8zy-6am$

Exam. 9. By ſuppoſing $3a$ to be $-3a$, then $12a$ added to $-3a$ the Sum is $9a$, by Art. 3. and again, ſuppoſing $2b$ to be $-2b$, then $7b$ added to $-2b$ the Sum is $5b$ by the ſame, and connecting theſe Quantities we have $9a+5b$, the Remainder required.

Exam. 10. $6ma$ being ſuppoſed to be negative, or to be $-6ma$, then $7ma$ added to $-6ma$ the Sum is ma, and $4y$ being ſuppoſed to be $-4y$, then $5y$ added to $-4y$ the Sum is y, hence $ma+y$, is the Remainder required.

Exam. 11. $3zy$ being ſuppoſed to be $-3zy$, and adding this to $-5zy$ the Sum is $-8zy$, by Art. 1. and $4am$ being ſuppoſed to be $-4am$, by adding that to $-2am$ the Sum by Art. 1. is $-6am$, hence $-8zy-6am$, is the Remainder required.

	Exam. 12.	*Exam.* 13.	*Exam.* 14.
From	$14a-5y$	$-7mn+2yd$	$2a+m$
Take	$-3a-5y$	$-3mn+3yd$	$5a-7$
Remains	$17a$	$-4mn-yd$	$-3a+m+7$

Exam. 12. The $-3a$ being ſuppoſed by the general Rule to be $3a$, and adding that to $14a$ the Sum is $17a$, by Art. 1. and the $-5y$ being ſuppoſed to be $5y$, if we add $5y$ to $-5y$, the Sum is a *Cypher*, or *nothing*, by Art. 5. hence $17a$ is the difference required.

Exam. 13. The $-3mn$ being ſuppoſed to be $3mn$, then by adding $3mn$ to $-7mn$ the Sum is $-4mn$, by Art. 3. and $3yd$ being ſuppoſed to be $-3yd$, and adding $-3yd$ to $2yd$ the Sum is $-yd$, by Art. 3. hence $-4mn-yd$, is the Remainder required.

Exam. 14. The $5a$ being ſuppoſed to be $-5a$, if that is added to $2a$ the Sum is $-3a$, by Art. 3. but the m, and 7 being different Quantities, ſet them down by Art. 6. only take particular Care to change the Sign of 7, according to the general Rule for Subſtraction, then will $-3a+m+7$, be the Remainder required.

	Exam. 15.	*Exam.* 16.	*Exam.* 17.
From	$-am+y$	$15yd+20$	$14d+7-a$
Subſtract	$+am+y$	$-3yd-16$	$-d+7-8a$
Remains	$-2am$	$18yd+36$	$15d+7a$

The Truth of theſe compound Operations, is proved in the ſame Manner as the ſimple Operations, by adding the Remainder to the Quantity which is ſubſtracted, and obſerving if that Sum is the ſame with thoſe Quantities from which the Subſtraction was made. Thus,

Exam. 15. $-2am$ added to am, the Sum is $-am$, by Art. 3. to which, connecting y with the Sign $+$, we find that by adding $am+y$ to $-2am$, the Sum is $-am+y$, the Quantity from which the Subſtraction was made.

Exam. 16. Now $18yd$ added to $-3yd$, the Sum is $15yd$, by Art. 3. and 36 added to -16 the Sum is 20 by the ſame, hence the Sum of $-3yd-16$ added to $18yd+36$, is $15yd+20$, the Quantity from which the Subſtraction was made.

Exam. 17. By adding $15d$ to $-d$ the Sum is $14d$, by Art. 3. and by adding $7a$ to $-8a$, the Sum is $-a$ by the ſame, to which putting down the $+7$, there being no Quantity to be added to that, hence $-d+7-8a$ being added to $15d+7a$, the Sum is $14d+7-a$, the Quantity from which the Subſtraction was made.

MULTIPLICATION.

In Multiplication there are three Caſes.

(9.) *Caſe* 1. WHEN the Quantities to be multiplied have like Signs, that is, they are both *affirmative*, or both *negative*, then ſet or join the Letters together, and to them prefix the Sign $+$, which will be the Product required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Multiply	a	y	$-z$	$-da$
By	d	m	$-a$	$-x$
Product	da	my	az	dax

Exam. 1. Having joined the Letters da and each of them having the *affirmative* Sign, therefore by the Rule da or $+da$, is the Product required.

Exam. 2. Having joined the Letters m and y and each of them having the affirmative Sign, therefore by the Rule my or $+my$, is the Product required.

Exam. 3. Having joined the Quantities z and a and each of them having the ſame Sign, therefore by the Rule az, or $+az$, is the Product required.

Exam. 4. Having joined the Quantities da and x and both of them having the ſame Sign, therefore dax, or $+dax$, is the Product required.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
Multiply	a	$-am$	$-y$	am
By	a	$-d$	$-dp$	an
Product	aa	amd	dpy	$aman$

Exam. 5. Having joined aa, and both the Quantities being affirmative or having the ſame Sign, therefore aa is the Product required.

Exam. 6. Having joined the Letters am and d, and each of them having the Sign —, hence amd is the Product required.

Exam. 7. Having joined the Letters y and dp, and becauſe each of them have the ſame Sign, therefore dpy, is the Product required.

Exam. 8. Having joined the Letters am and an, and becauſe each of them have the ſame Sign, therefore $aman$ is the Product required.

10. But if the Multiplicand conſiſts of two or more Quantities connected by the Signs + or —, then the Multiplier muſt be multiplied into each of thoſe Quantities, prefixing to each particular Multiplication its proper Sign, which will give the Product. Thus,

	Exam. 9.	*Exam.* 10.	*Exam.* 11.
Multiply	$a+d$	$z+y$	$-mx-n$
By	m	d	$-p$
Product	$ma+md$	$dz+dy$	$pmx+pn$

Exam. 9. If we multiply a by m, the Product is $m a$, by what has been already taught, Art. 9. and multiplying d by m, the Product is $m d$, but as m and d have both the *affirmative* Signs, therefore prefix the Sign $+$ before $m d$, and hence $m a + m d$, is the Product required.

Exam. 10. Multiplying z by d, the Product is $d z$, and multiplying y by d, the Product is $d y$, but as d and y have both the Sign $+$ before them, therefore prefixing the Sign $+$ before $d y$ we have $d z + d y$, the Product required.

Exam. 11. Multiplying $-m x$ by $-p$, the Product is $p m x$, for $m x$ and p have their Signs alike, being both negative, hence by Art. 9. the Product $p m x$, muſt have the affirmative Sign before it, and for the ſame Reaſon $-n$ multiplied by $-p$, the Product is $p n$, then placing the affirmative Sign before $p n$ we have $p m x + p n$, for the Product required.

	Exam. 12.	*Exam.* 13.	*Exam.* 14.
Multiply	$-m-y$	$-a-zy$	$ad+z$
By	$-d$	$-x$	y
Product	$dm+dy$	$ax+xzy$	$ady+yz$

Exam. 12. Multiplying $-m$ by $-d$, the Product is $m d$ by what was ſaid at Example 11, and multiplying $-d$ by $-y$, the Product is for the ſame Reaſon $d y$, and prefixing the Sign $+$ before $d y$, we have $d m + d y$, for the Product required.

Exam. 13. Multiplying $-a$ by $-x$, we have $a x$ for the Product, as in the laſt Example, and from multiplying $-zy$ by $-x$, we have for the ſame Reaſon $x z y$ for this Product, and to this Quantity $x z y$ prefixing the Sign $+$, by Art. 9. we have $a x + x z y$ for the Product required.

Exam. 14. Multiplying $a d$ by y, the Product is $a d y$, and multiplying z by y, the Product is $y z$; but as the Signs of y and z are both alike, therefore prefixing the Sign $+$ to $y z$, we have $a d y + y z$, the Product required.

11. It may not be improper to caution the Learner, that in Multiplication it is quite indifferent which Letter he places firſt, or laſt, for if I was to multiply $a m$ by d, the Product is $a m d$, or $m d a$, or $d m a$, or $a d m$, or any of the different poſitions in which the three Letters can be placed; this may be more compleatly and fully underſtood when we come to apply the Science to the Solution of Problems: But, that the Learner may form ſome Idea of the Truth of this, ſuppoſe we were to multi-

ply 3, 5, and 7 together, the Product will be the ſame in whatever Order theſe three Numbers are multiplied. Thus,

$$\begin{array}{r} 3 \\ 5 \\ \hline 15 \\ 7 \\ \hline 105 \end{array} \qquad \begin{array}{r} 7 \\ 5 \\ \hline 35 \\ 3 \\ \hline 105 \end{array} \qquad \begin{array}{r} 3 \\ 7 \\ \hline 21 \\ 5 \\ \hline 105 \end{array}$$

This Obſervation I adviſe the Learner to fix in his Mind, to prevent concluding he has done any of the following Examples erroneouſly, by happening to place the Letters different from what they are in the Book.

12. But if the Multiplier and Multiplicand conſiſts of two or more Quantities, then begin and multiply the Multiplicand by any one Quantity in the Multiplier, according to the Directions in Art. 9. after that multiply the Multiplicand by another Quantity in the Multiplier, and put this Product under the other, and continue doing this till the Multiplicand has been multiplied by every Quantity in the Multiplier; then under theſe Products draw a Line, and add them together by the ſeveral Caſes of Addition, and this will be the Product required.

Exam. 1.

Multiply $a+b$
By $m+n$

$ma+mb$ the Product of $a+b$ multiplied by m, by Art. 10.
$na+nb$ the Product of $a+b$ multiplied by n, by the ſame.

$ma+mb+na+nb$ the Sum by Art. 6. which is the Product required by Art. 12.

Multiply $m+y$
By $a+d$

$am+ay$ the Product of $m+y$ multiplied by a, by Art. 10.
$md+yd$ the Product of $m+y$ multiplied by d, by the ſame.

$am+ay+md+yd$ the Sum by Art. 6. and this is the Product required by Art. 12.

Multiply $-a-d$
By $-m-z$

$ma+md$ the Product of $-a-d$ multiplied by $-m$, by Art. 10.
$az+dz$ the Product of $-a-d$ multiplied by $-z$, by the ſame.

$ma+md+az+dz$ the Sum by Art. 6. and this is the Product required, by Art. 12.

Multiply $a+b$
By $a+b$

$aa+ab$ the Product of $a+b$ multiplied by a, by Art. 10.
$ab+bb$ the Product of $a+b$ multiplied by b, by the ſame.

$aa+2ab+bb$ the Product of $a+b$ multiplied by $a+b$.

Now in the Addition of the above Products I obſerve that there is ab in each of the two Lines, and there being no Co-efficient prefixt, *Unity* or 1 being then always underſtood to be the Co-efficient, hence $1ab$ added to $1ab$ is $2ab$; the other Quantities aa and bb in the two Products are ſet down as in the former Examples, therefore $aa+2ab+bb$, is the Product required.

And in ſuch Additions as theſe I would recommend it to the Learner, before he begins to add, to examine the ſeveral Quantities and ſee if the Letters in any two of them are alike, and if they are to collect them into one Sum, according to Art. 1 and 3, remembering that though the Letters which compoſe the two Quantities are not in the ſame Order in each; yet if they are but the ſame Letters, and no more in one, than there is in the other Quantity they are the ſame, and may be added by Art. 1 and 3.

The four following Examples are for the Exerciſe of the Scholar.

Multiply	$a+b$	$a+y$
By	$m+y$	$a+y$
	$am+mb$	$aa+ay$
	$ay+by$	$ay+yy$
Product	$am+mb+ay+by$	$aa+2ay+yy$

Multiply

Multiply	$y+m$	$-a-d$
By	$y+m$	$-a-d$
	$yy+ym$	$aa+ad$
	$ym+mm$	$ad+dd$
Product	$yy+2ym+mm$	$aa+2ad+dd$

13. *Caſe* 2. If there are Co-efficients or prefixt Numbers, then multiply the Numbers as in common Arthmetic, and to their Products join the Products of the Quantities found by the laſt *Caſe.*

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Multiply	$2a$	$5m$	$-7ad$	$-2y$
By	$3m$	$3y$	$-3m$	$-a$
Product	$6am$	$15my$	$21adm$	$2ay$

Exam. 1, The Product of the Co-efficients 2 and 3 is 6: the Product of a multiplied by m is am, joining theſe together it is $6am$ the Product required, for the Signs of $2a$ and $3m$ are alike.

Exam. 2. The Product of 5 by 3 is 15: the Product of m by y is my, joining theſe together it is $15my$ the Product required, for $5m$ and $3y$ have the ſame Signs.

Exam. 3. The Product of 7 by 3 is 21: the Product of ad by m is adm, and joining the 21 and adm it is $21adm$ the Product required, for the Signs of $7ad$ and $3m$ are both alike being negative.

Exam. 4. The Product of 2, and 1 the Co-efficient of a is 2, to which joining ay, the Product of a and y it is $2ay$ the Product required, for the Signs of $2y$ and a are alike being both negative.

Multiply	$7am$	$6dz$	$-3yp$	$-2dz$
By	$2d$	$2a$	$-3y$	d
Product	$14amd$	$12dza$	$9yyp$	$2ddz$

14. And if there are two or more Quantities with Co-efficients connected by the Signs + or —, to be multiplied by any Quantity and its Co-efficient, they are multiplied as in the laſt Article, only connecting the ſeveral particular Products together with their proper Signs as was done at Art. 10.

Exam.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
Multiply	$2a+3b$	$3y+5d$	$-2y-2a$
By	$3m$	$5m$	$-3z$
Product	$6am+9bm$	$15ym+25dm$	$6yz+6za$

Exam. 1. Multiplying $2a$ by $3m$ the Product is $6am$, by what has been already taught, Art. 13. and then multiplying $3m$ by $3b$ the Product is $9bm$, to which prefixing the *affirmative* Sign, as the Signs of $3b$ and $3m$ are alike, and we have $6am+9bm$ for the Product required.

Exam. 2. Multiplying $3y$ by $5m$ the Product is $15ym$, by the laſt Article; then multiplying $5m$ by $5d$ the Product is $25dm$, to which prefixing the affirmative Sign, as the Signs of $5d$ and $5m$ are both alike, and we have $15ym+25dm$ for the Product required.

Exam. 3. Multiplying $2y$ by $3z$ the Product is $6yz$, by the laſt Article, and becauſe the Signs of $2y$ and $3z$ are alike, being both negative, hence by Art. 9. the Product of $-2y$ and $-3z$ is $6yz$ or $+6yz$. Again, Multiplying $2a$ by $3z$ the Product is $6az$, to which prefixing the affirmative Sign, as the Signs of $2a$ and $3z$ are both alike, we have $6yz+6za$ for the Product required.

	Exam. 4.	*Exam.* 5.	*Exam.* 6.
Multiply	$3m+2y$	$-2d-3m$	$-3y-7my$
By	$6a$	$-4a$	$-2a$
Product	$18ma+12ya$	$8da+12ma$	$6ya+14amy$

Exam. 4. Multiplying $3m$ by $6a$ the Product is $18ma$, and multiplying $2y$ by $6a$ the Product is $12ya$, and placing the Sign $+$ before the $12ya$, becauſe the Signs of $6a$ and $2y$ are alike, we have $18ma+12ya$ for the Product required.

Exam. 5. Multiplying $2d$ by $4a$ the Product is $8da$, and becauſe the Signs of $2d$ and $4a$ are alike, being both negative, therefore $8da$ or $+8da$ is the Product of theſe Quantities. Now multiplying $4a$ by $3m$ the Product is $12ma$, to which prefixing the affirmative Sign, as $3m$ and $4a$ have the ſame Sign, both being negative, and we have $8da+12ma$ for the Product required.

Exam. 6. The Product of $-3y$ by $-2a$ is $6ya$, or $+6ya$, for the Reaſon in the laſt Example; and for the ſame Reaſon the Product of $-7my$ by $-2a$ is $14amy$, or $+14amy$, hence we have $6ya+14amy$ for the Product required.

Multiply	$-3m-2d$	$-2z-3y$	$-4d-5m$
By	$-4a$	$-4a$	$-2b$
Product	$12ma+8da$	$8za+12ay$	$8db+10mb$

15. And if there are two or more Quantities with Co-efficients connected by the Signs + or —, to be multiplied by two or more Quantities with Co-efficients connected in the ſame Manner, the Quantities are to be multiplied as at Art. 12. taking due Care to multiply the Co-efficients as has been taught, Art. 14. Thus,

Multiply $2a+3b$
By $3a+5m$

$6aa+9ab$ the Product of $2a+3b$ multiplied by $3a$, by Art. 14.
$10ma+15bm$ the Product of $2a+3b$ multiplied by $5m$, by the ſame.

$6aa+9ab+10ma+15bm$ the Product required, being the Sum of the two particular Products which are added together by Art. 6.

Multiply $3m+5y$
By $2a+3n$

$6am+10ay$ the Product of $3m+5y$ multiplied by $2a$, by Art. 14.
$9mn+15yn$ the Product of $3m+5y$ multiplied by $3n$, by Art. 14.

$6am+10ay+9mn+15yn$ the Product required, being the Sum of the two Products which are added together by Art. 6.

Multiply $2a+3b$
By $2a+2b$

$4aa+6ab$ the Product of $2a+3b$ multiplied by $2a$, by Art. 14.
$4ab+6bb$ the Product of $2a+3b$ multiplied by $2b$, by the ſame.

$4aa+10ab+6bb$ the Product required. In this

Addition the Reader is to obſerve that in one Line there is $6ab$, and in the other Line there is $4ab$, which two Quantities being added together the Sum is $10\,ab$, by Art. 1. but the $4aa$ and $6bb$ being different Quantities, they are ſet down by Art. 6. hence the Product of $2a+3b$ multiplied by $2a+2b$, is $4aa+10ab+6bb$.

$$\begin{array}{ll} \text{Multiply} & 3a+7b \\ \text{By} & 2a+5n \\ \hline & 6aa+14ba \\ & 15an+35bn \\ \hline \text{Product} & 6aa+14ba+15an+35bn \end{array} \qquad \begin{array}{l} 3a+2b \\ a+4b \\ \hline 3aa+2ba \\ 12ba+8bb \\ \hline 3aa+14ba+8bb \end{array}$$

16. *Caſe* 3. When the Signs of the two Quantities that are to be multiplied are one affirmative and the other negative, then multiply the Quantities as before directed, but to their Product prefix the negative Sign, or $-$.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Multiply	$-b$	$-a$	$-am$	$-dm$
By	a	d	y	z
Product	$-ba$	$-ad$	$-amy$	$-dmz$

Exam. 1. The Product of b by a is ba, for Multiplication is no more then joining the Letters, but as the Sign of b is $-$, and that of a is $+$, therefore to ba prefix the Sign $-$, ſo is $-ba$ the Product required.

Exam. 2. The Product of a by d is ad, but as the Signs of a and d are different, therefore prefix the Sign $-$ to ad, and then will $-ad$ be the Product required.

Exam. 3. The Product of am by y is amy, but as the Signs of am and y are different, therefore prefix the Sign $-$ to amy, ſo is $-amy$ the Product required.

Exam. 4. The Product of dm by z is dmz, but as the Signs of dm and z are different, therefore prefix the Sign $-$ to dmz, ſo is $-dmz$ the Product required.

This Operation being the ſame as at Art. 9. only taking Care to make the Sign $-$. I ſhall only ſubjoin the following Examples for the Exerciſe of the Learner.

$$\begin{array}{lcccc}
\text{Multiply} & xm & -ym & -ax & ma \\
 & -y & d & dy & -p \\
\hline
\text{Product} & -xmy & -ymd & -axdy & -map
\end{array}$$

17. And if two or more Quantities with Co-efficients are to be multiplied into any one Quantity, they are multiplied as at Art. 14. only due Care muſt be taken of the Signs ariſing in the Product.

$$\begin{array}{lccc}
 & \textit{Exam. } 1. & \textit{Exam. } 2. & \textit{Exam. } 3. \\
\text{Multiply} & -3a-2z & 2y+5d & -5m-2y \\
\text{By} & 3m & -3a & 3d \\
\hline
 & -9am-6zm & -6ay-15ad & -15md-6dy
\end{array}$$

Exam. 1. Multiplying $3a$ by $3m$, the Product by Art. 13. is $9am$, but as $3m$ has the Sign $+$ prefixed to it, and $3a$ has the Sign $-$ prefixed to it, therefore to the Product $9am$ prefix the Sign $-$, by Art. 16. Again, $2z$ multiplied by $3m$ the Product is $6zm$, but as $2z$ has the Sign $-$ to it, and $3m$ the Sign $+$, therefore to $6zm$ prefix the Sign $-$, by Art. 16. and we have $-9am-6zm$ for the Product required.

Exam. 2. Multiplying $2y$ by $3a$, the Product is $6ay$, but as the Sign of $2y$ is $+$, and that of $3a$ is $-$, therefore to $6ay$ prefix the Sign $-$, by Art. 16. Then multiplying $5d$ by $3a$ the Product is $15ad$, but as the Sign of $5d$ is $+$, and that of $3a$ is $-$, therefore prefix the Sign $-$ to $15ad$, by Art. 16. and we have $-6ay-15ad$ for the Product required.

Exam. 3. Multiplying $5m$ by $3d$, the Product is $15md$, but as the Sign of $5m$ is $-$, and that of $3d$ is $+$, therefore prefix the Sign $-$ to $15md$. Again, multiplying $2y$ by $3d$, the Product is $6yd$, but becauſe the Signs of $2y$ and $3d$ are different, therefore prefix the Sign $-$ to $6dy$, and we have $-15md-6dy$ for the Product required.

Examples for the Exerciſe of the Learner.

$$\begin{array}{lccc}
\text{Multiply} & -2a-3b & 3m+7y & -5y-3b \\
\text{By} & 4z & -2d & 3m \\
\hline
\text{Product} & -8az-12bz & -6md-14dy & -15ym-9bm
\end{array}$$

18. And if two or more Quantities with Co-efficients are to be multiplied with two or more Quantities with Co-efficients, if their Signs are unlike yet they are multiplied as at Art. 15. taking due Care of the Signs of the Product, by Art. 9, and 16.

Multiply $-3a-2m$
By $4b+6y$

$-12ab-8bm$ the Product of $-3a-2m$ multiplied by $4b$, by Art. 17.
$-18ay-12ym$ the Product of $-3a-2m$ multiplied by $6y$, by Art. 17.

$-12ab-8bm-18ay-12ym$ the Product required, being the Sum of the two particular Products, which are added by Art. 6.

Multiply $5y+3m$
By $-7d-3a$

$-35yd-21dm$ the Product of $5y+3m$ multiplied by $-7d$, by Art. 17.
$-15ay-9am$ the Product of $5y+3m$ multiplied by $-3a$, by Art. 17.

$-35yd-21dm-15ay-9am$ the Product required, being the Sum of the two particular Products, which are added by Art. 6.

Multiply $2a+3b$
By $-2a-3b$

$-4aa-6ab$ the Product of $2a+3b$ multiplied by $-2a$, by Art. 17.
$-6ab-9bb$ the Product of $2a+3b$ multiplied by $-3b$, by Art. 17.

$-4aa-12ab-9bb$ the Product required: For in this Addition the Reader may obſerve that there is $-6ab$ in each of the two particular Products, which being added together by Art. 1, they will make $-12ab$, but $-4aa$ and $-9bb$ being different Quantities, they muſt be placed ſeparate from one another. There are Examples of this kind at Art. 15.

19. It may be for the Scholar's Advantage to be put in Mind that if any Algebraic Quantities are to be multiplied by a pure Number, that then this Number is to be multiplied into every one of the Co-efficients of the other Quantities, in all reſpects as before, and to each particular Product ſet or join that Quantity whoſe Co-efficient was multiplied. Thus,

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
Multiply	$2a+3b$	$-3m-4d$	$4d+3y$
By	6	7	9
Product	$12a+18b$	$-21m-28d$	$36d+27y$

Exam. 1. Multiplying 6 by 2 the Product is 12, to which joining a it is $12a$, then multiplying 3 by 6 it is 18, to which joining b it is $18b$, and becauſe 6 and $3b$ have both the Sign +, therefore by Art. 9. prefix the Sign + to $18b$, ſo is $12a+18b$ the Product required.

Exam. 2. Multiplying 3 by 7 it is 21, to which joining m it is $21m$, but as the Sign of $3m$ is —, and that of 7 is +, therefore by Art. 16. prefix the Sign — to $21m$, and it is $-21m$. Again, multiplying 4 by 7 it is 28, to which joining d it is $28d$, but as the Signs of $4d$ and 7 are likewiſe unlike, therefore to $28d$ prefix the Sign —, ſo will $-21m-28d$ be the Product required.

Exam. 3. Multiplying 4 by 9 the Product is 36, to which joining d it is $36d$, and becauſe the Signs of $4d$ and 9 are alike, therefore it will be $36d$, or $+36d$; and multiplying 9 by $3y$ the Product will be $27y$, to which muſt be prefixt the Sign +, becauſe $3y$ and 9 have the ſame Sign, ſo is $36d+27y$ the Product required.

Examples wherein all the three *Caſes* of Multiplication are promiſcuouſly uſed.

$$\begin{array}{ll}
\text{Multiply} & 2a-3b \\
\text{By} & 5m+2y \\
\hline
 & 10ma-15mb \\
 & \quad 4ay-6yb \\
\hline
\text{Product} & 10ma-15mb+4ay-6yb
\end{array}$$

The $2a$ being multiplied by $5m$ the Product is $10ma$, by Art. 13. and $-3b$ being multiplied by the ſame $5m$, the Product is $-15mb$, by Art. 16 and 17.

And the $2a$ being multiplied by $2y$ the Product is $4ay$, by Art. 13. and $-3b$ multiplied by $2y$ the Product is $-6yb$, by Art. 16 and 17.

Now draw the Line, and begin to add them, and becauſe the Quantities are all different, they are added by Art. 6. and therefore the Product will be $10ma-15mb+4ay-6yb$.

Multiply

$$\begin{array}{ll}
\text{Multiply} & -7m+2a \\
\text{By} & \quad 3y-4n \\ \hline
 & -21my+6ay \\
 & \quad 28mn-8na \\ \hline
\text{Product} & -21my+6ay+28mn-8na
\end{array}$$

The $-7m$ multiplied by $3y$ the Product is $-21my$, by Art. 16 and 17. and $2a$ multiplied by $3y$ the Product is $6ay$, by Art. 13.

And $-7m$ multiplied into $-4n$ the Product is $28mn$, by Art. 13. and $-4n$ multiplied by $2a$ the Product is $-8na$, by Art. 16 and 17.

Now begin the Addition, and becauſe the Quantities are all different, they are added by Art. 6. and the Product is found to be $-21my+6ay+28mn-8na$.

$$\begin{array}{ll}
\text{Multiply} & 2a+3b \\
\text{By} & 2a-3b \\ \hline
 & 4aa+6ab \\
 & -6ab-9bb \\ \hline
\text{Product} & 4aa-9bb
\end{array}$$

Multiplying the $2a$ by $2a$ the Product is $4aa$, and multiplying the $3b$ by $2a$ the Product is $6ab$.

And multiplying the $2a$ by $-3b$ the Product is $-6ab$, becauſe the Signs of the two Quantities are unlike, and for the ſame Reaſon the Product of $3b$ by $-3b$, is $-9bb$.

Now begin the Addition, and I obſerve in the firſt Line there is $+6ab$ or $6ab$, but in the ſecond Line there is $-6ab$, now becauſe the Co-efficients are equal and the Quantities alike, but the Signs being contrary, therefore by Art. 5. theſe Quantities will deſtroy one another, then putting down the $4aa$ and $9bb$, by Art. 6. we have for the Product required $4aa-9bb$.

$$\begin{array}{lll}
\text{Multiply} & 7m+4a & \\
 & -3a+5 & \\ \hline
 & -21ma-12aa & \text{the Product of } 7m+4a \text{ multiplied by } -3a. \\
 & \quad 35m+20a & \text{the Product of } 7m+4a \text{ multiplied by } 5\text{, by Art. 19.} \\ \hline
\text{Product} & -21ma-12aa+35m+20a &
\end{array}$$

Multiply $5a+b$
By $\underline{2a+3b}$

$10aa+2ab$ the Product of $5a+b$ multiplied by $2a$.
$\underline{15ab+3bb}$ the Product of $5a+b$ multiplied by $3b$.

Product $10a+17ab+3bb$

DIVISION,

In which there are four Caſes.

20. *Caſe* 1. WHEN the Signs of the Quantities to be divided are either both affirmative, or both negative, reject all thoſe Quantities in the Dividend and Diviſor that are alike, and ſet down the Remainder, prefixing to it the Sign +, which will be the Quotient required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Divide	ab	dm	$-mn$	$-ap$
By	a	d	$-m$	$-p$
Quotient	b	m	n	a

Exam. 1. Becauſe a is both in the Dividend and Diviſor, therefore I reject it, and there being then only b left, it is the Quotient ſought, and it has the Sign +, becauſe the Signs of ab and a are both alike.

Exam. 2. Becauſe d is both in the Dividend and Diviſor, therefore I reject it, and there being only m left, it is the Quotient ſought, and its Sign muſt be +, becauſe the Signs of dm and d are both alike.

Exam. 3. Becauſe m is both in the Dividend and Diviſor, therefore I reject it, and n being only left I write it down for the Quotient ſought, which muſt have the Sign +, becauſe mn and m have both the ſame Sign.

Exam. 4. Becauſe p is in both the Dividend and Diviſor, I reject it, and place down a the Quantity left, for the Quotient ſought which muſt have the Sign +, for the Signs of ap and p are both alike.

Exam.

	Exam. 5.	Exam. 6.	Exam. 7.	Exam. 8.
Divide	amd	$-apy$	mda	$-myz$
By	am	$-py$	ma	$-z$
Quotient	d	a	d	ym

Exam. 5. Becauſe am is both in the Dividend and Diviſor, I reject it, and place down d for the Quotient, which muſt have the affirmative Sign, for the Signs of amd and am are alike.

Exam. 6. Becauſe py is both in the Dividend and Diviſor, I reject it, and place down a the remaining Quantity for the Quotient, which muſt have the affirmative Sign, for the Signs of apy and py are alike.

Exam. 7. Becauſe ma is both in the Dividend and Diviſor, I reject it, and place down d for the Quotient, which muſt have the Sign +, becauſe the Signs of mda and ma are alike.

Exam. 8. Becauſe z is both in the Dividend and Diviſor, I reject it, and place down ym or my, which is the ſame thing for the Quotient ſought, and which muſt have the Sign +, becauſe the Signs of myz and z are alike.

Divide	apz	$-mnd$	$-abc$	$abdy$
By	az	$-md$	$-c$	ay
Quotient	p	n	ab	bd

The Truth of theſe Operations in Diviſion may be proved like thoſe in Multiplication, for the Quotient and Diviſor being multiplied, the Product will be the Dividend if the Work is true; thus in the ſecond Example of the laſt four, by multiplying n the Quotient into $-md$ the Diviſor, the Product is mdn, or mnd, to which muſt be prefixt the Sign —, by Art. 16. becauſe the Signs of md and n are unlike, hence the Product with its Sign is $-mnd$, the given Dividend.

And in the laſt Example, if we multiply bd the Quotient by ay the Diviſor, the Product is $bday$, or $abdy$ which is the ſame thing, by Art. 11. and this Quantity muſt have the affirmative Sign, by Art. 9. for the Signs of bd and ay are alike, hence $+abdy$, or $abdy$, is the Product with its Sign, the ſame as the given Dividend: And ſo of any of the other Examples.

21. But if all the Quantities in the Diviſor are not to be found in the Dividend, then you muſt only reject thoſe Quantities in

the Dividend and Diviſor that are alike, placing down the remaining Quantities of the Dividend, and under them thoſe of the Diviſor that are not to be rejected by this Rule, and this will be the Quotient ſought, and ſtand like a Vulgar Fraction in common Arithmetic.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Divide	amb	$-mndz$	$-dayp$	$pnqr$
By	ay	$-mna$	$-dpz$	pad
Quotient	$\frac{mb}{y}$	$\frac{dz}{a}$	$\frac{ay}{z}$	$\frac{nqr}{ad}$

Exam. 1. Becauſe a is in both the Dividend and Diviſor, reject it, and place down mb the remaining part of the Dividend, under which drawing a Line, and place y the remaining part of the Diviſor, ſo will $\frac{mb}{y}$ be the Quotient ſought, and which muſt have the Sign +, by Art. 20. as the Signs of the Quantities to be divided are alike.

Exam. 2. Becauſe mn is in both the Dividend and Diviſor, reject it, and place down dz the remaining part of the Dividend, under which drawing a Line, and place a the remaining part of the Diviſor, ſo is $\frac{dz}{a}$ the Quotient required, and it muſt have the Sign +, by Art. 20. as the Signs of the Quantities to be divided are alike.

Exam. 3. Becauſe dp is in both the Dividend and Diviſor, reject it, and write down ay the remaining part of the Dividend, under which place z the remaining part of the Diviſor, as in the two former Examples, ſo will $\frac{ay}{z}$, or $+\frac{ay}{z}$ be the Quotient required, for the Signs of the two Quantities to be divided are alike.

Exam. 4. Becauſe p is in both the Dividend and Diviſor, reject it, and write down nqr the remaining part of the Dividend, under which place ad the remaining part of the Diviſor, and $\frac{nqr}{ad}$ is the Quotient required, which will be affirmative by Art. 20. becauſe the Signs of $pnqr$ and pad are alike.

Divide	$-apqn$	adz	mnd	$-yzdb$
By	$-anm$	ap	ma	$-yza$
Quotient	$\frac{pq}{m}$	$\frac{dz}{p}$	$\frac{nd}{a}$	$\frac{db}{a}$

Now theſe Operations are proved as at Art. 20. by multiplying the Quotient by the Diviſor; for in the laſt Example the Quotient is $\frac{db}{a}$, which is a Fraction: the Diviſor is $-yza$, which by the Rules of Vulgar Fractions in common Arithmetic is made this improper Fraction $-\frac{yza}{1}$, then the two Fractions to be multiplied are $\frac{db}{a}$, and $-\frac{yza}{1}$, multiplying the Numerators we have $dbyza$ for the new Numerator, and multiplying the Denominators we have a for the Denominator, hence the Product is this Fraction $\frac{dbyza}{a}$, but as $\frac{yza}{1}$ has the *negative* Sign, and $\frac{db}{a}$ has the affirmative Sign, therefore by Art. 16. prefix the Sign — to $\frac{dbyza}{a}$ and it is $-\frac{dbyza}{a}$, the Product with its true Sign: But in this Fraction as $-dbyza$ is to be divided by a, rejecting a both in Dividend and Diviſor by Art. 20. we have $-dbyz$ or $-yzdb$, the ſame with the Dividend in the given Example, in like Manner may any of the other be proved.

22. And if there are two or more Quantities connected by the Signs + or — to be divided by any ſingle Quantity, every Quantity in the Dividend muſt be divided by the Diviſor, ſetting down the particular Quotients, as at Art. 20. which muſt be connected by the Sign +, when the Signs of the Quantities to be divided are both alike.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
Divide	$ab+am$	$md+mz$	$-da-dpq$
By	a	m	$-d$
Quotient	$b+m$	$d+z$	$a+pq$

Exam. 1. Dividing ab by a the Quotient is b, by Art. 20, and dividing am by a the Quotient is m, by the ſame Art. but as am and a have both the affirmative Sign, therefore to m prefix the Sign $+$, ſo is $b+m$ the Quotient required.

Exam. 2. md being divided by m the Quotient is d, by Art. 20. and dividing mz by m the Quotient is z, to which prefixing the Sign $+$, as mz and m have both the ſame Sign, we have $d+z$ for the Quotient required.

Exam. 3. da being divided by d the Quotient is a, and becauſe da and a have both the negative Sign, or the Signs are alike, therefore a muſt have the Sign $+$, whence it is $+a$ or a, and dividing dpq by d the Quotient is pq, to which muſt be prefixed the Sign $+$, for the Signs of dpq and d are alike, hence we have $a+pq$ for the Quotient required.

	Exam. 4.	*Exam.* 5.	*Exam.* 6.
Divide	$-ab-am$	$bm+bn$	$-zyp-zya$
By	$-a$	b	$-zy$
Quotient	$b+m$	$m+n$	$p+a$

Exam. 4. Dividing ab by a the Quotient is b, by Art. 20. and it muſt be $+b$ or b, as the Signs of ab and a are alike: then dividing am by a the Quotient is m, and becauſe the Signs of am and a are alike, therefore by Art. 20. prefix the Sign $+$ to m, and $b+m$ is the Quotient required.

Exam. 5. Dividing bm by b the Quotient is m, by Art. 20. as before, and dividing bn by b the Quotient is n, and as bn and b have both the ſame Sign, therefore prefix the Sign $+$ to n, ſo is $m+n$ the Quotient required.

Exam. 6. Dividing $-zyp$ by $-zy$ the Quotient is p, by Art. 20. and dividing $-zya$ by $-zy$ the Quotient is a, to which prefixing the Sign $+$, for the Signs of zya and zy are alike, we have $p+a$ for the Quotient required.

Divide	$-dnz-zad$	$am+ad$	$-dy-dz$
	$-zd$	a	$-d$
Quotient	$n+a$	$m+d$	$y+z$

The Truth of theſe Operations are proved by multiplying the Quotient by the Diviſor, if that Product is the dividend the Work is true, otherwiſe not. Now in the laſt Example the Quotient is $y+z$, and the Diviſor $-d$, which being multiplied together by Art. 14. they produce $-dy-dz$ the given Dividend.

23. *Caſe.* 2. When the Signs of the Quantities to be divided are one *affirmative* and the other *negative*, find the Quotient of the Quantities as before; but to them prefix the *negative* Sign, or the Sign —.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Divide	$-am$	$-mnp$	ayz	dmb
By	a	m	$-ay$	$-db$
Quotient	$-m$	$-np$	$-z$	$-m$

Exam. 1. Becauſe a is in both the Dividend and Diviſor, therefore reject it, and place down m the remaining part of the Dividend, but as the Signs of am and a are different, therefore to m prefix the Sign —, and it will be $-m$ the Quotient required.

Exam. 2. Becauſe m is both in the Dividend and Diviſor, therefore reject it, and place down np the remaining part of the Dividend, but as the Signs of mnp and m are different, therefore to np prefix the Sign —, and it will be $-np$ the Quotient required.

Exam. 3. Becauſe ay is both in the Dividend and Diviſor, therefore reject it, and place down z the remaining part of the Dividend, but as the Signs of ayz and ay are different, therefore prefix the Sign — to z, and it will be $-z$, the Quotient required.

Exam. 4. Becauſe db is both in the Dividend and Diviſor, therefore reject it, and place down m the remaining part of the Dividend; but the Signs of the Quantities that are divided being different, therefore to m prefix the Sign —, and it will be $-m$ the Quotient required.

Divide	$-mnp$	$-mnp$	dyp	dab
By	m	mn	$-dy$	$-db$
Quotient	$-np$	$-p$	$-p$	$-a$

24. And if there are two or more Quantities connected by the Signs + or —, to be divided by any ſingle Quantity, the Operation is the ſame as at Art. 22. only taking due Care that when the Signs of thoſe Quantities to be divided are different, to prefix the Sign — before thoſe Quotients.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
Divide	$-mn-md$	$ad+ab$	$-dnz-dzy$
By	m	$-a$	dz
Quotient	$-n-d$	$-d-b$	$-n-y$

Exam. 1. Dividing mn by m the Quotient is n, by Art. 20. but as the Signs of mn and m are different, therefore by Art. 23. I prefix the Sign — to n, and it is $-n$. And dividing md by m, the Quotient is d; but as the Signs of md and m are different, therefore by Art. 23. prefix the Sign — to d, hence $-n-d$ is the Quotient required.

Exam. 2. Dividing ad by a the Quotient is d, to which the Sign — muſt be prefixt, by Art. 23. which makes it $-d$: then dividing ab by a the Quotient is b; but as the Signs of ab and a are different, therefore by Art. 23. prefix the Sign — to b, ſo $-d-b$ is the Quotient required.

Exam. 3. Dividing $-dnz$ by dz the Quotient is $-n$, by Art. 20 and 23. and for the ſame Reaſon dividing $-dzy$ by dz, the Quotient is $-y$, which placing after $-n$, we have $-n-y$ for the Quotient required.

Divide	$maz+mxd$	$-dab-dby$	$-azx-azb$
By	$-mz$	db	az
Quotient	$-a-d$	$-a-y$	$-x-b$

The Truth of theſe Operations are likewiſe proved from multiplying the Quotient by the Diviſor, and if it makes the Dividend, the Work is true, otherwiſe not.

25. *Caſe* 3. But when there are Co-efficients joined to the Quantities, divide the Co-efficients as in common Arithmetic; and to their Quotients join the Quotient of the Quantities found by the foregoing Directions; but cautiouſly remember that if the Signs of the Quantities that are divided are alike, the Quotient muſt have the affirmative Sign, as at Art. 20. but if the Signs of the Quantities that are divided are unlike, then the Quotient muſt have the Sign — prefixt to it, by Art. 23.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Divide	$16am$	$8yz$	$-24dm$	$-18ma$
By	$2a$	$2z$	$-6d$	$-6a$
Quotient	$8m$	$4y$	$4m$	$3m$

Exam. 1. Dividing 16 by 2 the Quotient is 8, and am divided by a the Quotient is m, joining 8 to the m it is $8m$, and as $16am$ and $2a$ have both the ſame Sign, hence by Art. 20. the Sign $+$ muſt be prefixt to $8m$, therefore the Quotient is $+8m$ or $8m$.

Exam. 2. Dividing 8 by 2 the Quotient is 4, and dividing yz by z the Quotient is y, joining the 4 to the y it is $4y$; but as $8yz$ and $2z$ have both the ſame Sign, therefore by Art. 20. prefix the Sign $+$ to $4y$, hence $+4y$ or $4y$ is the Quotient required.

Exam. 3. Dividing 24 by 6 the Quotient is 4, and dividing dm by d the Quotient is m, joining 4 to the m it is $4m$; but as $24dm$ and $6d$ have both the ſame Sign, therefore prefix the Sign $+$ to $4m$, hence $+4m$ or $4m$ is the Quotient required.

Exam. 4. Dividing 18 by 6 the Quotient is 3, and dividing ma by a the Quotient is m, joining 3 to the m it is $3m$, and as $18ma$ and $6a$ have both the ſame Sign, therefore by Art. 20. the Quotient is $+3m$ or $3m$.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
Divide	$-15ay$	$-8dm$	$28yz$	$-12da$
By	$3a$	$-4d$	$-7y$	$3a$
Quotient	$-5y$	$2m$	$-4z$	$-4d$

Exam. 5. Dividing 15 by 3 the Quotient is 5, and dividing ay by a the Quotient is y, joining 5 to the y it is $5y$, but as the Signs of $15ay$ and $3a$ are different, therefore by Art. 23. prefix the Sign $-$ to $5y$, and then $-5y$ is the Quotient required.

Exam. 6. Dividing 8 by 4 the Quotient is 2, and dividing dm by d the Quotient is m, joining the 2 and m it is $2m$; but as $8dm$ and $4d$ have both the ſame Sign, therefore by Art. 20. prefix the Sign $+$ to $2m$, and then $+2m$ or $2m$ is the Quotient required.

Exam. 7. Dividing 28 by 7 the Quotient is 4, and dividing yz by y the Quotient is z, joining the 4 and z it is $4z$; but as $28yz$ and $7y$ have different Signs, therefore by Art. 23. prefix the Sign $-$ to $4z$, ſo will $-4z$ be the Quotient required.

Exam. 8. Dividing 12 by 3 the Quotient is 4, and dividing da by a the Quotient is d, joining the 4 and d it is $4d$; but as the Signs of $12da$ and $3a$ are different, therefore by Art. 23. prefix the Sign $-$ to $4d$, and then $-4d$ is the Quotient required.

Divide	$-32am$	$18dza$	$-22ymn$	$16az$
By	$-8m$	$9a$	$11yn$	$-8z$
Quotient	$4a$	$2dz$	$-2m$	$-2a$

26. And if there are two or more Quantities connected together with Coefficients, to be divided by any ſingle Quantity and its Co-efficient, the Operation is ſtill performed in the ſame Manner, connecting the particular Quotients as at Art. 22, and 24; ſtill carefully remembering that when the Quantities that are divided have like Signs, whether they be *affirmative* or *negative*, the Quotient muſt have the *affirmative* Sign; but if the Signs of the Quantities that are divided are unlike, then the Quotient muſt have the Sign — prefixt to it.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
Divide	$4am+12ad$	$-16my+24mz$	$28dn-21db$
By	$2a$	$-4m$	$7d$
Quotient	$2m+6d$	$4y-6z$	$4n-3b$

Exam. 1. Dividing $4am$ by $2a$ the Quotient is $2m$, by Art. 25. and dividing $12ad$ by $2a$ the Quotient is $6d$, and becauſe the Signs of $2a$ and $12ad$ are alike, prefix the Sign $+$ to $6d$, and we have $2m+6d$ for the Quotient required.

Exam. 2. Dividing $-16my$ by $-4m$ the Quotient is $4y$, by Art. 25. for the Signs of $16my$ and $4m$ are alike, and dividing $24mz$ by $-4m$ the Quotient is $-6z$, for $6z$ muſt have the negative Sign prefixt to it, the Signs of $24mz$ and $4m$ being unlike; hence $4y-6z$ is the Quotient required.

Exam. 3. Dividing $28dn$ by $7d$ the Quotient is $4n$, or $+4n$, for the Signs of $28dn$ and $7d$ are alike: and dividing $-21db$ by $7d$ the Quotient is $-3b$, for $3b$ muſt have the negative Sign prefixt to it, as the Signs of $21db$ and $7d$ are unlike, hence $4n-3b$ is the Quotient required.

Divide	$16pa-28pd$	$-24nm+36mz$	$16zu-4zd$
By	$-4p$	$-4m$	$2z$
Quotient	$-4a+7d$	$6n-9z$	$8u-2d$

The Truth of theſe Operations are proved likewiſe from multiplying the Quotient by the Diviſor, for if the Work is true, the Product will be the Dividend: In the laſt Example the Diviſor is $2z$, and the Quotient is $8u-2d$, now if we

Multiply $8u - 2d$
By $2z$

$16zu - 4zd$ the Product the ſame as the given Dividend, and ſo may the other Examples be proved.

27. *Caſe* 4. But when the Quantities in the Dividend are not the ſame with thoſe in the Diviſor, then place down the Dividend with its Signs and Co-efficients, under which drawing a Line, and after the Manner of Vulgar Fractions place the Diviſor with the ſame Signs it had, and Co-efficients, and this will be the Quotient required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Divide	b	am	$3my$	$2dy$
By	a	d	z	b
Quotient	$\frac{b}{a}$	$\frac{am}{d}$	$\frac{3my}{z}$	$\frac{2dy}{b}$

Exam. 1. Becauſe b and a are different Quantities, therefore I place down the Dividend b, under which I draw a Line, and place the Diviſor a, ſo is $\frac{b}{a}$ the Quotient required.

Exam. 2. Becauſe am and d are different Quantities, therefore place down am the Dividend, draw a Line under it, and place the Diviſor d, ſo is $\frac{am}{d}$ the Quotient required.

Exam. 3. Becauſe $3my$ and z are different Quantities, therefore place down $3my$ the Dividend, under it draw a Line, and place z the Diviſor, ſo is $\frac{3my}{z}$ the Quotient required.

Exam. 4. Becauſe $2dy$ and b are different Quantities, therefore place down $2dy$ the Dividend, draw a Line under it, and place b the Diviſor, and $\frac{2dy}{b}$ is the Quotient required.

Divide	$2ma$	$4dz$	$21ma$	$8yd$
By	$3y$	$2y$	$5d$	$3z$
Quotient	$\frac{2ma}{3y}$	$\frac{4dz}{2y}$	$\frac{21ma}{5d}$	$\frac{8yd}{3z}$

Divide	ma	$7d$	$3mbc$	$24d$
By	$7y$	mz	yd	$7yz$
Quotient	$\frac{ma}{7y}$	$\frac{7d}{mz}$	$\frac{3mbc}{yd}$	$\frac{24d}{7yz}$

28. And if there are two or more Quantities connected by the Signs + or — to be divided by any ſingle Quantity, and the Quantities in the Dividend are different from thoſe in the Diviſor, then having ſet down all the Quantities in the Dividend with their Signs and Co-efficients as before, draw a Line under them all, under which place the Diviſor as before, and this will be the Quotient required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
Divide	$2a+3b$	$7y-2m$	$15z-7da$
By	$5m$	$3n$	$4y$
Quotient	$\frac{2a+3b}{5m}$	$\frac{7y-2m}{3n}$	$\frac{15z-7da}{4y}$

Exam. 1. Becauſe $2a+3b$ the Dividend and $5m$ the Diviſor are different, therefore place down $2a+3b$, under which draw a Line, and place the Diviſor $5m$, ſo is $\frac{2a+3b}{5m}$ the Quotient required.

Exam. 2. Becauſe $7y-2m$ the Dividend and $3n$ the Diviſor are different, therefore place down $7y-2m$, under which draw a Line, and place the Diviſor $3n$, ſo is $\frac{7y-2m}{3n}$ the Quotient required.

Exam. 3. Becauſe $15z-7da$ the Dividend and $4y$ are different, therefore place down $15z-7da$ the Dividend, under which draw a Line, and place $4y$ the Diviſor, ſo is $\frac{15z-7da}{4y}$ the Quotient required.

Divide	$4ma-3d$	$7db-5xz$	$19m-15p$
By	$5z$	$3y$	$7y$
Quotient	$\frac{4ma-3d}{5z}$	$\frac{7db-5xz}{3y}$	$\frac{19m-15p}{7y}$

Divide	$3dz-5b$	$7ym-3dn$	$25dp+5yz-17m$
By	$2y$	$5u$	$7a$
Quotient	$\frac{3dz-5b}{2y}$	$\frac{7ym-3dn}{5u}$	$\frac{25dp+5yz-17m}{7a}$

29. And if there are two or more Quantities connected by the Signs + or —, to be divided by two or more Quantities connected by the Signs + or —, but the Quantities in the Dividend are different from thoſe in the Diviſor, it is only placing down the Dividend as before, under which drawing a Line, and place in like Manner the Diviſor, and this will be the Quotient required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
Divide	$2a+m$	$5y-7d$	$-14m+5z-11x$
By	$5d+3y$	$3a+2m$	$3y-2d$
Quotient	$\frac{2a+m}{5d+3y}$	$\frac{5y-7d}{3a+2m}$	$\frac{-14m+5z-11x}{3y-2d}$

Exam. 1. Becauſe the Quantities in the Dividend and Diviſor are unlike, therefore place down $2a+m$ the Dividend with its Co-efficients and Signs, under which draw a Line, and place $5d+3y$ the Diviſor, ſo is $\frac{2a+m}{5d+3y}$ the Quotient required.

Exam. 2. Becauſe $5y-7d$ the Dividend is different from $3a+2m$ the Diviſor, therefore place down $5y-7d$ the Dividend, under which draw a Line, and place $3a+2m$ the Diviſor, ſo is $\frac{5y-7d}{3a+2m}$ the Quotient required.

Exam. 3. Becauſe $-14m+5z-11x$ the Dividend is different from $3y-2d$ the Diviſor, therefore place down $-14m+5z-11x$ the Dividend, draw a Line under it, and place $3y-2d$ the Diviſor, and $\frac{-14m+5z-11x}{3y-2d}$ will be the Quotient required.

Divide	$4m-5y$	$-21pm+19zy$	$14yz-9dx$
By	$3x+2z$	$5d-2b$	$-3m+5n$
Quotient	$\frac{4m-5y}{3x+2z}$	$\frac{-21pm+19zy}{5d-2b}$	$\frac{14yz-9dx}{-3m+5n}$

Divide	$-4a+5m-3d$	$4a+3y-5x$	$2a+3y$
By	$7z-8y$	$-7d+11m$	$-5z-7n$
Quotient	$\frac{-4a+5m-3d}{7z-8y}$	$\frac{4a+3y-5x}{-7d+11m}$	$\frac{2a+3y}{-5z-7n}$

30. It may be juſt obſerved for the Eaſe of the Learner, that when any Quantity is divided by itſelf, or the Dividend and Diviſor are alike, that then the Quotient will be *Unity*, or 1.

Divide	$2ab$	$14mn$	$-5dz$	$-7y$
By	$2ab$	$14mn$	$-5dz$	$-7y$
Quotient	1	1	1	1

For by Art. 25. If we divide the Co-efficients, the Quotient will be Unity, or 1; then, by Art. 20. rejecting all thoſe Quantities that are alike, both in the Dividend and Diviſor, the Quantities all vaniſh, and there will be none to be joined to the *Unity*, or 1; whence, in ſuch Caſes as theſe, *Unity*, or 1, is the Quotient required.

31. It may be further obſerved that if an abſolute or pure Number is the Diviſor, the Co-efficients in the Dividend if there are more than one, muſt be divided by the Diviſor, and to each of theſe Quotients join the reſpective Quantities of the Dividend, as at Art. 26.

Divide	$24ma+18yz$	$16za+24ym$	$+14yd+35z$
By	6	8	-7
	$4ma+3yz$	$2za+3ym$	$-2yd-5z$

But if the Diviſor will not exactly divide the Co-efficients of the Dividend, then place the Dividend and Diviſor in the Manner of Vulgar Fractions, as in the foregoing Articles.

The Method of dividing Quantities by one another, where the Quotient will conſiſt of ſeveral Quantities, being generally perplexing to Learners, it will be explained when we ſhew the Method of ſolving *Quadratic* Equations, it being not neceſſary to the preſent Deſign to explain it before we come to that part of the Work.

INVOLUTION.

31. THIS is only the raiſing of Powers from any given Root, and therefore is performed by Multiplication: For the Quantity which is given being multiplied by itſelf will be the Square of that Quantity, that Product being multiplied by the given Quantity, this Product will be the Cube of that Quantity, and that Product multiplied again by the given Quantity, will be the fourth Power of that Quantity; and ſo on as in common Arithmetic.

To find the Cube of a		To find the Cube of b	
	a		b
The Square of a	aa	The Square of b	bb
	a		b
The Cube of a	aaa	The Cube of b	bbb

To find the Cube of — — —	$2y$
	$2y$
Now $2y$ multiplied by $2y$ the Product will be by Art. 13.	$4yy$ the Square of $2y$
	$2y$
And $4yy$ multiplied by $2y$, the Product will be by Art. 13.	$8yyy$ the Cube of $2y$
To find the Cube of — — —	$3z$
	$3z$
Now $3z$ multiplied by $3z$, the Product will be by Art. 13.	$9zz$ the Square of $3z$
	$3z$
And $9zz$ multiplied by $3z$, the Product will be by Art. 13.	$27zzz$ the Cube of $3z$

To find the 4th Power of $-2x$
$-2x$

Now $-2x$ multiplied by $-2x$, the Product is by Art. 13. } $4xx$ the Square of $-2x$
$-2x$

And $4xx$ multiplied by $-2x$, the Product is by Art. 16. } $-8xxx$ the Cube of $-2x$
$-2x$

And $-8xxx$ multiplied by $-2x$, the Product is by Art. 13. } $16xxxx$ the 4th Power of $-2x$.

In like Manner any other ſingle Quantity may be raiſed to any required Power, and if the given Quantity is compounded of more Letters than one, it is done in the ſame Manner.

To find the 4th Power of $2ab$
$2ab$

$4aabb$ the Square of $2ab$
$2ab$

$8aaabbb$ the Cube of $2ab$
$2ab$

$16aaaabbbb$ the 4th Power of $2ab$.

32. And if there are two or more Quantites connected by the Signs $+$ or $-$, to be raiſed to any given Power, it is ſtill performed by common Multiplication. Two Quantities when connected by the Sign $+$, is commonly called a *Binomial.*

To raiſe the *Binomial*,
or $a+b$ to the third Power or Cube.
$a+b$

$aa+ab$ the Prod. of $a+b$ multip. by a, by Art. 10.
$ab+bb$ the Prod. of $a+b$ multip. by b, by Art. 10.

$aa+2ab+bb$ Sum of theſe two Products, which is
$a+b$ the Square of $a+b$.

$aaa+2aab+abb$ the Product of $aa+2ab+bb$ multiplied by a, by Art. 10.
$aab+2abb+bbb$ the Product of $aa+2ab+bb$ multiplied by b, by Art. 10.

$aaa+3aab+3abb+bbb$ the Sum of theſe two Products, which is the Cube of $a+b$.

When two Quantities are connected by the Sign $-$, it is commonly called a *Reſidual.*

To raiſe the *Reſidual*

$x-y$ to the third Power or Cube.
$x-y$

$xx-xy$ the Product of $x-y$ multiplied by x.
$-xy+yy$ the Product of $x-y$ multiplied by $-y$.

$xx-2xy+yy$ the Sum of theſe two Products which is the Square of $x-y$.
$x-y$

$xxx-2xxy+xyy$ the Product of $xx-2xy+yy$ multiplied by x.
$-xxy+2xyy-yyy$ the Product of $xx-2xy+yy$ multiplied by $-y$.

$xxx-3xxy+3xyy-yyy$ the Sum of theſe two Products, which is the Cube of $x-y$.

And if theſe compound Quantities have Co-efficients, the Work ſtill proceeds as at Art. 18.

To raiſe the *Binomial*

$2a+3b$ to the third Power.
$2a+3b$

$4aa+6ab$ the Product of $2a+3b$ multiplied by $2a$.
$6ab+9bb$ the Product of $2a+3b$ multiplied by $3b$.

$4aa+12ab+9bb$ the Sum of theſe two Products, which is the Square of $2a+3b$.
$2a+3b$

$8aaa+24aab+18abb$ the Product of $4aa+12ab+9bb$ multiplied by $2a$.
$12aab+36abb+27bbb$ the Product of $4aa+12ab+9bb$ multiplied by $3b$.

$8aaa+36aab+54abb+27bbb$ the Sum of theſe two Products, which is the Cube of $2a+3b$.

To raiſe $3m+2y$ to the third Power.

$$\begin{array}{l}
3m+2y \\
\hline
9mm+6my \\
\quad 6my+4yy \\
\hline
9mm+12my+4yy \text{ the Square of } 3m+2y \\
3m+2y \\
\hline
27mmm+36mmy+12myy \\
\quad 18mmy+24myy+8yyy \\
\hline
27mmm+54mmy+36myy+8yyy \text{ the Cube of } 3m+2y.
\end{array}$$

To raiſe $a-2b$ to the third Power.

$$\begin{array}{l}
a-2b \\
\hline
aa-2ab \\
\quad -2ab+4bb \\
\hline
aa-4ab+4bb \text{ the Square of } a-2b \\
a-2b \\
\hline
aaa-4aab+4abb \\
\quad -2aab+8abb-8bbb \\
\hline
aaa-6aab+12abb-8bbb \text{ the Cube of } a-2b.
\end{array}$$

In this Example I have placed the ſame Quantities under each other, for the more commodious adding them, though this is not neceſſary, and is a Knowledge the Learner will acquire from his own Obſervation.

EVOLUTION,

WHICH is the Extraction of Roots, and therefore oppoſite to Involution, and as Equations in which the unknown Quantity riſes above the Square are generally adfected and reſolved by the Method of *Converging Series*, we ſhall take into Conſideration the Square Root only; and endeavour to give ſuch Directions that the Learner may generally know, whether the Square Root of any given Quantity can be extracted or not.

33. Now ſo many Times as any Letter is repeated ſo high is the Power of that Letter ſaid to be. Thus, *a* is *a* to the firſt Power: *a a* is *a* to the ſecond Power or Square, and *a a a a* is *a* to the fourth Power, *&c.* as in Involution.

And to extract the Root of any ſimple Quantity, conſider how many Times the Quantity is repeated, or how high the Power of it is, and if it appears to be the ſecond, third, fourth, or any other Power, divide that Figure which expreſſes the highth of the Power by 2, and if it does not divide it exactly it is a *Surd* Quantity, and has no Square Root, but if it divides it exactly, ſet down the Quantity whoſe Root you are extracting as many Times as the Quotient of the above Diviſion directs, and that will be the Square Root required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
To extract the Square Root of	aa	$bbbb$	$bbbbbb$
The Square Root is	a	bb	bbb

Exam. 1. Here a is repeated twice, or to the ſecond Power; now dividing 2 by 2 the Quotient is 1, therefore ſetting down a once, or a, it is the Square Root required.

Exam. 2. Here b is repeated fourtimes, or to the fourth Power, now dividing 4 by 2 the Quotient is 2, therefore ſetting down b twice, or bb, it is the Square Root required.

Exam. 3. Here b is repeated ſix-times, or to the ſixth Power; now dividing 6 by 2 the Quotient is 3, therefore ſetting down b three times, or bbb, it is the Square Root required.

The Truth of theſe Operations may be proved by Multiplication, for if the Work is right the Square Root being multiplied by itſelf will produce the Quantity from which the Root was extracted. Thus in Example 2.

The Square Root is	bb
Which being multiplied by itſelf	bb
The Product is the given Square	$bbbb$

And ſo of any other Example.

	Exam. 4.	*Exam.* 5.
To extract the Square Root of	$aaaa$	$dddddd$
The Square Root is	aa	ddd

Exam. 4. Here a is repeated four times, or to the fourth Power; now dividing 4 by 2 the Quotient is 2, which ſhews that a muſt be repeated twice, that is, aa is the Square Root required.

Exam. 5. Here d is repeated ſix times, or to the ſixth Power; now dividing 6 by 2 the Quotient is 3, which ſhews that d muſt be repeated three times, and conſequently ddd is the Square Root required.

And if the Quantity, whoſe Root is to be extracted, has different Letters, then we muſt conſider if the Number of Times each Letter is repeated can be divided by 2 without any Remainder, and if they can, join the Letters together, repeating each ſo many Times as the Quotient of the reſpective Diviſion directs, and this will be the Square Root required; but if the Number of Times any one Letter is repeated cannot be divided by 2, then the whole Quantity has no Square Root.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
To extract the Square Root of	$aabbbb$	$aaaadddd$	$mmpp$
The Square Root is	abb	$aadd$	mp

Exam. 1. Here a is repeated twice, and 2 being divided by 2 the Quotient is 1, which ſhews a muſt be taken only once, or a. Now b is repeated four times, or to the fourth Power, and 4 being divided by 2 the Quotient is 2, which ſhews b muſt be repeated twice, or bb, now joining a to bb, I ſay, abb is the Square Root required.

Exam. 2. Here a is repeated to the fourth Power, and dividing 4 by 2 the Quotient is 2, which ſhews that a muſt be repeated twice, that is, it muſt be aa: Again, d is repeated to the fourth Power, that dividing 4 by 2 the Quotient is 2, which ſhews d muſt be repeated to the ſecond Power, or dd: Now joining aa to dd, we have $aadd$ for the Square Root required.

By the ſame Method of reaſoning we ſhall find in Example 3, that the Square Root of $mmpp$ is mp.

But when it is found that the given Quantity has not ſuch a Root as is required, then the Square Root of it is expreſſed by prefixing this Sign $\surd$ before it.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
Required the Square Root of	a	bbb	$ddddd$
The Square Root is	$\surd a$	$\surd bbb$	$\surd ddddd$

Exam. 1. Becauſe a is only repeated once, and as we cannot divide 1 by 2 and have the Quotient a whole Number, therefore I conclude a is a *Surd* Quantity, and accordingly, to expreſs the

Square Root of a, I prefix the Sign $\surd$ to it, ſo is $\surd a$ the Square Root required.

Exam. 2. Here b is repeated three times, and becauſe 3 cannot be divided exactly by 2, and have no remainder; therefore I conclude bbb is a *Surd* Quantity, and to expreſs the Square Root of it, I prefix the Sign $\surd$ to it, ſo is $\surd bbb$ the Square Root required.

Exam. 3. Here d being repeated five times, and as we cannot divide 5 by 2, and have no remainder, therefore I conclude that $ddddd$ is a *Surd* Quantity, and to expreſs the Square Root of it, I prefix the Sign $\surd$ to it, ſo is $\surd ddddd$ the Square Root required.

34. But to extract the Square Root of compound Quantities, or thoſe connected by the Signs $+$ or $-$, obſerve,

Firſt, That there muſt be three Quantities to make it a Square, for if there is only two Quantities it is a *Surd*. I take no Notice of any greater Number of Quantities than three, which may compoſe a Square, as they ſeldom or ever occur in any Operation.

Secondly, That theſe three Quantities have generally two different Letters only; there may be Caſes in which there are more than two different Letters in theſe three Quantities, but as they ſeldom happen, I chooſe not to perplex the Learner with them.

Thirdly, That two of theſe three Quantities are pure Powers of thoſe two Letters.

Fourthly, That both theſe pure Powers of the two different Letters, muſt have the Sign $+$ before them.

Fifthly, That the third of the above three Quantities is always twice the Product of the Square Root of the two pure Powers of the two different Letters, and may have either the Sign $+$ or $-$. Perhaps the Learner may not ſo eaſily diſcover this Particular, therefore he may ſuſpend the Conſideration of it till a little further.

Now if the given Quantity, whoſe Root is to be extracted, anſwers theſe Particulars, its Square Root may be extracted thus.

Sixthly, Extract the Square Root of the two pure Powers of the two different Letters, according to the Directions at Article 33.

Seventhly, If the Quantity mentioned at the *fifth* Particular has the *negative* Sign, connect the two Roots mentioned in the laſt Particular with the Sign $-$, and it will be the Square Root required.

Eighthly, But if the Quantity mentioned at the *fifth* Particular has the Sign $+$, then connect the two Roots with the Sign $+$, and this will be the Square Root required.

Now let it be required to extract the Square Root of $aa+2ab+bb$.

Here are three Quantities by the *firſt* Particular.

They have likewiſe two different Letters, *viz.* a and b, by the *ſecond* Particular.

Two of theſe Quantities, *viz.* aa and bb are pure Powers of the two Letters a and b, by the *third* Particular.

And both theſe pure Powers, *viz.* aa and bb have the Sign $+$, by the *fourth* Particular.

Now ſuppoſe we neglect the Conſideration of the *fifth* Particular, and attempt the Extraction of the Root by the *ſixth* Particular.

Then the Square Root of aa, is by Art. 33. — — a
And the Square Root of bb, is by the ſame — — b
And becauſe the third Quantity $2ab$ has the Sign $+$,
Therefore by the *eighth* Particular, I connect a and b with the Sign $+$, then it is — — — — $a+b$
Hence I ſuppoſe $a+b$ to be the Square Root of $aa+2ab+bb$.

Now we may give the Learner a better Idea of the *fifth* Particular, for the Square Roots of aa and bb the pure Powers of the different Letters are a and b, which being multiplied together gives ab, and this multiplied by 2 gives $2ab$, which is the third Quantity in the Example, and is, according to the *fifth* Particular, twice the Product of the Square Roots of the two pure Powers of the different Letters a and b, which are in the given Quantity $aa+2ab+bb$.

But to prove the Truth of the Operation, multiply the Root by itſelf, and if the Product agrees with the given Quantity, in its *Quantities*, *Signs*, and *Co-efficients*, you may conclude the Work to be right; but if it does not, you may conclude that the Work is erroneous or elſe that it has no Square Root, and is a *Surd* Quantity.

The Root of the laſt Example was ſuppoſed to be } $a+b$
Which being multiplied by itſelf $a+b$

$$\begin{array}{r}
a+b \\
a+b \\
\hline
aa+ab \qquad \\
ab+bb \\
\hline
aa+2ab+bb
\end{array}$$

The Product is the given Quantity, which proves that $a+b$ is the Square Root of $aa+2ab+bb$.

Required the Square Root of $aa+2za+zz$.

Here are three Quantities by the *firſt* Particular.

They have likewiſe two different Letters a and z, by the *ſecond* Particular.

Two of theſe three Quantities, *viz.* aa and zz, are pure Powers of a and z, by the *third* Particular.

And both theſe Powers have the Sign $+$ by the *fourth* Particular.

Now the third Quantity $2za$ is twice the Product of the Square Roots of the two pure Powers aa and zz.

Then to extract the Square Root of this $aa+2za+zz$ by the *ſixth* Particular.

The Square Root of aa by Art. 33. is — — a

The Square Root of zz by the ſame is — — z

Becauſe the third Quantity $2az$ has the Sign $+$, therefore by the *eighth* Particular, connect a and z with the Sign $+$, and, I ſay, $a+z$ is the Square Root required.

To try if the Square Root is $a+z$
Multiply it by itſelf $a+z$

$$\begin{array}{r} a+z \\ a+z \\ \hline aa+az \\ az+zz \\ \hline aa+2az+zz \end{array}$$

The Product is $aa+2az+zz$, which agreeing with that in the Example, in the Quantities, Signs, and Co-efficients, it appears that $a+z$ is the Square Root required.

To extract the Square Root of $mm-2mp+pp$.

Here are three Quantities by the *firſt* Particular.

They have likewiſe two different Letters m and p, by the *ſecond* Particular.

Two of theſe three Quantities, *viz.* mm and pp are pure Powers of m and p, by the *third* Particular.

And both theſe Powers have the Sign $+$, by the *fourth* Particular.

Likewiſe the third Quantity $-2mp$ is twice the Product of the Square Roots of the two pure Powers mm and pp.

Then according to the *ſixth* Particular, the Square Root of mm is — — — — — — m

By the ſame, the Square Root of pp is — — p

But as the third Quantity $2mp$ has the Sign $-$, therefore by the *ſeventh* Particular connect m and p with the Sign $-$, and, I ſay, $m-p$ is the Square Root required.

To try if the Square Root is $m-p$
Multiply it by itſelf $m-p$

$$\begin{array}{r} m-p \\ m-p \\ \hline mm-mp \\ -mp+pp \\ \hline mm-2mp+pp \end{array}$$

The Product $mm-2mp+pp$, agreeing in every thing with the given Quantity, it proves $m-p$ is the Square Root required.

By the ſame Method of reaſoning it will be found that the Square Root of $zz+2zx+xx$, is $z+x$.

And that the Square Root of $aa-2ad+dd$, is $a-d$.

And that the Square Root of $xx-2xm+mm$, is $x-m$.

And if it was required to extract the Square Root of $aa+ba+\frac{bb}{4}$.

Here the two pure Powers are aa and $\frac{bb}{4}$.

But the Square Root of aa is — — — a

And the Square Root of $\frac{bb}{4}$, is — — — $\frac{b}{2}$

And connecting theſe we have — — $a+\frac{b}{2}$

Which being multiplied by itſelf, produces $aa+ba+\frac{bb}{2}$, hence $a+\frac{b}{2}$ is the Square Root required.

Therefore when any one of the Quantities appears in a *Fractional* Manner, we muſt extract the Square Root of both the Numerator and Denominator, placing the Square Root of the Numerator for a new Numerator, and the Square Root of the Denominator for a new Denominator, and try the Work as before.

But if we cannot extract the Square Root of both the Numerator and Denominator, then we may conclude, we cannot extract the Square Root of the given Quantity.

Now by this reaſoning we ſhall find the Square Root of $xx+xa+\frac{aa}{4}$, to be $x+\frac{a}{2}$.

And that the Square Root of $mm - my + \frac{yy}{4}$ is $m - \frac{y}{2}$.

Now ſuppoſe it was required to extract the Square Root of $xx + 2xn - nn$.

Here are three Quantities by the *firſt* Particular.

They have likewiſe two different Letters x and n by the *ſecond* Particular.

Two of theſe three Quantities, *viz.* nn and pp are pure Powers of n and p.

But both theſe Powers have not the Sign $+$ for it is $-nn$, therefore by the *fourth* Particular, I conclude that the given Quantity $xx + 2xn - nn$ is a *Surd* Quantity, and its Square Root cannot be extracted any otherwiſe then by prefixing the Sign $\surd$ to it, as in Art. 33. Thus, $\sqrt{xx + 2xn - nn}$ is, or expreſſes, the Square Root of $xx + 2xn - nn$.

Let it be required to extract the Square Root of $aa + 5ab + bb$.

Here are three given Quantities by the *firſt* Particular.

They have likewiſe two different Letters a and b, by the *ſecond* Particular.

Two of theſe Quantities, *viz.* aa and bb are pure Powers of a and b.

And both theſe Powers have the Sign $+$ by the *fourth* Particular.

But then the third Quantity $5ab$ is not twice the Product of the Square Roots of aa and bb for their Roots being a and b; if they are multiplied the Product is ab, and that being multiplied by 2 it is $2ab$: Whereas the third Quantity in the given Example is $5ab$. Hence, I conclude that $aa + 5ab + bb$ is a *Surd* Quantity, and to expreſs its Square Root I prefix to it the Sign $\surd$, ſo will $\sqrt{aa + 5ab + bb}$ be the Square Root of $aa + 5ab + bb$.

And if it was required to extract the Square Root of $aa + 2ab + \frac{bb}{5}$, it will be found a *Surd* Quantity, it being impoſſible to extract the Square Root of 5, therefore prefix the Sign $\surd$ to $aa + 2ab + \frac{bb}{5}$, and then $\sqrt{aa + 2ab + \frac{bb}{5}}$ is the Square Root required.

For the ſame Reaſon the Square Root of $xx + 2xa + \frac{aa}{3}$ is $\sqrt{xx + 2xa + \frac{aa}{3}}$, it being impoſſible to extract the Square Root of 3.

When the Radical Sign or $\surd$ is prefixt to any compound Quantity, draw the Top of the Sign over all thoſe Quantities, which ſhews that they are all included under that Sign; for if the Sign was not to be drawn over all of them, it may be thought that the Square Root of that Quantity was only to be extracted which ſtands next the Radical Sign.

To extract the Square Root of $aaaa+2aab+bb$.

Here are three given Quantities by the *firſt* Particular.

They have likewiſe two different Letters a and b, by the *ſecond* Particular.

Two of theſe Quantities, *viz.* $aaaa$ and bb are pure Powers of a and b, by the *third* Particular.

And both theſe Powers have the Sign $+$, by the *fourth* Particular.

And the third Quantity $2aab$ is twice the Product of the Square Roots of $aaaa$ and bb, for their Roots by Art. 33. are aa and b.

Now by the *ſixth* Particular, the Square Root of $aaaa$ is aa

And by the ſame, the Square Root of bb is — — b

And as the third Term $2aab$ in the given Quantity has the Sign $+$, by the *eighth* Particular connect aa and b, the two Roots of $aaaa$ and bb with the Sign $+$, ſo is $aa+b$ the Square Root of $aaaa+2aab+bb$.

To prove which put down the ſuppoſed Square Root } $aa+b$

Which multiplied by itſelf $aa+b$

$$\begin{array}{l} aa+b \\ aa+b \\ \hline aaaa+aab \\ aab+bb \\ \hline aaaa+2aab+bb \end{array}$$

Which Product $aaaa+2aab+bb$ agreeing with the given Quantity in every Particular, proves the Square Root to be as above.

To extract the Square Root of $yyyy-2yyx+xx$.

Here the given Quantities agreeing with the firſt five Particulars as before.

By the *ſixth* Particular I find the Square Root of $yyyy$ is - yy

By the ſame, that the Square Root of xx is — — x

But as the third Term $-2yyx$ in the given Quantity has the Sign —, therefore by the *ſeventh* Particular I connect yy and x the two Roots with the Sign —, and ſay, or ſuppoſe $yy-x$ to be the Square Root required.

To prove which put down the ſuppoſed Root $yy - x$

Which multiply by itſelf $yy - x$

$$yyyy - yyx$$
$$- yyx + xx$$

The Product agreeing with the given Quantity $yyyy - 2yyx + xx$

By the ſame Method we ſhall find the Square Root of $nnnn + 2nnd + dd$ to be $nn + d$.

And that the Square Root of $xxxx + 2xxyy + yyyy$ is $xx + yy$.

And we ſhall find that $dddd + 3ddy + yy$ is a *Surd* Quantity, and it's Square Root muſt be expreſſed by prefixing the radical Sign to it, thus $\sqrt{dddd + 3ddy + yy}$.

We ſhall likewiſe find that $- pppp + 2ppy + yy$ is a *Surd* Quantity, and to extract it's Square Root, is only to prefix to it the radical Sign, thus $\sqrt{- pppp + 2ppy + yy}$.

Of Surd QUANTITIES.

THESE are ſuch Quantities whoſe Roots cannot be exactly extracted, and as they ariſe in the Reſolution of *Algebraic* Queſtions, we ſhall explain ſo much of theſe Quantities only as is neceſſary to the preſent Deſign; for a particular and copious Explanation of them in all their Varieties and Circumſtances would itſelf make a Treatiſe.

Addition of Surd Quantities.

35. *Caſe* 1. When the Quantities under the Radical Signs are alike, add the rational Quantities, or thoſe which are without the Radical Sign, together, by the Rules of Addition, at *Art.* 1, 2, 3, 4, 5, 6. and to this Sum join the *Surd* Quantities, and this will be the Sum required.

And if there appears to be no rational Quantities without the radical Sign, then *Unity* or 1 is always ſuppoſed to be the rational Quantity.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
To	$\sqrt{am}$	$2\sqrt{dy}$	$6m\sqrt{d+a}$	$5y\sqrt{dm+z}$
Add	$\sqrt{am}$	$\sqrt{dy}$	$4m\sqrt{d+a}$	$y\sqrt{dm+z}$
Sum	$2\sqrt{am}$	$3\sqrt{dy}$	$10m\sqrt{d+a}$	$6y\sqrt{dm+z}$

Exam. 1. There being no rational Quantities, therefore Unity or 1 is the rational Quantity to each. Now 1 added to 1 makes 2, to which joining the Surd $\sqrt{am}$, we have $2\sqrt{am}$, the Sum required.

Exam. 2. The rational Quantities being 2 and 1, their Sum is 3, to which joining $\sqrt{dy}$ we have $3\sqrt{dy}$, the Sum required.

Exam. 3. The rational Quantities are $6m$ and $4m$, which being added make $10m$, to which joining the Surd $\sqrt{d+a}$ we have $10m\sqrt{d+a}$, the Sum required.

Exam. 4. The rational Quantities are $5y$ and y, which being added make $6y$, to which joining the Surd $\sqrt{dm+z}$ we have $6y\sqrt{dm+z}$, the Sum required.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.
To	$13yd\sqrt{z-x}$	$15z\sqrt{da+p}$	$-7m\sqrt{da-y}$
Add	$5yd\sqrt{z-x}$	$-3z\sqrt{da+p}$	$-2m\sqrt{da-y}$
Sum	$18yd\sqrt{z-x}$	$12z\sqrt{da+p}$	$-9m\sqrt{da-y}$

Exam. 5. The rational Quantities $13yd$, and $5yd$ being added, make $18yd$, to which joining the Surd Quantity $\sqrt{z-x}$, we have $18yd\sqrt{z-x}$, the Sum required.

Exam. 6. The rational Quantities $15z$, and $-3z$ being added, their Sum by *Art.* 3. is $12z$, to which joining the Surd $\sqrt{da+p}$, we have $12z\sqrt{da+p}$, the Sum required.

Exam. 7. The rational Quantities $-7m$, and $-2m$ being added make $-9m$, to which joining $\sqrt{da-y}$, we have $-9m\sqrt{da-y}$, the Sum required.

To	$-2y\sqrt{ma+m}$	$-15m\sqrt{da-zp}$	$16dp\sqrt{14+p}$
Add	$-3y\sqrt{ma+m}$	$7m\sqrt{da-zp}$	$-12dp\sqrt{14+p}$
Sum	$-5y\sqrt{ma+m}$	$-8m\sqrt{da-zp}$	$4dp\sqrt{14+p}$

To	$4y\sqrt{zd-za}$	$5y\sqrt{mp+x}$	$7z\sqrt{ma-d}$
Add	$3y\sqrt{zd-za}$	$-4y\sqrt{mp+x}$	$-8z\sqrt{ma-d}$
	$7y\sqrt{zd-za}$	$y\sqrt{mp+x}$	$-z\sqrt{ma-d}$

36. *Caſe* 2. When the Letters under the *radical* Sign are *different*, then place them down one after the other with the ſame Signs they had in the Queſtion, in the Manner as at Art. 6. and this will be the Sum required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
To	$\sqrt{a}$	$\sqrt{b+m}$	$m\sqrt{da+y}$
Add	$\sqrt{b}$	$\sqrt{d+y}$	$m\sqrt{z}$
Sum	$\sqrt{a}+\sqrt{b}$	$\sqrt{b+m}:+\sqrt{d+y}$	$m\sqrt{da+y}:+m\sqrt{z}$

Exam. 1. The Letters under the radical Signs being different put down $\sqrt{a}$, then becauſe $\sqrt{b}$ has the Sign $+$, therefore after $\sqrt{a}$ put $+$ after which put $\sqrt{b}$, and we have $\sqrt{a}+\sqrt{b}$ for the Sum required.

Exam. 2. The Letters under the radical Signs being different put down $\sqrt{b+m}$: after which place two dots to ſhew that *Surd* goes no further, then becauſe $\sqrt{d+y}$ has the Sign $+$, therefore after the Quantity $\sqrt{b+m}$: put $+$ and after that the Surd $\sqrt{d+y}$, and we have $\sqrt{b+m}:+\sqrt{d+y}$ for the Sum required.

Exam. 3. The Letters under the radical Signs being different put down $m\sqrt{da+y}$: and becauſe the Quantity $m\sqrt{z}$ has the Sign $+$, therefore after $\sqrt{da+y}$: put the Sign $+$, after which put the Quantity $m\sqrt{z}$, and we have $m\sqrt{da+y}:+m\sqrt{z}$ for the Sum required.

	Exam. 4.	*Exam.* 5.
To	$y\sqrt{da}$	$-5\sqrt{da-y}$
Add	$-z\sqrt{m}$	$-2m\sqrt{zm}$
Sum	$y\sqrt{da}-z\sqrt{m}$	$-5\sqrt{da-y}:-2m\sqrt{zm}$

Exam. 6.

$$\begin{array}{l} -2m\sqrt{bz+n} \\ 3y\sqrt{dz-b} \\ \hline -2m\sqrt{bz+n}:+3y\sqrt{dz-b} \end{array}$$

Exam. 4. The Letters under the radical Signs being different put down $y\sqrt{da}$, and becauſe $-z\sqrt{m}$ has the Sign $-$, therefore after $y\sqrt{da}$ put the Sign $-$, and after that the Quantity $z\sqrt{m}$, and we have $y\sqrt{da}-z\sqrt{m}$ for the Sum required.

Exam. 5. The Letters under the radical Signs being different put down $-5\sqrt{da-y}:$ and becauſe $-2m\sqrt{zm}$ has the Sign $-$, therefore after $-5\sqrt{da-y}:$ put the Sign $-$, and after that the Quantity $2m\sqrt{zm}$, and we have $-5\sqrt{da-y}:$ $-2m\sqrt{zm}$ for the Sum required.

Exam. 6. Becauſe the Letters under the radical Signs are different I put down $-2m\sqrt{bz+n}$, but $3y\sqrt{dz-b}$ having the Sign $+$, therefore after $-2m\sqrt{bz+n}:$ put the Sign $+$, and after that the Quantity $3y\sqrt{dz-b}:$ and we have $-2m\sqrt{bz+n}:+3y\sqrt{dz-b}$ for the Sum required.

To	$-5\sqrt{da}$	$m\sqrt{bma}$
Add	$7\sqrt{m}$	$3\sqrt{yp+q}$
Sum	$-5\sqrt{da}+7\sqrt{m}$	$m\sqrt{bma}+3\sqrt{yp+q}$
To	$-3y\sqrt{p+r}$	$14m\sqrt{da+pz}$
Add	$m\sqrt{d}$	$7\sqrt{z+p}$
Sum	$-3y\sqrt{p+r}:+m\sqrt{d}$	$14m\sqrt{da+pz}:+7\sqrt{z+p}$
To	$-5y\sqrt{dp-z}$	
add	$+7y\sqrt{zm+a}$	
Sum	$-5y\sqrt{dp-z}:+7y\sqrt{zm+a}$	

Subſtraction of Surd Quantities.

37. *Caſe* 1. When the Letters under the *Radical* Signs are *alike*, ſubſtract the rational Quantities from the rational Quantities by Art. 7. and to the difference join the Surd Quantities, which will be the remainder required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
From	$5\sqrt{da}$	$5m\sqrt{mz}$	$14y\sqrt{d+z}$	$21pm\sqrt{db-r}$
Subſtract	$3\sqrt{da}$	$2m\sqrt{mz}$	$3y\sqrt{d+z}$	$19pm\sqrt{db-r}$
Remains	$2\sqrt{da}$	$3m\sqrt{mz}$	$11y\sqrt{d+z}$	$2pm\sqrt{db-r}$

Exam. 1. The rational Quantities are 5 and 3, ſubſtracting 3 from 5 there remains 2, to which joining the Surd $\sqrt{da}$ we have $2\sqrt{da}$ the remainder required.

Exam. 2. The rational Quantities are $5m$ and $2m$, ſubſtracting $2m$ from $5m$ there remains $3m$, to which joining the Surd $\sqrt{mz}$ we have $3m\sqrt{mz}$ the remainder required.

Exam. 3. The rational Quantities are $14y$ and $3y$, ſubſtracting $3y$ from $14y$ there remains $11y$, to which joining the Surd $\sqrt{d+z}$ we have $11y\sqrt{d+z}$ for the remainder required.

Exam. 4. The rational Quantities are $21pm$ and $19pm$, ſubſtracting $19pm$ from $21pm$ there remains $2pm$, to which joining the Surd $\sqrt{db-r}$, we have $2pm\sqrt{db-r}$ for the remainder required.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.
From	$17d\sqrt{ba}$	$-5y\sqrt{d+a}$	$-5m\sqrt{d+ab}$
Subſtract	$-4d\sqrt{ba}$	$3y\sqrt{d+a}$	$-6m\sqrt{d+ab}$
Remains	$21d\sqrt{ba}$	$-8y\sqrt{d+a}$	$m\sqrt{d+ab}$

Exam. 5. The rational Quantities are $17d$ and $-4d$: Now to ſubſtract $-4d$ from $17d$, by the Rule for Subſtraction at Art. 7. change the Sign of $-4d$, or ſuppoſe it to be changed, then $-4d$ becomes $+4d$ or $4d$; then by Art. 7. if we add $17d$ to $4d$ it is $21d$, which is the remainder that ariſes by ſubſtracting $-4d$ from $17d$; now to this $21d$ join the Surd $\sqrt{ba}$, and we have $21d\sqrt{ba}$ for the remainder required.

Exam. 6. Now to ſubſtract the rational Quantity $3y$ from $-5y$, we muſt by Art. 7. change or ſuppoſe the Sign of $3y$ to be changed, which will make it $-3y$: then by the ſame Art. $-3y$ added to $-5y$ it is $-8y$, which is the remainder that ariſes from the Subſtraction of the rational Quantities, therefore to this $-8y$ join the Surd Quantity $\sqrt{d+a}$, and we have $-8y\sqrt{d+a}$ for the remainder required.

Exam. 7. Here the rational Quantities are $-5m$ and $-6m$, and by the Rule for Subſtraction Art. 7. if we ſuppoſe the Sign of $-6m$ to be changed, it becomes $+6m$ or $6m$, and then adding $-5m$ to $6m$ it is m, the remainder ariſing from the Subſtraction of the rational Quantities; and if to this m we join the Surd $\sqrt{d+ab}$ we have $m\sqrt{d+ab}$ the remainder required.

	Exam. 8.	*Exam.* 9.	*Exam.* 10.
From	$21m\sqrt{d+a}$	$-9d\sqrt{mn+p}$	$12y\sqrt{d-an}$
Subſtract	$9m\sqrt{d+a}$	$-2d\sqrt{mn+p}$	$-3y\sqrt{d-an}$
Remains	$12m\sqrt{d+a}$	$-7d\sqrt{mn+p}$	$15y\sqrt{d-an}$

	Exam. 11.	*Exam.* 12.	*Exam.* 13.
From	$-4a\sqrt{m-p}$	$14p\sqrt{d-y}$	$-5a\sqrt{x+y}$
Subſtract	$2a\sqrt{m-p}$	$-3p\sqrt{d-y}$	$3a\sqrt{x+y}$
Remains	$-6a\sqrt{m-p}$	$17p\sqrt{d-y}$	$-8a\sqrt{x+y}$

	Exam. 14.	*Exam.* 15.	*Exam.* 16.
From	$7a\sqrt{ap}$	$-21\sqrt{ap-ax}$	$-14\sqrt{da-z}$
Subſtract	$2a\sqrt{ap}$	$-9\sqrt{ap-ax}$	$7\sqrt{da-z}$
Remains	$5a\sqrt{ap}$	$-12\sqrt{ap-ax}$	$-21\sqrt{da-z}$

The Truth of theſe Operations are proved as in Subſtraction of common Numbers. Thus at Example 1, the remainder is $2\sqrt{da}$, and the Quantity ſubſtracted was $3\sqrt{da}$, now if we add theſe together by Art. 34. the Sum is $5\sqrt{da}$, which being the ſame Quantity from which $3\sqrt{da}$ was ſubſtracted, it proves the Work to be true.

Again at Example 6. the remainder is $-8y\sqrt{d+a}$: the Quantity ſubſtracted was $3y\sqrt{d+a}$: Now by Art. 34. if to $-8y\sqrt{d+a}$ we add $3y\sqrt{d+a}$, the Sum is $-5y\sqrt{d+a}$, which being the Quantity from which $3y\sqrt{d+a}$ was ſubſtracted, it proves the Work to be true.

38. *Caſe* 2. When the Letters under the radical Signs are different, ſet them down one after the other, as at Art. 36; but in ſetting them down take Care to change the Signs of thoſe Quantities that are to be ſubſtracted, by Art. 7. and this will be the remainder required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
From	$2\sqrt{da}$	$2m\sqrt{dp}$	$5y\sqrt{a}$
Subſtract	$3\sqrt{m}$	$y\sqrt{z}$	$-3\sqrt{b}$
Remains	$2\sqrt{da}-3\sqrt{m}$	$2m\sqrt{dp}-y\sqrt{z}$	$5y\sqrt{a}+3\sqrt{b}$

Exam. 1. The Letters under the radical Signs being different therefore place down $2\sqrt{da}$, and becauſe $3\sqrt{m}$ the Quantity to be ſubſtracted has the Sign $+$, therefore after the $2\sqrt{da}$ place the Sign $-$, and after that the Quantity $3\sqrt{m}$, and we have $2\sqrt{da}-3\sqrt{m}$ for the remainder required.

Exam. 2. Becauſe the Letters under the radical Signs are different therefore put down $2m\sqrt{dp}$, and becauſe $y\sqrt{z}$ the Quantity to be ſubſtracted has the Sign $+$, therefore after $2m\sqrt{dp}$ put the Sign $-$, and after that $y\sqrt{z}$, and we have $2m\sqrt{dp}-y\sqrt{z}$ for the remainder required.

Exam. 3. Becauſe the Letters under the radical Signs are different, therefore put down $5y\sqrt{a}$, but as $-3\sqrt{b}$ the Quantity to be ſubſtracted has the Sign $-$, therefore after $5y\sqrt{a}$ put the Sign $+$, and after that $3\sqrt{b}$, and we have $5y\sqrt{a}+3\sqrt{b}$ for the remainder required.

	Exam. 4.	*Exam.* 5.
From	$m\sqrt{da+p}$	$-5y\sqrt{a}$
Subſtract	$2\sqrt{a}$	$-d\sqrt{b}$
Remains	$m\sqrt{da+p}:-2\sqrt{a}$	$-5y\sqrt{a}+d\sqrt{b}$

Exam. 6.

From	$5m\sqrt{a}$
Subſtract	$-z\sqrt{p+q}$
Remains	$5m\sqrt{a}+z\sqrt{p+q}$

Exam. 4. Becauſe the Letters under the radical Signs are different, put down $m\sqrt{da+p}$, but as $2\sqrt{a}$ the Quantity to be ſubſtracted has the Sign $+$, therefore after $m\sqrt{da+p}$ put the Sign $-$, and after that $2\sqrt{a}$, and we have $m\sqrt{da+p}:-2\sqrt{a}$ for the remainder required.

Exam. 5. Becauſe the Letters under the radical Signs are different, put down $-5y\sqrt{a}$, but as $-d\sqrt{b}$ the Quantity to be ſubſtracted has the Sign $-$, therefore after the $-5y\sqrt{a}$ put the Sign $+$, and after that $d\sqrt{b}$, and we have $-5y\sqrt{a}+d\sqrt{b}$ for the remainder required.

Exam. 6. Becauſe the Letters under the radical Signs are different, put down $5m\sqrt{a}$, but as $-z\sqrt{p+q}$ has the Sign $-$ before it, therefore after $5m\sqrt{a}$ put the Sign $+$, and after that $z\sqrt{p+q}$, and we have $5m\sqrt{a}+z\sqrt{p+q}$ for the remainder required.

From	$5\sqrt{a+p}$	$m\sqrt{p}$
Subſtract	$m\sqrt{y}$	$-y\sqrt{da-p}$
Remains	$5\sqrt{a+p}:-m\sqrt{y}$	$m\sqrt{p}+y\sqrt{da-p}$
From	$3u\sqrt{d+p}$	$-5n\sqrt{da}$
Subſtract	$2n\sqrt{z-y}$	$-3\sqrt{m}$
Remains	$3u\sqrt{d+p}:-2n\sqrt{z-y}$	$-5n\sqrt{da}+3\sqrt{m}$
From	$-5\sqrt{p+z}$	$14\sqrt{da}$
Subſtract	$3n\sqrt{m}$	$7\sqrt{p+y}$
Remains	$-5\sqrt{p+z}:-3n\sqrt{m}$	$14\sqrt{da}:-7\sqrt{p+y}$

The Truth of theſe Operations are proved in the ſame Manner as in the laſt Article, by adding the remainder to the Quantity that was ſubſtracted; and if their Sum makes the Quantity from which the other was taken, the Work is true, if not there is a miſtake.

Thus at *Example* 1, the remainder is $2\sqrt{da}-3\sqrt{m}$
To which if we add the Quantity that was ſubſtracted $3\sqrt{m}$

The Sum is $2\sqrt{da}$, the ſame in the given Exmaple. For in this $2\sqrt{da}$
Addition adding $+3\sqrt{m}$ to $-3\sqrt{m}$, the Co-efficients and Quantities being the ſame and the Signs contrary, they deſtroy one another or go out of the Work, by Art. 5.

Again at *Example* 5, the remainder is $-5y\sqrt{a}+d\sqrt{b}$
To which if we add the Quantity that was ſubſtracted $-d\sqrt{b}$

The Sum is $-5y\sqrt{a}$, the ſame as in the given Example. For here $-5y\sqrt{a}$
$-d\sqrt{b}$ being added to $d\sqrt{b}$ or $+d\sqrt{b}$, they deſtroy one another as in the laſt Inſtance. In like Manner the Reader may prove any of the other Examples.

Multiplication of Surd Quantities.

39. *Caſe* 1. When there are no rational Quantities joined to the Surd Quantities, then multiply the Surd Quantities, as in Multiplication of *Algebra*, but to their Product prefix the radical Sign.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Multiply	$\sqrt{a}$	$\sqrt{mn}$	$\sqrt{py}$	$\sqrt{zx}$
By	$\sqrt{m}$	$\sqrt{d}$	$\sqrt{z}$	$\sqrt{a}$
Product	$\sqrt{am}$	$\sqrt{mnd}$	$\sqrt{pyz}$	$\sqrt{zxa}$

Exam. 1. Multiplying a by m, the Product is am, to which prefixing the Sign $\surd$, we have $\sqrt{am}$ the Product required.

Exam. 2. Multiplying mn by d, the Product is mnd, to which prefixing the Sign $\surd$, we have $\sqrt{mnd}$ the Product required.

Exam. 3. Multiplying py by z, the Product is pyz, to which prefixing the Sign $\surd$, we have $\sqrt{pyz}$ the Product required.

Exam. 4. Multiplying zx by a, the Product is zxa, to which prefixing the Sign $\surd$, we have $\sqrt{zxa}$ the Product required.

	Exam. 5.	*Exam.* 6.
Multiply	$\sqrt{pa}$	$\sqrt{zy}$
By	$\sqrt{z}$	$\sqrt{x}$
Product	$\sqrt{paz}$	$\sqrt{zyx}$

	Exam. 7.	*Exam.* 8.
Multiply	$\sqrt{a+b}$	$\sqrt{mn-z}$
By	$\sqrt{y}$	$\sqrt{a}$
Product	$\sqrt{ay+yb}$	$\sqrt{mna-az}$

Exam. 7. Multiplying $a+b$ by y, the Product is $ay+yb$, by Art. 10. to which prefixing the Sign $\surd$, and drawing it over all the Quantities, we have $\sqrt{ay+yb}$ the Product required.

Exam. 8. Multiplying $mn - z$ by a, the Product is $mna - az$, by Art. 10 and 16. to which prefixing the radical Sign as in the laſt Example, we have $\sqrt{mna - az}$ the Product required.

	Exam. 9.	*Exam.* 10.	*Exam.* 11.
Multiply	$\sqrt{ap + z}$	$\sqrt{az - ap}$	$\sqrt{d - y}$
By	$\sqrt{y}$	$\sqrt{m}$	$\sqrt{p}$
Product	$\sqrt{apy + yz}$	$\sqrt{azm - apm}$	$\sqrt{dp - py}$

Exam. 9. Multiplying $ap + z$ by y, the Product is $apy + yz$, to which prefixing the radical Sign, we have $\sqrt{apy + yz}$ the Product required.

Exam. 10. Multiplying $az - ap$ by m, the Product is $azm - apm$, to which prefixing the radical Sign, we have $\sqrt{azm - apm}$ the Product required.

Exam. 11. Multiplying $d - y$ by p, the Product is $dp - py$, to which prefixing the radical Sign, we have $\sqrt{dp - py}$ the Product required.

Multiply	$\sqrt{ab}$	$\sqrt{zy}$	$\sqrt{ap - z}$
By	$\sqrt{a - p}$	$\sqrt{d + y}$	$\sqrt{d}$
Product	$\sqrt{aab - abp}$	$\sqrt{zyd + zyy}$	$\sqrt{apd - dz}$

Multiply	$\sqrt{ap + z}$	$\sqrt{ay}$	$\sqrt{m}$
By	$\sqrt{m}$	$\sqrt{d - z}$	$\sqrt{a - py}$
Product	$\sqrt{apm + zm}$	$\sqrt{ayd - ayz}$	$\sqrt{ma - mpy}$

40. *Caſe* 2. When there are rational Quantities joined to the Surds, then multiply the rational Quantities together as in Multiplication of *Algebra*, after which multiply the Surd Quantities together by the laſt *Article*, and joining theſe two Products, this will be the Product required.

And when there are no rational Quantities prefixt, then *Unity*, or 1, is always ſuppoſed to be the rational Quantity prefixt.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Multiply	$a\sqrt{m}$	$ap\sqrt{z}$	$3\sqrt{mn}$	$\sqrt{mp}$
By	$d\sqrt{y}$	$2\sqrt{a}$	$a\sqrt{p}$	$y\sqrt{d}$
Product	$ad\sqrt{my}$	$2ap\sqrt{za}$	$3a\sqrt{mnp}$	$y\sqrt{mpd}$

Exam. 1. Multiplying the rational Quantities a and d, the Product is $a\,d$, and multiplying the Surds $\sqrt{m}$ by $\sqrt{y}$, the Product is $\sqrt{my}$; joining this to the rational Quantity $a\,d$, we have $a\,d\sqrt{my}$ for the Product required.

Exam. 2. Multiplying the rational Quantities ap by 2, the Product is $2ap$, and multiplying the Surds $\sqrt{z}$ by $\sqrt{a}$, the Product is $\sqrt{za}$; joining theſe we have $2ap\sqrt{za}$ for the Product required.

Exam. 3. Multiplying the rational Quantities 3 and a, the Product is $3a$, and multiplying the Surds $\sqrt{mn}$ by $\sqrt{p}$, the Product is $\sqrt{mnp}$; joining theſe we have $3a\sqrt{mnp}$ the Product required.

Exam. 4. Multiplying the rational Quantities y and 1, (for 1 is the rational Quantity of $\sqrt{mp}$, there being no rational Quantity prefixt) the Product is y, and multiplying the Surds $\sqrt{mp}$ by $\sqrt{d}$, the Product is $\sqrt{mpd}$, and joining theſe we have $y\sqrt{mpd}$ the Product required.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
Multiply	$am\sqrt{p}$	$y\sqrt{pq}$	$m\sqrt{2p}$	$2a\sqrt{3z}$
By	$z\sqrt{d}$	$d\sqrt{z}$	$4y\sqrt{y}$	$3d\sqrt{4y}$
Product	$amz\sqrt{pd}$	$yd\sqrt{pqz}$	$4my\sqrt{2py}$	$6ad\sqrt{12zy}$

Exam. 5. The Product of the rational Quantities is amz, and the Product of the Surds is $\sqrt{pd}$, theſe being joined we have $amz\sqrt{pd}$ the Product required.

Exam. 6. The Product of the rational Quantities is yd, and the Product of the Surds is $\sqrt{pqz}$, theſe being joined we have $yd\sqrt{pqz}$ the Product required.

Exam. 7. The Product of the rational Quantities is $4my$, and the Product of the Surds is $\sqrt{2py}$, theſe being joined we have $4my\sqrt{2py}$ the Product required.

Exam. 8. The Product of the rational Quantities is $6ad$, and the Product of the Surds is $\sqrt{12zy}$, theſe being joined we have $6ad\sqrt{12zy}$ the Product required.

	Exam. 9.	*Exam.* 10.	*Exam.* 11.	*Exam.* 12.
Multiply	$y\sqrt{p}$	$\sqrt{mn}$	$2\sqrt{dx}$	$3\sqrt{2z}$
By	$a\sqrt{z}$	$a\sqrt{y}$	$\sqrt{z}$	$5\sqrt{7y}$
Product	$ya\sqrt{pz}$	$a\sqrt{mny}$	$2\sqrt{dxz}$	$15\sqrt{14zy}$

	Exam. 13.	*Exam.* 14.	*Exam.* 15.
Multiply	$m\sqrt{a+y}$	$d\sqrt{m-pz}$	$a\sqrt{ap+z}$
By	$a\sqrt{p}$	$y\sqrt{d}$	$x\sqrt{y}$
Product	$ma\sqrt{ap+py}$	$dy\sqrt{md-pzd}$	$ax\sqrt{apy+yz}$

Exam. 13. Multiplying the rational Quantities m and a, the Product is ma, and multiplying $a+y$ by p, the Product is $ap+py$, but prefixing to this the Sign $\sqrt{}$, becauſe they are Surds, we have $\sqrt{ap+py}$ for the Product of the Surds, which joining to ma the Product of the rational Quantities, we have $ma\sqrt{ap+py}$ the Product required.

Exam. 14. Multiplying the rational Quantities d and y, the Product is dy, and multiplying the Surds $\sqrt{m-pz}$ by $\sqrt{d}$, the Product is $\sqrt{md-pzd}$, which being joined to dy, the Product of the rational Quantities, we have $dy\sqrt{md-pzd}$ the Product required.

Exam. 15. The Product of the rational Quantities is ax, and the Product of the Surds is $\sqrt{apy+yz}$, and theſe being joined we have $ax\sqrt{apy+yz}$ the Product required.

	Exam. 16.	*Exam.* 17.
Multiply	$am\sqrt{py+d}$	$2\sqrt{am-y}$
By	$y\sqrt{z}$	$a\sqrt{p}$
Product	$amy\sqrt{pyz+zd}$	$2a\sqrt{amp-py}$

	Exam. 18.
Multiply	$m\sqrt{pd}$
By	$a\sqrt{d-a}$
Product	$ma\sqrt{pdd-pda}$

Exam. 16. The rational Quantities am and y, being multiplied, the Product is amy, and $py+d$ being multiplied by z, the Product is $pyz+zd$; but before it prefix the radical Sign, becauſe theſe Quantities are Surds, then it is $\sqrt{pyz+zd}$, joining this to the Product amy, we have $amy\sqrt{pyz+zd}$ the Product required.

Exam. 17. The Product of the rational Quantities 2 and a, is $2a$, and the Product of the Surds is $\sqrt{amp-py}$: joining theſe we have $2a\sqrt{amp-py}$ the Product required.

Exam. 18. The Product of the rational Quantities m and a, is ma, and pd multiplied into $d-a$, is $pdd-pda$, to which prefix the radical Sign becauſe theſe are Surds, and this becomes $\sqrt{pdd-pda}$, now joining it to ma we have $ma\sqrt{pdd-pda}$ the Product required.

Multiply	$a\sqrt{p-y}$	$3\sqrt{m-n}$	$\sqrt{n+b}$
By	$2b\sqrt{m}$	$2\sqrt{5a}$	$a\sqrt{z}$
Product	$2ab\sqrt{pm-my}$	$6\sqrt{5ma-5na}$	$a\sqrt{zn+zb}$

Multiply	$2a\sqrt{3y+z}$	$y\sqrt{p-z}$
By	$b\sqrt{d}$	$3m\sqrt{a}$
Product	$2ab\sqrt{3yd+dz}$	$3ym\sqrt{pa-az}$

Multiply	$5\sqrt{y-x}$
By	$3a\sqrt{2b}$
Product	$15a\sqrt{2by-2bx}$

Diviſion of Surd Quantities.

41. *Caſe* 1. When there are no rational Quantities joined with the Surd Quantities, reject all thoſe Quantities in the Dividend and Diviſor that are alike, as at Art. 20. and ſet down the remainder, to which prefix the radical Sign.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Divide	$\sqrt{mn}$	$\sqrt{ma}$	$\sqrt{abd}$	$\sqrt{abd}$
By	$\sqrt{m}$	$\sqrt{a}$	$\sqrt{ab}$	$\sqrt{a}$
Quotient	$\sqrt{n}$	$\sqrt{m}$	$\sqrt{d}$	$\sqrt{bd}$

Exam. 1. Becauſe m is in both the Dividend and Diviſor, therefore reject it, and place down n, the remaining part of the Dividend, to which prefixing the radical Sign, we have $\sqrt{n}$ the Quotient required.

Exam. 2. Becauſe a is in both the Dividend and Diviſor, therefore reject it, and place down m, the remaining part of the Dividend, to which prefixing the radical Sign, we have $\sqrt{m}$ the Quotient required.

Exam. 3. Becauſe ab is in both the Dividend and Diviſor, reject it, and place down d, the remaining part of the Dividend, to which prefixing the radical Sign, we have $\sqrt{d}$ the Quotient required.

Exam. 4. Becauſe a is in both Dividend and Diviſor, reject it, and place down bd, the remaining part of the Dividend, to which prefixing the radical Sign, we have $\sqrt{bd}$ the Quotient required.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
Divide	$\sqrt{mdy}$	$\sqrt{bzd}$	$\sqrt{bzd}$	$\sqrt{ypa}$
By	$\sqrt{y}$	$\sqrt{bd}$	$\sqrt{zd}$	$\sqrt{yp}$
Quotient	$\sqrt{md}$	$\sqrt{z}$	$\sqrt{b}$	$\sqrt{a}$

Exam. 5. Becauſe y is in both the Dividend and Diviſor, reject it, and place down md, with the Sign $\sqrt{}$ before it, and we have $\sqrt{md}$ the Quotient required.

Exam. 6. Becauſe bd is in both the Dividend and Diviſor, reject it, and place down z, with the Sign $\sqrt{}$ before it, and we have $\sqrt{z}$ the Quotient required.

Exam. 7. Becauſe zd is in both the Dividend and Diviſor, reject it, and place down b, with the Sign $\sqrt{}$ before it, and we have $\sqrt{b}$ the Quotient required.

Exam. 8. Becauſe yp is in both the Dividend and Diviſor, reject it, and place down a, with the Sign $\sqrt{}$ before it, and we have $\sqrt{a}$ the Quotient required.

	Exam. 9.	*Exam.* 10.	*Exam.* 11.	*Exam.* 12.
Divide	$\sqrt{max}$	$\sqrt{ndy}$	$\sqrt{ayz}$	$\sqrt{abd}$
By	$\sqrt{x}$	$\sqrt{ny}$	$\sqrt{a}$	$\sqrt{ad}$
Quotient	$\sqrt{ma}$	$\sqrt{d}$	$\sqrt{yz}$	$\sqrt{b}$

	Exam. 13.	*Exam.* 14.	*Exam.* 15.
Divide	$\sqrt{am+ap}$	$\sqrt{py-pn}$	$\sqrt{bd-bm}$
By	$\sqrt{a}$	$\sqrt{p}$	$\sqrt{b}$
Quotient	$\sqrt{m+p}$	$\sqrt{y-n}$	$\sqrt{d-m}$

Exam. 13. If we divide $am+ap$ by a, the Quotient is $m+p$, by Art. 22. but becauſe they are Surds, prefix the Sign $\surd$ to $m+p$, and we have $\sqrt{m+p}$ the Quotient required.

Exam. 14. Dividing $py-pn$ by p, the Quotient is $y-n$, by Art. 22 and 24. to which prefixing the Sign $\surd$, we have $\sqrt{y-n}$ the Quotient required.

Exam. 15. Divinding $bd-bm$ by b, the Quotient is $d-m$, by Art. 22, and 24, to which prefixing the Sign $\surd$, we have $\sqrt{d-m}$ the Quotient required.

Divide	$\sqrt{bn+ba}$	$\sqrt{mx-md}$	$\sqrt{nz-np}$
By	$\sqrt{b}$	$\sqrt{m}$	$\sqrt{n}$
Quotient	$\sqrt{n+a}$	$\sqrt{x-d}$	$\sqrt{z-p}$

Divide	$\sqrt{zx-zy}$	$\sqrt{ad+ay}$	$\sqrt{bd-bm}$
By	$\sqrt{z}$	$\sqrt{a}$	$\sqrt{b}$
Quotient	$\sqrt{x-y}$	$\sqrt{d+y}$	$\sqrt{d-m}$

The Truth of theſe Operations are proved by multiplying the Quotient by the Diviſor, for if that produces the Dividend, the Work is true, otherwiſe ſome error has been made. Thus in Example 2, the Diviſor is $\sqrt{a}$, and the Quotient is $\sqrt{m}$, which being multiplied by Art. 39. the Product is $\sqrt{ma}$ the given Dividend.

And at Example 6, the Diviſor is $\sqrt{bd}$, and the Quotient is $\sqrt{z}$, which being multiplied by Art. 39. the Product is $\sqrt{bzd}$ the given Dividend.

And at Example 13, the Diviſor is $\sqrt{a}$, and the Quotient is $\sqrt{m+p}$, which being multiplied by Art. 39. the Product is $\sqrt{am+ap}$ the given Dividend; in the ſame Manner may any of the other Examples be proved.

42. *Caſe* 2. When there are rational Quantities joined with the Surds, divide the rational Quantities by the rational Quantities, by the Rules in Diviſion of *Algebra*; and to their Quotient, join the Quotient of the Surds found by the laſt Article, which will be the Quotient required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Divide	$ay\sqrt{mn}$	$bm\sqrt{yz}$	$yd\sqrt{az}$	$ma\sqrt{ayn}$
By	$a\sqrt{m}$	$m\sqrt{z}$	$y\sqrt{a}$	$a\sqrt{ay}$
Quotient	$y\sqrt{n}$	$b\sqrt{y}$	$d\sqrt{z}$	$m\sqrt{n}$

Exam. 1. Dividing the rational Quantities ay by a, the Quotient is y, by Art. 20. and dividing $\sqrt{mn}$ by $\sqrt{m}$, the Quotient is $\sqrt{n}$, by Art. 41. now joining y to $\sqrt{n}$, we have $y\sqrt{n}$ the Quotient required.

Exam. 2. Dividing the rational Quantities bm by m, the Quotient is b, by Art. 20. and dividing $\sqrt{yz}$ by $\sqrt{z}$, the Quotient is $\sqrt{y}$, now joining b and $\sqrt{y}$, we have $b\sqrt{y}$ the Quotient required.

Exam. 3. Dividing the rational Quantities yd by y, the Quotient is d, by Art. 20. and dividing $\sqrt{az}$ by $\sqrt{a}$, the Quotient is $\sqrt{z}$, by Art. 41. now joining d and $\sqrt{z}$, we have $d\sqrt{z}$ the Quotient required.

Exam. 4. Dividing the rational Quantities ma by a, the Quotient is m, by Art. 20. and dividing $\sqrt{ayn}$ by $\sqrt{ay}$, the Quotient is $\sqrt{n}$, by Art. 41. now joining m and $\sqrt{n}$, we have $m\sqrt{n}$ the Quotient required.

	Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
Divide	$ayn\sqrt{mn}$	$mn\sqrt{xay}$	$xa\sqrt{nd}$	$dz\sqrt{anp}$
By	$ay\sqrt{m}$	$n\sqrt{xy}$	$a\sqrt{n}$	$z\sqrt{an}$
Quotient	$n\sqrt{n}$	$m\sqrt{a}$	$x\sqrt{d}$	$d\sqrt{p}$

Exam. 5. Dividing the rational Quantities ayn by ay, the Quotient is n, by Art. 20. and dividing $\sqrt{mn}$ by $\sqrt{m}$, the Quotient is $\sqrt{n}$, by Art. 41. now joining n and $\sqrt{n}$, we have $n\sqrt{n}$ the Quotient required.

Exam. 6. Dividing the rational Quantities mn by n, the Quotient is m, by Art. 20. and dividing $\sqrt{xay}$ by $\sqrt{xy}$, the Quotient is $\sqrt{a}$, by Art. 41. now joining m and $\sqrt{a}$, we have $m\sqrt{a}$ the Quotient required.

Exam. 7. Dividing the rational Quantities xa by a, the Quotient is x, and dividing $\sqrt{nd}$ by $\sqrt{n}$, the Quotient is $\sqrt{d}$, by Art. 41. now joining x and $\sqrt{d}$, we have $x\sqrt{d}$ the Quotient required.

Exam. 8. Dividing the rational Quantities dz by z, the Quotient is d, and dividing $\sqrt{anp}$ by $\sqrt{an}$, the Quotient is $\sqrt{p}$, by Art. 41. now joining d and $\sqrt{p}$, we have $d\sqrt{p}$ the Quotient required.

	Exam. 9.	*Exam.* 10.	*Exam.* 11.	*Exam.* 12.
Divide	$4mn\sqrt{ab}$	$my\sqrt{az}$	$dn\sqrt{xy}$	$8an\sqrt{rd}$
By	$2m\sqrt{a}$	$y\sqrt{z}$	$n\sqrt{x}$	$4a\sqrt{r}$
Quotient	$2n\sqrt{b}$	$m\sqrt{a}$	$d\sqrt{y}$	$2n\sqrt{d}$

Exam.

	Exam. 13.	*Exam.* 14.	*Exam.* 15.	*Exam.* 16.
Divide	$mx\sqrt{pq}$	$4an\sqrt{rd}$	$xz\sqrt{myp}$	$rm\sqrt{dz}$
By	$x\sqrt{p}$	$a\sqrt{d}$	$z\sqrt{my}$	$r\sqrt{d}$
Quotient	$m\sqrt{q}$	$4n\sqrt{r}$	$x\sqrt{p}$	$m\sqrt{z}$

	Exam. 17.	*Exam.* 18.	*Exam.* 19.
Divide	$mn\sqrt{ap+ax}$	$yp\sqrt{zd+zm}$	$day\sqrt{ym+yr}$
By	$m\sqrt{a}$	$p\sqrt{z}$	$da\sqrt{y}$
Quotient	$n\sqrt{p+x}$	$y\sqrt{d+m}$	$y\sqrt{m+r}$

Exam. 17. Dividing the rational Quantities mn by m, the Quotient is n, by Art. 20. and dividing $\sqrt{ap+ax}$ by $\sqrt{a}$, we have $\sqrt{p+x}$, by Art. 41. and joining n and $\sqrt{p+x}$, we have $n\sqrt{p+x}$ the Quotient required.

Exam. 18. Dividing the rational Quantities yp by p, the Quotient is y, by Art. 20. and dividing $\sqrt{zd+zm}$ by $\sqrt{z}$, the Quotient is $\sqrt{d+m}$, by Art. 41. and joining y and $\sqrt{d+m}$, we have $y\sqrt{d+m}$ the Quotient required.

Exam. 19. Dividing the rational Quantities day by da, the Quotient is y, by Art. 20. and dividing $\sqrt{ym+yr}$ by $\sqrt{y}$, the Quotient is $\sqrt{m+r}$, by Art. 41. and joining y and $\sqrt{m+r}$, we have $y\sqrt{m+r}$ the Quotient required. The following Examples are done in the ſame Manner.

Divide	$4an\sqrt{dy+dn}$	$an\sqrt{pz-pb}$	$6bdy\sqrt{pm+pd}$
By	$2a\sqrt{d}$	$a\sqrt{p}$	$3bd\sqrt{p}$
Quotient	$2n\sqrt{y+n}$	$n\sqrt{z-b}$	$2y\sqrt{m+d}$
Divide	$pn\sqrt{dx-db}$	$12ba\sqrt{py-px}$	$anx\sqrt{pd-pm}$
By	$n\sqrt{d}$	$3a\sqrt{p}$	$ax\sqrt{p}$
Quotient	$p\sqrt{x-b}$	$4b\sqrt{y-x}$	$n\sqrt{d-m}$

The Truth of theſe Operations are proved likewiſe from multiplying the Quotient by the Diviſor, and if that Product makes the Dividend the Work is true, if not there is a miſtake. Thus in

Exam. 1. the Quotient is $y\sqrt{n}$, and the Diviſor is $a\sqrt{m}$; now multiplying $y\sqrt{n}$ by $a\sqrt{m}$, by Art. 39. firſt multiply the rational Quantities y and a, this Product is ay, and multiplying $\sqrt{n}$ by $\sqrt{m}$, this Product is $\sqrt{mn}$, and joining this to ay we have $ay\sqrt{mn}$ the Product, which being the ſame as the given Dividend, proves the Work to be true.

And at *Example* 5. the Diviſor is $ay\sqrt{m}$, and the Quotient is $n\sqrt{n}$, now multiplying $ay\sqrt{m}$ by $n\sqrt{n}$ according to Art. 39. we firſt multiply the rational Quantities ay by n, and this Product is ayn; then multiplying $\sqrt{m}$ by $\sqrt{n}$, this Product is $\sqrt{mn}$, and joining this to ayn, we have for the Product $ayn\sqrt{mn}$, which being the ſame with the given Dividend, the Work is true.

And at *Example* 17. the Diviſor is $m\sqrt{a}$, and the Quotient is $n\sqrt{p+x}$, and multiplying theſe by Art. 39. we firſt multiply the rational Quantities m and n together, and this Product is mn, then multiplying $\sqrt{a}$ by $\sqrt{p+x}$, this Product is $\sqrt{ap+ax}$, which being joined to mn, the Product is $mn\sqrt{ap+ax}$, the ſame as the given Dividend, and ſo may any of the other Examples be proved.

Involution of Surd Quantities.

43. *Caſe* 1. When there are no rational Quantities joined with the Surds, it is only ſetting the Quantities down without their radical Sign, which raiſes the given Root as high as is the Index of the radical Sign.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Raiſe to the Square or ſecond Power	$\sqrt{a}$	$\sqrt{mn}$	$\sqrt{na}$	$\sqrt{b}$
The Square	a	mn	na	b

This being no more according to the Rule but to ſet down the Quantities without their radical Sign, it is ſo eaſy as not to need any farther Explanation.

The Reaſon on which the Operation is founded is that any Quantity or Number being multiplied by itſelf, will produce the Square of that Quantity or Number, thus $2 \times 2 = 4$, whence 4 is the Square of 2, and $a \times a = aa$, which is the Square of a, and ſo of any other Quantity. Now ſuppoſing the Square Root of a, was to be extracted, which by Art. 33. is $\sqrt{a}$. But as $\sqrt{a}$ is the Root, and a was the Square from which that Root was extracted, hence $\sqrt{a}$ multiplied into $\sqrt{a}$, muſt produce a, by what has been juſt ſaid: Now $\sqrt{a}$ multiplied by $\sqrt{a}$, is $\sqrt{aa}$ by Art. 39. and as $\sqrt{aa}$ ſignifies the Square Root of aa, which is a, by Art. 33. it follows that to involve any Surd that has no rational Quantities joined with it, is only to ſet down the Quantities without their radical Sign.

To find the Square or ſecond Power of	$\sqrt{ax}$	$\sqrt{nd}$	$\sqrt{pr}$	$\sqrt{z}$
The Square	ax	nd	pr	z

And if there are ſeveral Quantities connected by the Signs + or —, and are all under the radical Sign, they are involved in the ſame Manner.

Raiſe to the ſecond Power or Square	$\sqrt{\overline{a+b}}$	$\sqrt{\overline{an-d}}$	$\sqrt{\overline{p+ny}}$
The Square	$a+b$	$an-d$	$p+ny$

Raiſe to the ſecond Power or Square	$\sqrt{\overline{pd-n}}$	$\sqrt{\overline{dz+zy}}$	$\sqrt{\overline{pm-nd}}$
The Square	$pd-n$	$dz+zy$	$pm-nd$

Raiſe to the 2d Power or Squ.	$\sqrt{\overline{a+y-d}}$	$\sqrt{\overline{am-n+db}}$	$\sqrt{\overline{pz+zx-xd}}$
The Square	$a+y-d$	$am-n+db$	$pz+zx-xd$

44. *Caſe* 2. When there are rational Quantities joined with the Surds, then involve the rational Quantities as high as the Index of the Surd denotes, and multiply theſe involved Quantities into the Surd Quantities, after the radical Sign is taken away.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.
Raiſe to the Square	$a\sqrt{m}$	$b\sqrt{nz}$	$d\sqrt{y}$	$zz\sqrt{b}$
The Square	aam	$bbnz$	ddy	$zzzzb$

Ex. 1. The rational Quantity a being ſquared is by Art. 31. aa

The Surd Quantity $\sqrt{m}$ being put down without the radical Sign is — — m

Theſe being multiplied the Prod. is the Square required aam

Ex. 2. The rational Quantity b being ſquared is by Art. 31. bb
The Surd Quantity $\sqrt{nz}$ without the radical Sign is - - nz
Theſe being mutiplied the Prod. is the Square required $bbnz$

Exam. 3. The rational Quantity d being ſquared is - - dd
The Surd Quantity $\sqrt{y}$ without the radical Sign is - - y
Theſe being multiplied the Prod. is the Square required ddy

Exam. 4. The rational Quantity zz being ſquared is $zzzz$
The Surd Quantity $\sqrt{b}$ without the radical Sign is - - b
Theſe being multiplied the Prod. is the Squ. required $zzzzb$

	Exam. 5.	*Exam.* 6.	*Exam.* 7.	*Exam.* 8.
Raiſe to the Squ.	$an\sqrt{p}$	$dz\sqrt{yx}$	$p\sqrt{xy}$	$da\sqrt{z}$
The Square	$aannp$	$ddzzyx$	$ppxy$	$ddaaz$

Exam. 5. The rational Quantity an being ſquared is $aann$
The Surd Quantity $\sqrt{p}$ without the radical Sign is - - p
Theſe being multiplied the Prod. is the Squ. required $aannp$

Exam. 6. The rational Quantity dz being ſquared is $ddzz$
The Surd Quantity $\sqrt{yx}$ without the radical Sign is - - yx
Theſe being multiplied the Prod. is the Squ. required $ddzzyx$

Exam. 7. The rational Quantity p being ſquared is - - pp
The Surd Quantity $\sqrt{xy}$ without the radical Sign is - - xy
Theſe being multiplied the Prod. is the Squ. required $ppxy$

Exam. 8. The rational Quantity da being ſquared is $ddaa$
The Surd Quantity $\sqrt{z}$ without the radical Sign is - - z
Theſe being multiplied the Prod. is the Squ. required $ddaaz$

Raiſe to the Squ.	$m\sqrt{pz}$	$mn\sqrt{d}$	$a\sqrt{rd}$	$py\sqrt{m}$
The Square	$mmpz$	$mmnnd$	$aard$	$ppyym$

Raiſe to the Square	$x\sqrt{pd}$	$xn\sqrt{a}$	$z\sqrt{px}$	$az\sqrt{d}$
The Square	$xxpd$	$xxnna$	$zzpx$	$aazzd$

And if there are more Quantities than one under the radical Sign, connected with the Signs + or —, then after the rational Quantities are involved, or raiſed as high as the Index of the

Surd

Surd requires; place theſe under the radical Quantities, taking away their Sign, then multiply them by the Rules of Multiplication at Art. 9, *&c.* and this will be the Square required.

	Exam. 1.	*Exam.* 2.	*Exam.* 3.
Raiſe to the Square	$a\sqrt{m+y}$	$b\sqrt{d+z}$	$m\sqrt{z-x}$
The Square	$aam+aay$	$bbd+bbz$	$mmz-mmx$

Exam. 1. The Surd Quantity $\sqrt{m+y}$ without the radical Sign is - - - $m+y$
The rational Quantity a being ſquared is - - - - aa
Theſe being multiplied according to Art. 10. the Product is the Square required. $aam+aay$

Exam. 2. The Surd Quantity $\sqrt{d+z}$ without the radical Sign is - - - $d+z$
The rational Quantity b being ſquared is - - - bb
Theſe being multip. the Prod. is the Squ. required $bbd+bbz$

Exam. 3. The Surd Quantity $\sqrt{z-x}$ without the radical Sign is - - - $z-x$
The rational Quantity m being ſquared is - - - mm
Theſe being multiplied the Product is the Square required - - $mmz-mmx$

	Exam. 4.	*Exam.* 5.	*Exam.* 6.
Raiſe to the Square	$z\sqrt{a+n}$	$x\sqrt{b-d}$	$d\sqrt{z+y}$
The Square	$zza+zzn$	$xxb-xxd$	$ddz+ddy$

Exam. 4. The Surd Quantity $\sqrt{a+n}$ without the radical Sign is - - - $a+n$
The rational Quantity z being ſquared is - - - zz
Theſe being multiplied the Product is the Square required - - $zza+zzn$

Exam. 5. The Surd Quantity $\sqrt{b-d}$ without the radical Sign is - - - $b-d$
The rational Quantity x being ſquared is - - - - xx
Theſe being multiplied the Product is the Square required - - $xxb-xxd$

Exam. 6. The Surd Quantity $\sqrt{z+y}$ without the radical Sign is - - - $z+y$

The rational Quantity d being ſquared is - - - - dd

Theſe being multiplied the Product is the Square required - - $ddz+ddy$

Raiſe to the Square	$y\sqrt{a-n}$	$n\sqrt{a+d}$	$d\sqrt{p-z}$
The Square	$yya-yyn$	$nna+nnd$	$ddp-ddz$

Raiſe to the Square	$e\sqrt{p-r}$	$d\sqrt{e+y}$	$z\sqrt{n-y}$
The Square	$eep-eer$	$dde+ddy$	$zzn-zzy$

45. *Caſe* 3. But if there are rational Quantities connected with the Surd Quantities by the Signs + or —, they are involved in the ſame Manner as compound Quantities, at Art. 32. and carefully obſerving the Directions concerning the Multiplication of Surd Quantities, at Art. 40. and their Involation at Art. 43.

To raiſe to the Square or ſecond Power - - - $a+\sqrt{b}$

Putting down again the ſame Quantity - - - $a+\sqrt{b}$

The Prod. from multiplying $a+\sqrt{b}$ by a, for a multiplied by a, the Prod. is aa, and $\sqrt{b}$ multiplied by a, the Prod. is $a\sqrt{b}$, by Art. 40. $aa+a\sqrt{b}$

The Prod. from multiplying $a+\sqrt{b}$ by $\sqrt{b}$, for a multiplied by $\sqrt{b}$, the Prod. is $a\sqrt{b}$, by Art. 40. and $\sqrt{b}$ multip. by $\sqrt{b}$, the Prod. is b, by Art. 43. $a\sqrt{b}+b$

The Sum is $aa+2a\sqrt{b}+b$: for $a\sqrt{b}$ added to $a\sqrt{b}$ is $2a\sqrt{b}$, by Art. 35. that the Square of $a+\sqrt{b}$ is $aa+2a\sqrt{b}+b$

To raiſe to the Square or ſecond Power - - - $d+\sqrt{z}$

Putting down again the ſame Quantity - - - $d+\sqrt{z}$

The Prod. from multiplying $d+\sqrt{z}$ by d, by what is mentioned in the laſt Example is $dd+d\sqrt{z}$

The Prod. from multiplying $d+\sqrt{z}$ by $\sqrt{z}$, by what is mentioned in the laſt Example is $d\sqrt{z}+z$

Their Sum added as in the laſt Example is the Square of $d+\sqrt{z}$ $dd+2d\sqrt{z}+z$

To raiſe to the Square or ſecond Power - - - - $x - \sqrt{a}$

Putting down again the ſame Quantity - - - - $x - \sqrt{a}$

The Prod. from multiplying $x - \sqrt{a}$ by x, for x multiplied by x, the Prod. is xx, and $-\sqrt{a}$ multiplied by x, the Prod. is $-x\sqrt{a}$, by Art. 40. the Signs of $-\sqrt{a}$ and x being different $xx - x\sqrt{a}$

The Product from multiplying $x - \sqrt{a}$ by $-\sqrt{a}$, for x multiplied by $-\sqrt{a}$, the Product is $-x\sqrt{a}$, the Signs being unlike, but $-\sqrt{a}$ multiplied by $-\sqrt{a}$, the Signs being alike the Product is $\sqrt{aa}$ or a, by Art. 39 and 43. $-x\sqrt{a} + a$

Their Sum is the Square of $x - \sqrt{a}$ $xx - 2x\sqrt{a} + a$

To raiſe to the Square or ſecond Power - - - - $y - \sqrt{x}$

Putting down again the ſame Quantity - - - - $y - \sqrt{x}$

The Product from multiplying $y - \sqrt{x}$ by y, from what is mentioned in the laſt Example is $yy - y\sqrt{x}$

The Prod. from multiplying $y - \sqrt{x}$ by $-\sqrt{x}$, from what is ſaid in the laſt Example $-y\sqrt{x} + x$

Their Sum is the Square of $y - \sqrt{x}$ - - $yy - 2y\sqrt{x} + x$

To raiſe to the Square or ſecond Power - - - - $b + \sqrt{xa}$

Putting down the ſame Quantity - - - - - $b + \sqrt{xa}$

The Prod. from multiplying $b + \sqrt{xa}$ by b $bb + b\sqrt{xa}$

The Prod. from mutiplying $b + \sqrt{xa}$ by $\sqrt{xa}$ $b\sqrt{xa} + xa$

The Sum being the Square of $b + \sqrt{xa}$ $bb + 2b\sqrt{xa} + xa$

To raiſe to the Square or ſecond Power - - - - $m + \sqrt{dz}$

Putting down the ſame Quantity - - - - - $m + \sqrt{dz}$

The Prod. from multiplying $m + \sqrt{dz}$ by m $mm + m\sqrt{dz}$

The Prod. from multiplying $m + \sqrt{dz}$ by $\sqrt{dz}$ $m\sqrt{dz} + dz$

The Sum being the Square of $m + \sqrt{dz}$ $mm + 2m\sqrt{dz} + dz$

To raiſe to the Square or ſecond Power - - - - $z - \sqrt{dn}$

Putting down the ſame Quantity - - - - $z - \sqrt{dn}$

$$zz - z\sqrt{dn}$$

$$-z\sqrt{dn} + dn$$

The Square of $z - \sqrt{dn}$ - - - $zz - 2z\sqrt{dn} + dn$

To raiſe to the Square or ſecond Power - - - $p-\sqrt{yz}$
Putting down again the ſame Quantity - - - $p-\sqrt{yz}$

$$\begin{array}{r} pp-p\sqrt{yz} \\ -p\sqrt{yz}+yz \\ \hline \end{array}$$

The Square of $p-\sqrt{yz}$ - - - $pp-2p\sqrt{yz}+yz$

Having thus copiouſly explained all the Rules neceſſary to be known in order to the Solution of Queſtions, we now come to their Uſe and Application in the Reduction of Equations, or the Method by which Problems are ſolved, and Queſtions anſwered.

When any Problem or Queſtion is propoſed to be anſwered in an *Analytic* Manner, for the ſeveral Numbers that are in the Queſtion we generally put Letters, repreſenting likewiſe the Numbers which are to be found by Letters, and for Diſtinction ſake uſe the *Vowels* for the unknown Numbers, and *Conſonants* for thoſe that are known.

Then we begin to expreſs all the Conditions of the Queſtion, by ranging and connecting the Letters in ſuch a Manner that they ſhall repreſent all the Circumſtances of the Queſtion, this being only to tranſlate the Queſtion from *Engliſh* into *Algebra*.

Thus 6 being added to 5, the Sum is *equal* to 11.
Now ſuppoſe $b=6$ $d=5$ $m=11$
Then the above Propoſition will be expreſſed in *Algebra*, Thus, $b+d=m$

And when any Letters or Numbers are ſo connected, that between any of them there appears this Sign $=$, it is called an Equation, for the Sign $=$ ſignifies *Equality* or *Equation*, and in the due ordering and managing theſe Equations conſiſts the whole of the *Analytic* Science or *Algebra*.

Equations conſiſt of Quantities or Letters ſome known, and others unknown, and the grand Work is ſo to manage the Equation by the Rules of Certainty and Science, that all the known Quantities may at laſt be found on one ſide of the Equation, and the unknown Quantity by itſelf on the other Side of the Equation: For when this is done the Equation is brought to a Solution, and the Queſtion is anſwered.

And that Part of *Algebra* which teaches how to manage theſe Quantities, ſo as to carry all the known Quantities on one Side, leaving the unknown Quantity by itſelf on the other Side of the Equation, is called the *Reduction of Equations*, which is done by *Addition*, *Subſtraction*, *Multiplication*, *Diviſion*, *Involution*, and *Evolution*, according as the Caſe requires.

To

To reduce an Equation, by Addition, or Subſtraction.

46. WHEN any known Quantities are on the ſame Side of the Equation with the unknown Quantity, and connected by the Signs + or —, to reduce ſuch an Equation is only to *tranſpoſe or place the known Quantities on the other Side of the Equation, or Sign of Equality, prefixing to them their contrary Sign*, that is, *thoſe Quantities which have the Sign + after they are tranſpoſed muſt have the Sign —, and thoſe which have the Sign — muſt have the Sign +.*

Queſtion 1. *To find that Number to which* 6 *being added, and ſubſtracting* 15 *from this Sum, the remainder may be equal to* 11.

Now ſuppoſe $a =$ the Number ſought $b = 6$ $d = 15$ $m = 11$.

Then I am to find a Number, which I call	1	a
To which 6 or b being added, it is by Art. 6.	2	$a+b$
From which Sum, 15 or d is to be ſubſtracted, that is, to $a+b$ connect d by the Sign —	3	$a+b-d$
Which $a+b-d$ is to be equal to 11 or m, that is	4	$a+b-d=m$
Now to reduce this Equation, or to anſwer the Queſtion, I obſerve d, a known Quantity, is on the ſame Side of the Equation with the unknown Quantity a, therefore tranſpoſe d, that is, write down the remaining Part of that Side of the Equation without d, and place it on the other Side with the Sign +, it having before the Sign —, then we have	5	$a+b=m+d$
Again b is a known Quantity on the ſame Side of the Equation with a, then by taking it away from that Side of the Equation, and placing it on the other Side with the contrary Sign, or —, we have	6	$a=m+d-b$

Here the Queſtion is ſolved, for the unknown Number or Quantity a, is equal to the Number repreſented by m, added to the Number repreſented by d, from which Sum ſubſtracting the Number repreſented by b.

11 repreſented by m
15 repreſented by d
26 Sum of the Numbers repreſented by m and d
6 repreſented by b
Remains 20 which is a, or the Number ſought

And that this is the Number required, is thus proved, from the Conditions of the Queſtion.

I ſay the Number ſought is — — 20
For if to this I add — — — 6
The Sum is — — — 26
From which ſubſtracting — — — 15
There remains as the Queſtion required — 11

Queſtion 2. *A Man being asked how many Shillings he had, ſaid if you add 15 to them I have, and then ſubſtract 20 from this Sum, and then add 19 to the remainder, I ſhall have 64 Shillings. How many Shillings had he?*

Let $a =$ the Number of Shillings he had, $b = 15$, $d = 20$, $m = 19$, $n = 64$.

Then, A Man had a certain Number of Shillings which I call	1	a
To which 15 or b being added we have, by Art. 6.	2	$a + b$
From which Sum taking away 20 or d, that is, connect d by the Sign —	3	$a + b - d$
To which adding 19 or m we have, by Art. 6.	4	$a + b - d + m$
Which $a + b - d + m$ is to be equal to 64 or n, hence	5	$a + b - d + m = n$
Now to reduce this Equation, or Anſwer the Queſtion: I begin with tranſpoſing m a known Quantity, by putting down the remaining part of that Side of the Equation, and placing m on the other Side with the contrary Sign, which gives	6	$a + b - d = n - m$

And to tranſpoſe *d* another known Quantity, put down the remaining Part of that Side of the Equation, and *d* on the other Side with a contrary Sign, whence we have	7	$a+b=n-m+d$
And laſtly by tranſpoſing *b*, that is, placing it on the other Side of the Equation with a contrary Sign, we have	8	$a=n-m+d-b$

That is, if from the Number repreſented by n we ſubſtract that repreſented by m, and to the remainder add the Number repreſented by d, and from this Sum ſubſtract the Number repreſented by b, the remainder will be the Number ſought.

64 repreſented by n
19 repreſented by m

45 or $n-m$
20 repreſented by d

65 or $n-m+d$
15 repreſented by b

50 the Number ſought or a; and therefore I ſay the Man had 50 *s.* at firſt, which is thus proved, from the Conditions of the Queſtions.

I ſay he had at firſt — — —	50 *s.*
For if to them you add — —	15
	65
And from that Sum ſubſtract — —	20
	45
And then add to the remainder —	19
It makes what the Queſtion requires —	64

Queſtion 3. *A Countryman asked another how many Eggs he had, why ſays he, if you ſubſtract* 15 *from them I have, and then add* 21 *to thoſe that are left, and ſubſtract* 7 *from that Sum, but if you add* 19 *to what is then left I ſhall have* 43 *Eggs. How many Eggs had he?*

Let $a =$ the Number of Eggs, $b = 15$, $d = 21$, $m = 7$, $n = 19$, $p = 43$.

Now the Countryman had a Numof Eggs which I call - -	1	a
From which 15 or b being ſubſtracted, or connecting b by the Sign —, we have - -	2	$a - b$
To which $a - b$, if we add 21 or d, we have by Art. 6. -	3	$a - b + d$
From which Sum ſubſtracting 7 or m, or connecting m by the Sign —	4	$a - b + d - m$
To which adding 19 or n, we have by Art. 6. - -	5	$a - b + d - m + n$
And this $a - b + d - m + n$ is to be equal to 43 or p, hence	6	$a - b + d - m + n = p$
Now to reduce this Equation, or anſwer the Queſtion, I begin with tranſpoſing n, by putting down the remaining Part of that Side of the Equation, and n on the other Side with its contrary Sign, then - -	7	$a - b + d - m = p - n$
Now tranſpoſe m, by putting down the remaining Part of that Side of the Equation, and m on the other Side with its contrary Sign, and we have - -	8	$a - b + d = p - n + m$
Then tranſpoſe d, by putting down the remaining Part of that Side of the Equation, and d on the other Side with its contrary Sign, then - - -	9	$a - b = p - n + m - d$
Laſtly, tranſpoſe b, by putting down the remaining Part of that Side of the Equation, and b on the other Side with its contrary Sign, and it is - -	10	$a = p - n + m - d + b$

Hence a, the unknown Quantity or Number of Eggs, is equal to that Number repreſented by p, ſubſtracting from it the Number repreſented by n, adding to this remainder the Number repreſented by m, ſubſtracting from this Sum the Number repreſented by d, and adding to this remainder the Number repreſented by b.

43 repreſented by p
19 repreſented by n

24 or $p - n$
7 repreſented by m

31 or $p - n + m$
21 repreſented by d

10 or $p - n + m - d$
15 repreſented by b

25 the Number ſought or a; and therefore I ſay the Man had 25 Eggs, which is thus proved, from the Conditions of the Queſtion.

I ſay the Man had - - - 25 Eggs
For if from them you ſubſtract - - 15

10
And to that remainder add - - - 21

31
And from this Sum ſubſtract - - - 7

24
And to this remainder add - - - 19
It makes what the Queſtion requires - - 43

Queſtion 4. *To find that Number to which* 19 *being added, if from this Sum we ſubſtract* 50, *and add* 7 *to the remainder, and ſubſtract* 60 *from this Sum, and by adding* 6 *to that remainder, this Sum may be* 22.

Let $a =$ Number ſought, $b = 19$, $d = 50$, $m = 7$, $n = 60$, $p = 6$, $g = 22$.

Now I am to find a Number which I call	1	a
To which 19 or b, being added we have by Art. 6.	2	$a + b$
From which Sum ſubſtracting 50 or d, that is, connecting d, by the Sign $-$, and it is	3	$a + b - d$

And

And to this remainder adding 7 or *m*, we have by Art. 6.	4	$a+b-d+m$
From this Sum ſubſtracting 60 or *n*, that is, connecting *n*, by the Sign — - -	5	$a+b-d+m-n$
And to this remainder adding 6 or *p*, we have - -	6	$a+b-d+m-n+p$
And this $a+b-d+m-n+p$ is to be equal to 22 or *g*, hence	7	$a+b-d+m-n+p=g$
Now to anſwer the Queſtion, I tranſpoſe *p*, by putting down the remaining Part of that Side of the Equation, and *p* on the other Side with its contrary Sign, hence - -	8	$a+b-d+m-n=g-p$
Then tranſpoſe *n*, by putting down the remaining Part of that Side of the Equation, and *n* on the other Side with its contrary Sign, then - -	9	$a+b-d+m=g-p+n$
Then tranſpoſe *m*, by putting down the remaining Part of that Side of the Equation, and *m* on the other Side with its contrary Sign, whence	10	$a+b-d=g-p+n-m$
Then tranſpoſe *d*, by putting down the remaining Part of that Side of the Equation, and *d* on the other Side with its contrary Sign, and -	11	$a+b=g-p+n-m+d$
Laſtly, tranſpoſe *b*, by putting down the remaining Part of that Side of the Equation, and *b* on the other Side with its contrary Sign, we have	12	$a=g-p+n-m+d-b$

Hence *a* the unknown Number, is equal to that Number repreſented by *g*, ſubſtracting from it the Number repreſented by *p*, adding to this remainder the Number repreſented by *n*, ſubſtracting from this Sum the Number repreſented by *m*, adding to the remainder the Number repreſented by *d*, and ſubſtracting from this Sum the Number repreſented by *b*.

22 the Number repreſented by g
6 the Number repreſented by p

16 or $g - p$
60 the Number repreſented by n

76 or $g - p + n$
7 the Number repreſented by m

69 or $g - p + n - m$
50 the Number repreſented by d

119 or $g - p + n - m + d$
19 the Number repreſented by b

100 the Number ſought or a, which is thus proved, from the Conditions of the Queſtion.

I ſay the Number ſought was - - -	100
For if to that you add - - -	19
	119
And from that Sum ſubſtract 50 - -	50
	69
And to this remainder add - -	7
	76
And from the Sum ſubſtract - - -	60
	16
And add to this remainder - - -	6
It makes what the Queſtion requires -	22

The Directions to the two following Queſtions are not quite ſo copious, that the Judgment of the Learner may be a little more exerciſed.

Queſtion 5. *A Number of Men were walking on a Bowling-Green, one Man was wondering to another how many there were, the other replied, if you ſubſtract* 7 *from their Sum, and add* 15 *to the remainder, and ſubſtract* 9 *from this Sum, and add* 56 *to that remainder, and ſubſtract* 2 *from that Sum, this will leave* 100. *To find the Number of Men on the Bowling-Green.*

Let $a =$ Number of Men on the Bowling-Green, $b = 7$, $d = 15$, $g = 9$, $m = 56$, $n = 2$, $p = 100$.

I am to find the Number of Men on the Bowling-Green which I call - -	1	a
From which 7 or *b*, being ſubſtracted, which is only to connect *b* by the Sign $-$	2	$a-b$
To which remainder adding 15 or *d*, we have by Art. 6.	3	$a-b+d$
From which Sum ſubſtracting 9 or *g*, or connecting *g* by the Sign $-$, and we have	4	$a-b+d-g$
To this remainder add *m* or 56, by Art. 6. - -	5	$a-b+d-g+m$
From which Sum ſubſtracting 2 or *n*, that is, connecting *n* by the Sign $-$, it is	6	$a-b+d-g+m-n$
Which $a-b+d-g+m-n$ is to be equal to 100 or *p*, hence	7	$a-b+d-g+m-n=p$
By tranſpoſing *n* we have - -	8	$a-b+d-g+m=p+n$
By tranſpoſing *m* we have - -	9	$a-b+d-g=p+n-m$
By tranſpoſing *g* we have -	10	$a-b+d=p+n-m+g$
By tranſpoſing *d* we have - -	11	$a-b=p+n-m+g-d$
By tranſpoſing *b* we have - -	12	$a=p+n-m+g-d+b$

100 is the Number repreſented by p
2 or n to be added

102 or $p+n$
56 or m to be ſubſtracted

46 or $p+n-m$
9 or g to be added

55 or $p+n-m+g$
15 or d to be ſubſtracted

40 or $p+n-m+g-d$
7 or b to be added

47 the Number ſought or a, for $a=p+n-m+g-d+b$.

Now to prove if 47 was the Number of Men that were on the Bowling-Green, let us try if it will anſwer the Conditions of the Queſtion.

I ſay

I ſay the Number of Men was - -	47
For if from that you ſubſtract - -	7
	40
And add to the remainder - -	15
	55
And from this Sum ſubſtract - -	9
	46
And add to that remainder - -	56
	102
And from that Sum ſubſtract - -	2
It makes what the Queſtion requires - -	100

Queſtion 6. *One Man required another to tell him how many Shillings he had, by ſaying that if to them there was added 5, and from this Sum ſubſtracting 3, and adding 16 to that remainder, and from that Sum ſubſtracting 50, and adding 54 to that remainder, he ſhould then have 43 Shillings. How many Shillings had he?*

Let a = the Number of Shillings the Man had, $b = 5$, $d = 3$, $m = 16$, $n = 50$, $p = 54$, $q = 43$. -

The Man had a certain Number of Shillings which I call - - -	1	a
To which 5 or b, being added we have - -	2	$a+b$
From which Sum ſubſtracting 3 or d, we have -	3	$a+b-d$
To which remainder adding 16 or m, we have -	4	$a+b-d+m$
From which Sum ſubſtracting 50 or n, we have -	5	$a+b-d+m-n$
To which remainder adding 54 or p, we have -	6	$a+b-d+m-n+p$
Which $a+b-d+m-n+p$ is to be equal to 43 or q, hence - - -	7	$a+b-d+m-n+p=q$
The Queſtion being now expreſſed in *Algebra*, by tranſpoſing p, we have -	8	$a+b-d+m-n=q-p$

By

By tranſpoſing n we have -	9	$a+b-d+m=q-p+n$
By tranſpoſing m we have -	10	$a+b-d=q-p+n-m$
By tranſpoſing d we have -	11	$a+b=q-p+n-m+d$
Laſtly, by tranſpoſing b we have	12	$a=q-p+n-m+d-b$

43 is the Number repreſented by q
—54 from which ſubſtracting 54 or p, there remains —11

—11 or $q-p$
50 to which —11 adding 50 or n, the Sum is 39

39 or $q-p+n$
16 from which ſubſtracting 16 or m

23 or $q-p+n-m$
3 to which adding 3 or d

26 or $q-p+n-m+d$
5 from which ſubſtracting 5 or b

21 hence 21 is the Number ſought; which is thus proved,

I ſay the Man had - -	21 Shillings
For if to them you add - -	5
	26
And from that Sum ſubſtract - -	3
	23
And to that remainder add - -	16
	39
And if from this Sum we ſubſtract -	50
There remains a *negative* or - - - -	—11
And if to this remainder we add -	54
It makes what the Queſtion requires -	43

When a *negative* Number is to be ſubſtracted from an *affirmative* Number, and the negative Number is greateſt, as in the laſt Queſtion, it is only to take the difference of the two Numbers and place the Sign — before it, and if the next Number to be added is *affirmative*, and greater than the *negative* Remainder, then it is only ſubſtracting the *negative* Remainder from the *affirmative* Number which is to be added, which will be the Sum.

If the Learner finds any Difficulty in conceiving this, he may collect all the *affirmative* Numbers into one Sum; and all the *negative* Numbers into another, and ſubſtracting the Sum of the *negatives*, from the Sum of the *affirmatives*, the remainder is the Anſwer to the Queſtion.

In the laſt Queſtion,

The *affirmative* Quantities or Numbers are -

$$\begin{array}{r} q=43 \\ n=50 \\ d=\ \ 3 \\ \hline 96 \\ -75 \\ \hline 21=a \text{ as before} \end{array}$$

The *negative* Quantities or Numbers are -

$$\begin{array}{r} -p=-54 \\ -m=-16 \\ -b=-\ \ 5 \\ \hline -75 \end{array}$$

To reduce an Equation by Multiplication.

47. In the laſt Article, the unknown Quantity was connected with the known Quantities by the Signs + or — only, but it may happen that the unknown Quantity may be *divided* by ſome known Quantity; in this Caſe, *multiply every Part or all the Terms of the Equation by that known Quantity*; and the part of the Equation containing the unknown Quantity will now be multiplied and divided by the ſame Quantity, then begin and take down this Equation, rejecting this Quantity from that part of the Equation where it both multiplies and divides the unknown Quantity, by Art. 20. for it is in both the Dividend and Diviſor, after this Equation is ſet down, if there is any other Quantities connected with the unknown one by the Signs + or —, tranſpoſe them to the other Side of the Equation as in the laſt Article, by which Method we ſhall have all the known Quantities on one Side of the Equation, and the unknown one by itſelf on the other Side, which is the Solution of the Queſtion.

Queſtion 7. *A Gameſter challenging another to play for as many Guineas as he had in his Hand, the other required to know how many there were, he replied if you divide them by* 5 *and add* 19 *to that Quotient, I ſhall then have* 23 *Guineas in my Hand. How many Guineas had he?*

Let $a =$ the Number of Guineas the Gameſter had, $b = 5$, $d = 19$, $m = 23$.

Then the Gameſter had a certain Number of Guineas which I call -	1	a
Which being divided by 5 or b, the Quotient is by Art. 27. - -	2	$\frac{a}{b}$
To which Quotient $\frac{a}{b}$ if we add 19 or d, we have by Art. 6.	3	$\frac{a}{b} + d$
And this $\frac{a}{b} + d$, is to be equal to 23 or m, therefore we have -	4	$\frac{a}{b} + d = m$
The Queſtion being expreſſed in *Algebra* by the Equation $\frac{a}{b} + d = m$, in which Equation the unknown Quantity a being divided by b; now by the Rule multiply every Part or Quantity in the Equation by b, and in this Multiplication, multiply only the Numerator of the Quantity $\frac{a}{b}$, or a by b, by the Rule of Vulgar Fractions in Arithmetic, then we have. - -	5	$\frac{ab}{b} + bd = bm$
Becauſe b is both in the Dividend and Diviſor of the Quantity $\frac{ab}{b}$, hence by the Rule rejecting b from $\frac{ab}{b}$ only, and placing down the remaining Part a, and all the other Parts of the Equation, without any alteration, we have	6	$a + bd = bm$
Tranſpoſing bd by the laſt Article, it being a known Quantity, then - -	7	$a = bm - bd$

And the Queſtion is now anſwered, for a the unknown Quantity is equal to the Product of the two Numbers repreſented by b and m, ſubſtracting from it the Product of the two Numbers repreſented by b and d.

The Number repreſented by b is 5, the Number repreſented by m is 23, which two Numbers being multiplied is bm or - - - - -	115
The Number repreſented by b is 5, the Number repreſented by d is 19, which two Numbers being multiplied is bd or - - - -	95
Subſtracting bd from bm, that is, 95 from 115, leaves $bm-bd$ or - - - - -	20

Which is the Number ſought, or the Guineas the Gameſter had, and is proved from the Conditions of the Queſtion, thus,

I ſay the Gameſter had - - -	20 Guineas
For if that Number is divided by 5, the Quotient is	4
But if to this 4 we add - -	19
It makes what the Queſtion requires -	23

Queſtion 8. *To find that Number which being divided by* 15, *if to this Quotient we add* 27, *and ſubſtract* 13 *from this Sum, the remainder may be equal to* 18.

Let $a=$ the Number ſought, $b=15$, $d=27$, $m=13$, $p=18$.

Now I am to find a Number which I call - -	1	a
Which being divided by 15 or b, we have by Art. 27.	2	$\frac{a}{b}$
To which Quotient or $\frac{a}{b}$, if we add 27 or d, we have by Art. 6. - - -	3	$\frac{a}{b}+d$
From which Sum if we ſubſtract 13 or m, that is, connect m by the Sign $-$, it is -	4	$\frac{a}{b}+d-m$
Which $\frac{a}{b}+d-m$ is by the Queſtion, to be equal to 18 or p, hence we have -	5	$\frac{a}{b}+d-m=p$

The Queſtion being now expreſſed in *Algebra* by this Equation $\frac{a}{b}+d-m=p$, and the unknown Quantity *a* being divided by *b*, begin by multiplying every Part of the Equation by *b* as in the laſt Queſtion, and then we have - - -	6	$\frac{ab}{b}+db-mb=pb$
Becauſe *b*, is both in the Dividend and Diviſor of the Quantity $\frac{ab}{b}$, reject *b* from this Quantity only as in the laſt Queſtion, and place down *a* and the remaining Quantities in the Equation without any alteration, then we have	7	$a+db-mb=pb$
Becauſe *m b* is a known Quantity, tranſpoſe it by the Directions in the laſt Article, and we have - - -	8	$a+db=pb+mb$
Becauſe *d b* is a known Quantity, tranſpoſe it by the ſame Directions, and we have -	9	$a=pb+mb-db$

Now *a* the unknown Quantity being by itſelf on one Side of the Equation, the Queſtion is ſolved; for *a*, the unknown Quantity, is equal to the Product of the two Numbers repreſented by *p* and *b*, added to the Product of the two Numbers repreſented by *m* and *b*, ſubſtracting from this Sum the Product of the Numbers repreſented by the two Letters *d* and *b*.

The Number repreſented by *p* is 18, the Number repreſented by *b* is 15, the Product of theſe two Numbers is *p b* or - - - - - -	270
The Number repreſented by *m* is 13, the Number repreſented by *b* is 15, the Product of theſe two Numbers is *m b* or - - - - - -	195
Add theſe two Products together, the Sum is $pb+mb$ or	465

The Number repreſented by d is 27, the Number repreſented by b is 15, the Product of theſe two Numbers is 405, which being ſubſtracted from the Sum of the other two Products - - - - 405

Leaves $pb+mb-db$ or - - - - 60

Therefore I ſay 60 is equal to a, or 60 is the Number ſought, which is thus proved from the Queſtion.

I ſay the Number ſought is - -	60
For if that is divided by 15 the Quotient is -	4
To which Quotient or 4, if we add -	27
The Sum is - - -	31
And if from this Sum we ſubſtract -	13
There remains what the Queſtion requires -	18

Queſtion 9. *A Man being asked how many Shillings he had, replied, if you divide the Number I have by* 25, *and ſubſtract* 3 *from that Quotient, and then add* 51 *to this remainder, and from this Sum ſubſtracting* 40, *I ſhall have* 12 *Shillings left. How many Shillings had he?*

Let $a=$ the Number of Shillings the Man had, $b=25$, $d=3$, $m=51$, $p=40$, $z=12$.

Then the Man had a certain Number of Shillings which I call - -	1	a
Which being divided by 25 or b, we have by Art. 27. -	2	$\frac{a}{b}$
From which Quotient or $\frac{a}{b}$, if we ſubſtract 3 or d, that is, connecting d by the Sign —	3	$\frac{a}{b}-d$
To which if we add 51 or m, we have by Art. 6. -	4	$\frac{a}{b}-d+m$
From which ſubſtracting 40 or p, that is, connecting p by the Sign —, we have -	5	$\frac{a}{b}-d+m-p$
Which $\frac{a}{b}-d+m-p$ is by the Queſtion, to be equal to 12 or z, hence we have	6	$\frac{a}{b}-d+m-p=z$

The

The Queſtion being now expreſſed in *Algebra*, and the unknown Quantity a being divided by b, multiply every Quantity in the Equation by b, as in the two laſt Queſtions, then we have -	7	$\frac{ab}{b} - db + mb - pb = zb$
And rejecting b out of the Quantity $\frac{ab}{b}$ only, becauſe it is in both the Dividend and Diviſor, and ſet down the reſt as in the two laſt Queſtions, we have -	8	$a - db + mb - pb = zb$
Becauſe pb is a known Quantity, tranſpoſe it by Art. 46. and we have - -	9	$a - db + mb = zb + pb$
Becauſe mb is a known Quantity, tranſpoſe that in like Manner, then we have -	10	$a - db = zb + pb - mb$
Becauſe db is a known Quantity, by tranſpoſing it we have - -	11	$a = zb + pb - mb + db$

Now it appears the unknown Quantity or a, is equal to the Product of the two Numbers repreſented by z and b, added to the Product of the two Numbers repreſented by p and b, ſubſtracting from this Sum the Product of the two Numbers repreſented by m and b, and adding to this remainder the Product of the two Numbers repreſented by d and b.

The Number repreſented by z is 12, and that by b is 25, the Product of theſe two is zb, or -	300
The Number repreſented by p is 40, and that by b is 25, the Product of theſe two is pb or -	1000
Adding theſe two Products together the Sum is $zb + pb$, or	1300
The Number repreſented by m is 51, and that by b is 25, the Product of theſe two is mb or -	1275
Which being ſubſtracted from the Sum of the other two, leaves $zb + pb - mb$ or - -	25
The Number repreſented by d is 3, and that by b is 25, the Product of theſe two is db or -	75
Which added to the laſt remainder, the Sum is $zb + pb - mb + db$ or - -	100

Now I ſay the unknown Quantity a, or the Number of Shillings the Man had, is 100, which is thus proved, from the Conditions of the Queſtion.

I ſay the Man had - - -	100 Shillings
For if that Number is divided by 25, the Quotient is	4
From which Quotient if we ſubſtract -	3
Remains - - -	1
To which adding - -	51
The Sum is - - -	52
From which ſubſtracting - -	40
There remains what the Queſtion requires -	12

Queſtion 10. *A Country Servant, who underſtood* Algebra, *was asked by his Maſter how many Cows there were in the Field, he replied, if you add* 13 *to their Number, and divide that Sum by* 8, *and then add* 19 *to that Quotient, and ſubſtract* 11 *from this Sum, there will be* 12 *Cows left. How many Cows were there?*

Let $a =$ the Number of Cows, $b = 13$, $d = 8$, $m = 19$, $p = 11$, $x = 12$.

Now there were in the Field a certain Number of Cows which I call - -	1	a
To which 13 or b, being added we have, by Art. 6. -	2	$a+b$
Which $a+b$ being divided by 8 or d, we have by Art. 28.	3	$\frac{a+b}{d}$
To which if we add 19 or m, we have by Art. 6. -	4	$\frac{a+b}{d}+m$
From which if we ſubſtract 11 or p, we have by connecting p with the Sign $-$ -	5	$\frac{a+b}{d}+m-p$
Which $\frac{a+b}{d}+m-p$ is by the Queſtion, to be equal to 12 or x, hence we have	6	$\frac{a+b}{d}+m-p=x$

Becauſe a the unknown Term, is part of the Quantity $\frac{a+b}{d}$ which is divided by d, therefore multiply every Quantity by d, that is, multiply every Quantity in the Numerator by d, according to the Rule of Vulgar Fractions in Arithmetic, and we have -	7	$\frac{ad+bd}{d}+md-pd=xd$
Becauſe d is in every Term of the Dividend and Diviſor of the Quantity $\frac{ad+bd}{d}$, reject the d, from $\frac{ad+bd}{d}$ only, by Art. 22 and 24, and ſet down all the reſt as before, then - -	8	$a+b+md-pd=xd$
Now begin to tranſpoſe pd, it being a known Quantity, then we have - -	9	$a+b+md=xd+pd$
Becauſe md is a known Quantity, therefore tranſpoſe it, and we have - -	10	$a+b=xd+pd-md$
Becauſe b is a known Quantity, therefore tranſpoſe it, and we have - -	11	$a=xd+pd-md-b$

By this it appears that a, the unknown Quantity, is equal to the Product of the two Numbers repreſented by x and d, added to the Product of the two Numbers repreſented by p and d, ſubſtracting from this Sum the Product of the two Numbers repreſented by m and d, ſubſtracting ſtill from this remainder the Number repreſented by b.

The Product of the two Numbers repreſented by x and d, is xd or - - - 96
The Product of the two Numbers repreſented by p and d, is pd or - - - 88
The Sum of theſe two Products is $xd+pd$, or 184

The Product of the two Numbers repreſented by m and d, is md or - - -	152
Which being ſubſtracted from the Sum of the other two Products there remains $xd+pd-md$ -	32
From which ſubſtracting the Number repreſented by b	13
The remainder is $xd+pd-md-b$, which is equal to a, or the Number ſought - -	19

And that there were 19 Cows in the Field is thus proved, from the Conditions of the Queſtion.

I ſay the Number of Cows were - -	19
For if to them we add - -	13
	32
And divide that Sum or 32 by 8, the Quotient is	4
To which Quotient if we add - -	19
The Sum is - - -	23
From which Sum ſubſtracting - -	11
There remains what the Queſtion requires -	12

Queſtion 11. *Two young Gentlemen were diſputing how many Men were at a public Diverſion, but not agreeing, they referred it to a third Perſon, who, being ſkilled in Algebra, inſtead of a direct Anſwer, replied, that if you ſubſtract* 115 *from their Sum, and divide this remainder by* 50, *and add* 39 *to that Quotient, from which Sum ſubſtracting* 16, *and adding* 68 *to this remainder, this laſt Sum will be equal to* 101. *How many Men were there?*

Let $a=$ the Number of Men, $b=115$, $c=50$, $d=39$, $n=16$, $p=68$, $x=101$.

There were a certain Number of Men which I call - -	1	a
From which 115, or b, being ſubſtracted we have	2	$a-b$
Which remainder of $a-b$, being divided by 50 or c, we have by Art. 28.	3	$\frac{a-b}{c}$
To which Quotient if we add 39 or d, we have	4	$\frac{a-b}{c}+d$

From this Sum if we ſubſtract 16 or n, we have - -	5	$\frac{a-b}{c}+d-n$
To which remainder if we add 68 or p, we have	6	$\frac{a-b}{c}+d-n+p$
Which $\frac{a-b}{c}+d-n+p$ by the Queſtion, is to be equal to 101 or x, hence - -	7	$\frac{a-b}{c}+d-n+p=x$
Becauſe a, the unknown Quantity, is part of this $\frac{a-b}{c}$, which being divided by c, therefore multiplying every quantity by c, as in the laſt Queſtion, we have	8	$\frac{ca-cb}{c}+cd-cn+cp=cx$
Becauſe c is in every Term of the Dividend and Diviſor of $\frac{ca-cb}{c}$ reject the c, as in the laſt Queſtion, and ſet down all the reſt as before, and we have - -	9	$a-b+cd-cn+cp=cx$
Now tranſpoſe cp, it being a known Quantity, then it is - -	10	$a-b+cd-cn=cx-cp$
Tranſpoſe cn, it being a known Quantity, and we have - -	11	$a-b+cd=cx-cp+cn$
Tranſpoſe cd, it being a known Quantity, and we have - -	12	$a-b=cx-cp+cn-cd$
And tranſpoſing b, it being a known Quantity, we have - -	13	$a=cx-cp+cn-cd+b$

Hence it appears that a, the unknown Quantity, is equal to the Product of the two Numbers repreſented by c and x, ſubſtracting from it the Product of the two Numbers repreſented by c and p, adding to that remainder the Product of the two Numbers repreſented by c and n, ſubſtracting from this Sum, the Product of the two Numbers repreſented by c and d, and adding to this remainder the Number repreſented by b.

The Product of the two Numbers repreſented by c and x, is cx or - - - - -	5050
The Product of the two Numbers repreſented by c and p, is cp or - - - - -	3400
The remainder is $cx - cp$ or - -	1650
The Product of the two Numbers repreſented by c and n, is cn or - - - - -	800
Which added to the laſt remainder, the Sum is $cx - cp + cn$ or - - - - -	2450
The Product of the two Numbers repreſented by c and d, is cd or 1950, which being ſubſtracted -	1950
The remainder is $cx - cp + cn - cd$, or -	500
Adding the Number repreſented by b, or -	115
The Sum is $cx - cp + cn - cd + b$, which is equal to a, the Number ſought - - -	615

And that there were 615 Men is thus proved, from the Conditions of the Queſtion.

I ſay there were - - -	615 Men
For if from them we ſubſtract - - - -	115
	500
And divide the remainder 500 by 50, the Quotient is	10
To which adding - -	39
The Sum is - - -	49
From which ſubſtracting - -	16
The remainder is - - -	33
To which adding - - -	68
It makes what the Queſtion requires - -	101

Queſtion 12. *There is a certain Number to which 9 being added, and dividing this Sum by 5, if from this Quotient we ſubſtract 6, and add 101 to the remainder, from that Sum ſubſtracting 10 there remains 97. What is the Number ſought?*

Let $a=$ the Number ſought, $b=9$, $c=5$, $d=6$, $m=101$, $p=10$, $x=97$.

Now I am to find a certain Number which I call	1	a
To which 9 or *b*, being added we have by Art. 6.	2	$a+b$
Which being divided by 5 or *c*, we have by Art. 28. - -	3	$\frac{a+b}{c}$
From which being ſubſtracted 6 or *d*, we have	4	$\frac{a+b}{c}-d$
To which adding 101 or *m*, we have by Art. 6.	5	$\frac{a+b}{c}-d+m$
From which ſubſtracting 10 or *p*, that is, connecting *p*, by the Sign —	6	$\frac{a+b}{c}-d+m-p$
Which $\frac{a+b}{c}-d+m-p$ is to be equal to 97 or *x*, hence	7	$\frac{a+b}{c}-d+m-p=x$
The Queſtion being thus expreſſed in *Algebra*, begin and multiply by *c*, for the reaſon in the laſt Queſtion, then we have - - -	10	$\frac{ca+cb}{c}-cd+cm-cp=cx$
Rejecting *c* out of $\frac{ca+cb}{c}$, and ſetting down the reſt as before, then	11	$a+b-cd+cm-cp=cx$
Now tranſpoſing *cp*, it being a known Quantity, we have - -	12	$a+b-cd+cm=cx+cp$
Tranſpoſing *cm*, it being a known Quantity, we have - - -	13	$a+b-cd=cx+cp-cm$
Tranſpoſing *cd*, it being a known Quantity, we have - - -	14	$a+b=cx+cp-cm+cd$
Laſtly, tranſpoſing *b*, it being a known Quantity, we have -	15	$a=cx+cp-cm+cd-b$

The *Algebraic* Operation being finiſhed, the *Numerical* Work is thus.

The Product of the two Numbers repreſented by c and x, is. cx or	485
The Product of the two Numbers repreſented by c and p, is cp or	50
The Sum is $cx + cp$ or	535
The Product of the two Numbers repreſented by c and m, is cm or	505
Subſtracting, the remainder is $cx + cp - cm$ or	30
The Product of the two Numbers repreſented by c and d, is cd or	30
Adding, the Sum is $cx + cp - cm + cd$ or	60
The Number repreſented by b is 9, ſubſtracting	9
The remainder is $cx + cp - cm + cd - b$, which is equal to a, or the Number ſought	51

Which is thus proved from the Conditions of the Queſtion.

I ſay the Number ſought is	51
For if to this we add	9
The Sum is	60
Which Sum, or 60, being divided by 5, the Quotient is	12
From which Quotient if we ſubſtract	6
The remainder is	6
To which adding	101
The Sum is	107
From which ſubſtracting	10
There remains what the Queſtion requires	97

To reduce an Equation by Diviſion.

48. In the laſt Article the unknown Quantity was divided by a known Quantity, in this Article the unknown Quantity will be *multiplied* into a known Quantity; when this happens, *divide every Quantity on both Sides of the Equation, by the ſame known Quantity into which the unknown Quantity is multiplied*, then you

will find the unknown Quantity to be multiplied and divided by the ſame Quantity, now place down this Equation, rejecting only the Letter from that Quantity, where it multiplies and divides the unknown Quantity as in the laſt Article; then tranſpoſe the Quantities as before, but if there are none to be tranſpoſed the Queſtion is ſolved.

If any Quantities are connected with the unknown one by the Signs + or —, it will be moſt convenient for the Learner to tranſpoſe them before he begins to divide by the Rule juſt given.

Queſtion 13. *A Perſon required another to tell him how many Shillings he had, by ſaying that if their Number was multiplied by* 13, *and if from that Product was ſubſtracted* 25 *he ſhould then have* 170 *Shillings. How many Shillings had he?*

Let $a =$ the Number of Shillings he had, $b = 13$, $d = 25$, $m = 170$.

A Perſon had a certain Number of Shillings which I call - -	1	a
Which being multiplied by 13, or b, we have by Art. 9. - - -	2	ba
From which Product, or ba, if we ſubſtract 25, or d, we have - -	3	$ba - d$
Which remainder, or $ba - d$, is by the Queſtion to be equal to 170, or m, hence - - -	4	$ba - d = m$
Becauſe d is on the ſame Side of the Equation with the unknown Quantity, and connected by the Sign —, therefore tranſpoſe d, then -	8	$ba = m + d$
There being no more Quantities to be tranſpoſed, and the unknown Quantity being multiplied by b, therefore divide both Sides of the Equation by b. Now ba divided by b gives $\frac{ba}{b}$, and $m + d$ divided by b, gives $\frac{m+d}{b}$ by Art. 28. therefore we have - - - -	9	$\frac{ba}{b} = \frac{m+d}{b}$

Becauſe b is in both the Dividend and Diviſor of the Quantity $\frac{ba}{b}$ reject b, by Art. 20. and putting down the other Quantities without any alteration as in the foregoing Queſtion, we have - - - - } 10 | $a = \frac{m+d}{b}$

From hence it appears that a, the unknown Quantity, is equal to the Sum of the two Numbers repreſented by m and d, divided by the Number repreſented by b.

The Number repreſented by m is - - - 170
The Number repreſented by d is - - - 25
The Sum is $m+d$ or - - - 195

And dividing 195 or $m+d$ by 13, or b, the Quotient is $\frac{m+d}{b}$ or 15, which is a, or the Number ſought.

The Truth of which is thus proved, from the Conditions of the Queſtion.

I ſay the Perſon had - - - - 15 Shillings
For if that is multiplied by - - 13
45
15
The Product is - - - 195
From which ſubſtracting - - 25
It makes what the Queſtion requires - 170

Queſtion 14. *A Butcher ſeeing a Drover going to Market with a Number of Sheep, asked how many there were; the Drover, not being diſpoſed to inform him, anſwered, if you multiply their Number by* 9, *and ſubſtract* 157 *from that Product, and add* 168 *to this remainder, I ſhall then have* 2000 *Sheep. How many Sheep had he?*

Let $a =$ the Number of Sheep, $b = 9$, $d = 157$, $m = 168$, $p = 2000$.

Then

Then a Drover had a certain Number of Sheep which I call	1	a
Which being multiplied by 9 or b, we have by Art. 9.	2	ba
From which Product 157 or d, being ſubſtracted, that is, connecting d by the Sign —, we have	3	$ba-d$
To which remainder adding 168 or m, we have by Art. 6.	4	$ba-d+m$
Which $ba-d+m$ is by the Queſtion to be equal to 1000 or p, hence we have	5	$ba-d+m=p$
Now according to the Rule begin with tranſpoſing m, and we have	6	$ba-d=p-m$
Then tranſpoſing d, we have	7	$ba=p-m+d$
The Quantities being now all tranſpoſed that were connected by the Signs + or —, and the unknown Quantity being multiplied by b, therefore by the Rule divide both Sides of the Equation by b, but ba divided by b, gives $\frac{ba}{b}$, and $p-m+d$ divided by b, gives $\frac{p-m+d}{b}$, by Art. 28. hence we have	8	$\frac{ba}{b}=\frac{p-m+d}{b}$
Rejecting b, out of the Quantity $\frac{ba}{b}$, becauſe it is in both the Dividend and Diviſor, and placing down the remaining Parts of the Equation without any alteration as before, we have	9	$a=\frac{p-m+d}{b}$

The *Algebraic* Work is now finiſhed, for the unknown Quantity a is on one Side of the Equation by itſelf, and it appears that a the unknown Number is equal to the Number repreſented by p, ſubſtracting from it the Number repreſented by m, adding to this remainder the Number repreſented by d, and dividing this Sum by the Number repreſented by b.

The Number repreſented by p is - - 2000
From which ſubſtracting the Number repreſented by m, which is - - - - 168

There remains $p-m$ or - - 1832
To which adding the Number repreſented by d - 157

The Sum is $p-m+d$ or - - 1989

And dividing this 1989, or $p-m+d$, by 9. the Number repreſented by b, the Quotient is $\frac{p-m+d}{b}$, or 221, which is a, or the Number of Sheep the Drover had; and is proved by the Conditions of the Queſtion thus.

I ſay the Number of Sheep were - - 221
For that being multiplied by - - 9

The Product is - - - - 1989
From which ſubſtracting - - - 157

There remains - - - 1832
To which adding - - - 168

It makes what the Queſtion requires - - 2000

Queſtion 15. *A Man being asked what he gave for his Horſe, anſwered, if you multiply the Number of Pounds I gave by* 5, *and then add* 15 *to this Product, and from that Sum ſubſtract* 50, *and to this Remainder adding* 25, *from which Sum ſubſtracting* 15, *this Remainder will be equal to* 80. *What did he give for his Horſe?*

Let $a=$ what he gave for the Horſe, $b=5$, $d=15$, $c=50$, $p=25$, $m=15$, $x=80$.

The Perſon gave for the Horſe a certain Sum which I call	1	a
Which being multiplied by 5, or b, we have by Art. 9.	2	ba
To which Product, if we add 15, or d, we have by Art. 6.	3	$ba+d$
From which Sum ſubſtracting 50, or c, that is, connecting c by the Sign —, we have	4	$ba+d-c$

To

To which remainder adding 25, or p, we have - -	5	$ba+d-c+p$
From which Sum ſubſtracting 15, or m, we have -	6	$ba+d-c+p-m$
Which $ba+d-c+p-m$ is by the Queſtion, to be equal to 80, or x, hence we have - - -	7	$ba+d-c+p-m=x$
Now by tranſpoſing m, we have	8	$ba+d-c+p=x+m$
And by tranſpoſing p, we have	9	$ba+d-c=x+m-p$
And by tranſpoſing c, we have	10	$ba+d=x+m-p+c$
And by tranſpoſing d, we have	11	$ba=x+m-p+c-d$
The Quantities connected by the Signs + or —, being now all tranſpoſed, I obſerve the unknown Quantity, to be multiplied by b, therefore divide every Term in the Equation, or both Sides of the Equation, by b. Now dividing ba by b, it is $\frac{ba}{b}$, and dividing $x+m-p+c-d$ by b, we have $\frac{x+m-p+c-d}{b}$, as in the foregoing Queſtions, hence we have - -	12	$\frac{ba}{b}=\frac{x+m-p+c-d}{b}$
Rejecting the b, out of the Quantity $\frac{ba}{b}$, becauſe it is in both the Dividend and Diviſor, and placing down the reſt of the Equation without any alteration as before, and we have - -	13	$a=\frac{x+m-p+c-d}{b}$

That is, a, the unknown Quantity, is equal to the Number repreſented by x, added to the Number repreſented by m, ſubſtracting from that Sum the Number repreſented by p, adding to this Remainder the Number repreſented by c, ſubſtracting from this Sum the Number repreſented by d, and dividing this Remainder by the Number repreſented by b.

Now x is - - - - -	80
To which adding m or - - -	15
The Sum is $x+m$, or - -	95
From which ſubſtracting p, or - -	25
There remains $x+m-p$, or - -	70
To which adding c, or - - -	50
The Sum is $x+m-p+c$, or - -	120
From which ſubſtracting d, or - -	15
There remains $x+m-p+c-d$, or -	105

Now dividing this 105, or $x+m-p+c-d$ by b, or 5, the Quotient is $\frac{x+m-p+c-d}{b}$, or 21, which is equal to a or the Price of the Horſe.

And that the Horſe coſt 21 Pounds is proved from the Conditions of the Queſtion, thus.

I ſay the Horſe coſt - - -	21 Pounds
For if that is multiplied by - -	5
The Product is - - -	105
To which adding - -	15
The Sum is - - -	120
From which ſubſtracting - -	50
There remains - - -	70
To which adding - -	25
The Sum is - - -	95
From which ſubſtracting - -	15
There remains what the Queſtion requires -	80

Queſtion 16. *There is a certain Number which being multiplied by* 7, *if from this Product we ſubſtract* 21, *and to this Remainder add* 11, *and from this Sum ſubſtract* 23, *and add to this Remainder* 33, *this laſt Sum will be* 210. *What is the Number ſought?*

Let $a=$ the Number ſought, $b=7$, $d=21$, $x=11$, $c=23$, $p=33$, $r=210$.

Now there is a certain Number ſought which I call -	1	a
Which being multiplied by 7, or *b*, we have by Art. 9.	2	$b\,a$
From which ſubſtracting 21, or *d*, that is, connecting *d* by the Sign —, we have -	3	$b\,a - d$
To which adding 11, or *x*, we have by Art. 6.	4	$b\,a - d + x$
From which ſubſtracting 23, or *c*, that is, connecting *c* by the Sign —, we have -	5	$b\,a - d + x - c$
To which adding 33, or *p*, we have by Art. 6. - -	6	$b\,a - d + x - c + p$
And this $b\,a - d + x - c + p$ is by the Queſtion, to be equal to 210, or *r*, hence we have - - -	7	$b\,a - d + x - c + p = r$
The Queſtion being now expreſſed in *Algebra*, begin the Solution by tranſpoſing *p*, and then we have - -	8	$b\,a - d + x - c = r - p$
Tranſpoſing *c* we have -	9	$b\,a - d + x = r - p + c$
Tranſpoſing *x* we have -	10	$b\,a - d = r - p + c - x$
Tranſpoſing *d* we have -	11	$b\,a = r - p + c - x + d$
All the Quantities being now tranſpoſed that were connected by the Signs + or —, and the unknown Quantity, being multiplied by *b*, dividing every Term, or both Sides of the Equation by *b*, as in the laſt Example, then we have - -	12	$\frac{b\,a}{b} = \frac{r - p + c - x + d}{b}$
Now reject *b*, out of the Quantity $\frac{b\,a}{b}$, becauſe it is in both the Dividend and Diviſor, and ſeting down the remaining Parts of the Equation, as in the laſt Queſtion, and we have . - -	13	$a = \frac{r - p + c - x + d}{b}$

Now to find what a, is in Numbers.

The Number repreſented by r, is	210
From which ſubſtracting the Number repreſented by p, which is	33
There remains $r-p$, or	177
To which adding the Number repreſented by c	23
The Sum is $r-p+c$, or	200
From which ſubſtracting the Number repreſented by x	11
There remains $r-p+c-x$, or	189
To which adding the Number repreſented by d	21
The Sum is $r-p+c-x+d$, or	210

And dividing this 210 by b, or 7, the Quotient is $\frac{r-p+c-x+d}{b}$, or 30, which is equal to a, or the Number ſought, and is thus proved.

I ſay the Number ſought is	30
For if this is multiplied by	7
The Product is	210
From which ſubſtracting	21
There remains	189
To which adding	11
The Sum is	200
From which ſubſtracting	23
There remains	177
To which adding	33
The Sum is what the Queſtion requires	210

Queſtion 17. *A Gameſter challenged another to play with him for as many Guineas as were in his Hand; but being asked how many there were, anſwered, if you multiply them by* 10, *and ſubſtract* 100 *from this Product, and to this Remainder adding* 55, *and from this Sum ſubſtract* 31, *and adding to this Remainder* 115, *I ſhall then have* 539 *Guineas. How many had he at firſt?*

Let $a=$ the Number of Guineas he had, $b=10$, $c=100$, $d=55$, $m=31$, $x=115$, $p=539$.

Then

Then a Gameſter had a certain Number of Guineas which I call - - - -	1	a
Which being multiplied by 10, or *b*, we have by Art. 9.	2	ba
From which ſubſtracting 100, or *c*, we have - -	3	$ba-c$
To which adding 55, or *d*, we have by Art. 6. - -	4	$ba-c+d$
From which ſubſtracting 31, or *m*, we have - -	5	$ba-c+d-m$
To which adding 115, or *x*, we have - -	6	$ba-c+d-m+x$
Which by the Queſtion is to be equal to 539, or *p*, hence we have - -	7	$ba-c+d-m+x=p$
Then by tranſpoſing *x* we have	8	$ba-c+d-m=p-x$
Tranſpoſing *m* it is - -	9	$ba-c+d=p-x+m$
Tranſpoſing *d* we have - -	10	$ba-c=p-x+m-d$
And tranſpoſing *c* - -	11	$ba=p-x+m-d+c$
Now divide by *b*, as before directed, and we have -	12	$\frac{ba}{b}=\frac{p-x+m-d+c}{b}$
And rejecting *b*, from $\frac{ba}{b}$, and placing down the reſt as before, then - -	13	$a=\frac{p-x+m-d+c}{b}$

The Queſtion being now ſolved in *Algebra*, we are by that to find what *a* is equal to in Numbers.

Now *p* is equal to - - -	539
From which ſubſtracting *x*, which is equal to -	115
There remains $p-x$, or - - -	424
To this adding *m*, which is equal to -	31
The Sum is $p-x+m$, or - -	455
From this ſubſtracting *d*, which is equal to -	55
There remains $p-x+m-d$, or - -	400
To this adding *c*, which is equal to -	100
The Sum is $p-x+m-d+c$, or - -	500

But dividing this 500 by b, which is 10, the Quotient is 50, for the Number of Guineas the Gameſter had; and is thus proved from the Conditions of the Queſtion.

I ſay the Gameſter had - - -	50 Guineas
For if that Number is multiplied by -	10
The Product is - - -	500
From which ſubſtracting - - -	100
There remains - - - -	400
To which adding - - -	55
The Sum is - - -	455
From which ſubſtracting - - -	31
There remains - - -	424
To which adding - - -	115
The Sum is what the Queſtion requires -	539

Queſtion 18. *A Perſon being asked how many Hours it was paſt Noon, replied, if you multiply the Number of Hours paſt Noon by* 7, *and ſubſtract* 5 *from that Product, and to this Remainder add* 9, *and from this Sum ſubſtract* 3, *and to this Remainder adding* 4, *this Sum will be equal to* 12. *How many Hours was it paſt Noon, or what of the Clock was it?*

Let $a =$ the Number of Hours it was paſt Noon, or the Number ſought, $m = 7$, $p = 5$, $d = 9$, $c = 3$, $b = 4$, $x = 12$.

Then there is a certain Number of Hours paſt Noon, which I call - - -	1	a
Which being multiplied by 7 or m, we have by Art. 9.	2	ma
From which ſubſtracting 5 or p, we have - -	3	$ma - p$
To which adding 9 or d, we have by Art. 6. - -	4	$ma - p + d$
From which ſubſtracting 3 or c, that is, connecting c by the Sign —, we have -	5	$ma - p + d - c$
To which adding 4 or b, we have by Art. 6. - -	6	$ma - p + d - c + b$

Which by the Queſtion is to be equal to 12, or x, hence	7	$ma-p+d-c+b=x$
Now tranſpoſe b, and we have	8	$ma-p+d-c=x-b$
Tranſpoſing c - -	9	$ma-p+d=x-b+c$
Tranſpoſing d - -	10	$ma-p=x-b+c-d$
Tranſpoſing p - -	11	$ma=x-b+c-d+p$
Now dividing by m, as in the former Queſtions, and we have - - -	12	$\frac{ma}{m}=\frac{x-b+c-d+p}{m}$
Rejecting m from the Quantity $\frac{ma}{m}$, as before, and we have	13	$a=\frac{x-b+c-d+p}{m}$

The *Algebraic* Work being finiſhed, we are to find what a is in Numbers.

Now x is equal to - - - - -	12
From which ſubſtracting b, or - - -	4
There remains $x-b$, or - - - -	8
To which adding c, or - - - - -	3
The Sum is $x-b+c$, or - - -	11
From which ſubſtracting d, or - - -	9
There remains $x-b+c-d$, or - -	2
To which adding p, or 5, the Sum is $x-b+c-d+p$ or	7

And dividing this by m, or 7, the Quotient is 1, which is equal to a, or the Number of Hours it was paſt Noon, hence it was 1 of the Clock in the Afternoon.

Which is thus proved, from the Conditions of the Queſtion.

I ſay the Number of Hours paſt Noon were -	1
For if that is multiplied by - - - -	7
The Product is - - - -	7
From which ſubſtracting - - -	5
There remains - - - -	2
To which adding - - - -	9
The Sum is - - - -	11
From which ſubſtracting - - - -	3
There remains - - - - -	8
To which adding - - -	4
The Sum is what the Queſtion requires - -	12

To reduce an Equation by Involution.

49. Hitherto there has been no Equation in which the unknown Quantity has had the radical Sign prefixt before it, or has been connected with other known Quantities under the radical Sign, but as this is a Caſe which frequently happens, we are now to explain the Manner, how ſuch Equations are managed.

If any Part of an Equation is a *ſurd* Quantity, but the unknown Quantity is not under the radical Sign, then there is no occaſion to clear this Equation of its *Surds*, but if the unknown Quantity is under the radical Sign, then the Equation muſt be cleared of its *Surds*.

And when there is a given Equation where the *unknown Quantity is under the radical Sign*, and there are more Quantities without the radical Sign on that Side of the Equation, and connected by the Signs + or —, tranſpoſe all thoſe Quantities which are without the radical Sign, to the other Side of the Equation; then raiſe both Sides of the Equation to the Square, if the radical Sign expreſſes the Square Root, or to the Cube; if the radical Sign expreſſes the Cube Root, and ſo on, by which means the Equation will be cleared of its *Surds*.

Then if there are no known Quantities on the ſame Side of the Equation with the unknown one the Queſtion is ſolved, but if there are ſtill known Quantities on the ſame Side of the Equation with the unknown Quantity, the Equation is to be reduced by ſome of the Methods before explained, at Art. 46, 47, 48.

The Square Root is expreſſed by this Sign $\sqrt{}$, and the Cube Root by the ſame Sign with a 3 on the Top, thus $\sqrt[3]{}$ that if any Root is taken beſides the Square Root, the Figure over the Sign ſhews what Root it is, but when it is only the Square Root, then there is generally no Figure over the Sign.

Queſtion. 19. *Two Gentlemen were talking of the Number of Acres there were in a Park, the Park-Keeper being preſent, and diſpoſed to ſhew his Learning, told them, that if they extracted the ſquare Root of the Number of Acres there were in the Park, from which ſquare Root ſubſtracting* 5, *this Remainder will be equal to* 50. *How many Arces were there in the Park?*

Let $a =$ the Number of Acres there were in the Park, $b = 5$, $d = 50$.

Now there were a certain Number of Acres in the Park which I call	1	a
The Square Root of which, by Art. 33. is - - - - -	2	$\sqrt{a}$
From which if we ſubſtract 5, or b, that is, connecting b by the Sign —	3	$\sqrt{a}:-b$
Which $\sqrt{a}:-b$ by the Queſtion, is equal to 50, or d, hence -	4	$\sqrt{a}:-b=d$
The Queſtion being now expreſſed in *Algebra*, and obſerving that b, is not under the radical Sign, therefore tranſpoſe b, then -	5	$\sqrt{a}=d+b$
Now all the Quantities being tranſpoſed, which were not under the radical Sign, begin and ſquare both Sides of the Equation, as the radical Sign expreſſes the ſquare Root. But the Square of $\sqrt{a}$ is a, by Art. 43. and the Square of $d+b$ is $dd+2db+bb$, by Art. 32. and making theſe equal to one another, for the Square of equal Quantities or Numbers muſt be equal, and we have -	6	$a=dd+2db+bb$

Hence it appears that a, the unknown Quantity is equal to the Square of the Number repreſented by d, added to twice the Product of the two Numbers repreſented by d and b, and this Sum added to the Square of the Number repreſented by b.

The Square of the Number repreſented by d is dd, or	2500
The Product of the two Numbers repreſented by d and b, is db or 250, and twice that Product is $2db$, or - - - - -	500
The Sum is $dd+2db$, or - -	3000
The Square of the Number repreſented by b is bb, or	25
The Sum is $dd+2db+bb$ or 3025, which is $=a$, or the Number ſought - - -	3025

Hence, I ſay there were 3025 Acres in the Park, which is thus proved, from the Conditions of the Queſtion.

The Number of Acres in the Park were - - -	3025
Now the Square Root of that Number is -	55
From which ſubſtracting - - -	5
There remains what the Queſtion requires - -	50

Queſtion 20. *A Perſon, who had been fortunate at Gaming, was asked how many Guineas he had won, to which he anſwered, that if the ſquare Root of them was extracted, from which Root ſubſtracting 7 he ſhould then have 16 Guineas. What Number of Guineas did he win?*

Let $a =$ the Number of Guineas he won, $b = 7$, $d = 16$.

Now a Perſon won a Number of Guineas which I call -	1	a
The ſquare Root of which by Art. 33. is - - - -	2	$\sqrt{a}$
From which ſubſtracting 7 or b, we have - - -	3	$\sqrt{a} - b$
Which $\sqrt{a} - b$ by the Queſtion, is to be equal to 16 or d, hence	4	$\sqrt{a} - b = d$
Now becauſe b is a known Quantity, and not under the radical Sign, therefore tranſpoſe b, then -	5	$\sqrt{a} = d + b$
All the Quantities not under the radical Sign being now tranſpoſed, in order to clear the Equation of the Surds, raiſe both Sides of the Equation to the Square or ſecond Power. But the Square of $\sqrt{a}$ is a, by Art. 43. and the Square of $d + b$ is $dd + 2db + bb$, by Art. 32. and making theſe two equal to one another, for the Square of equal Quantities or Numbers muſt be equal, and we have	6	$a = dd + 2db + bb$

That is to ſay, the unknown Quantity or a, is equal to the Square of the Number repreſented by d, added to twice the Product of the two Numbers repreſented by d and b, which Sum is to be added to the Square of the Number repreſented by b.

The

The Square of the Number repreſented by d is dd, or	256
The Product of the two Numbers repreſented by d and b, is db, or 112, and twice that Product is $2db$, or	224
The Sum is $dd+2db$, or - - -	480
The Square of the Number repreſented by b is bb, or	49
The Sum is $dd+2db+bb$, or 529, which is equal to a, or the Number ſought - - -	529

Therefore the Perſon won 529 Guineas, and is thus proved, from the Conditions of the Queſtion.

I ſay the Number of Guineas he won was - -	529
For the ſquare Root of that Number is - -	23
And if from that Square Root we ſubſtract - -	7
There remains what the Queſtion requires - -	16

Queſtion 21. *A Gentleman having ſold his Eſtate, an impertinent illiterate Perſon asked him what he had ſold it for, why Sir, replied he, if you extract the ſquare Root of the Number of Guineas for which I ſold it, and then add* 17 *to that Number, this Sum will be equal to* 317. *How many Guineas had the Gentleman for his Eſtate?*

Let $a=$ the Number of Guineas for which the Gentleman ſold his Eſtate, $b=17$, $d=317$.

Now a Gentleman ſold his Eſtate for a Number of Guineas which I call	1	a
The ſquare Root of which by Art. 33. is - - - -	2	$\sqrt{a}$
To which 17, or b, being added we have - - - -	3	$\sqrt{a}+b$
Which $\sqrt{a}+b$ by the Queſtion, is to be equal to 317, or d, hence	4	$\sqrt{a}+b=d$
The Queſtion being now expreſſed in *Algebra*, and becauſe b is a known Quantity, and not under the radical Sign, therefore tranſpoſe b, and we have -	5	$\sqrt{a}=d-b$
Now ſquare both Sides of the Equation, and make them equal to one another, for the Reaſons mentioned in the two laſt Queſtions, and we have - -	6	$a=dd-2db+bb$

From hence, we know that a, the unknown Quantity is equal to the Square of the Number repreſented by d, ſubſtracting from it twice the Product of the Numbers repreſented by d and b, and adding to the Remainder the Square of the Number repreſented by b.

The Square of the Number repreſented by d is dd, or - - - - -	100489
The Product of the two Numbers repreſented by b and d, is db, or 5389, and twice that Product is $2db$, or - - - - - -	10778
Which being ſubſtracted the Remainder is $dd-2db$, or - - - - -	89711
The Square of the Number repreſented by b is bb, or - - - - -	289
The Sum is $dd-2db+bb$ or 90000, which is equal to a, and is the Number ſought -	90000

Now that the Eſtate was ſold for 90000 Guineas, is thus proved, from the Conditions of the Queſtion.

I ſay the Eſtate was ſold for - -	90000 Guineas
For the ſquare Root of that is - -	300
To which if we add - - -	17
The Sum is what the Queſtion requires -	317

Queſtion 22. *A young Gentleman, when he came of Age, asked his Guardian the annual Rent of the Eſtate his Father left him, to which he was anſwered, that if he extracted the ſquare Root of the Number of Pounds for which the Eſtate was rented, and to this Root, if he added 27 it would be equal to 100 Pounds. What was the annual Rent of the Eſtate?*

Let $a=$ the Rent of the Eſtate, $m=27$, $x=100$.

Now the Rent of the Eſtate was -	1	a
The ſquare Root of which by Art. 33. is - - -	2	$\sqrt{a}$
To which 27, or m, being added we have - - - -	3	$\sqrt{a}+m$
Which by the Queſtion is to be equal to 100, or x, hence - -	4	$\sqrt{a}+m=x$

The Queſtion being now expreſſed in *Algebra*, begin by tranſpoſing *m*, for the Reaſons mentioned in the former Queſtions, and then we have - - -	5	$\sqrt{a} = x - m$
Now ſquaring both Sides of the Equation, to take away the radical Sign, as was done in the foregoing Queſtion, and then we have -	6	$a = xx - 2xm + mm$

And there being no more Quantities to be tranſpoſed the Queſtion is ſolved, for we may find the value of *a*, in Numbers, from the *Algebraic* Work, thus

The Square of the Number repreſented by x is xx, or	10000
The Product of the two Numbers repreſented by the Letters x and m, is xm, or 2700, and twice that Product is $2xm$, or - - - - - -	5400
Which being ſubſtracted leaves $xx - 2xm$, or -	4600
The Square of the Number repreſented by m is mm, or	729
The Sum is 5329, or $xx - 2xm + mm$, which is equal to a, or the Number ſought - - -	5329

And that the annual Rent of the Eſtate was 5329 Pounds, may be thus proved, from the Conditions of the Queſtion.

I ſay the annual Rent of the Eſtate was -	5329 Pounds
For the ſquare Root of that is - -	73
To which there being added - - -	27
The Sum is what the Queſtion requires -	100

Queſtion 23. *To find that Number to which* 1290 *being added, if the ſquare Root of this Sum is extracted, from which Root ſubſtracting* 29 *the Remainder may be equal to* 71.

Let $a =$ the Number ſought, $b = 1290$, $d = 29$, $x = 71$.

There is a Number ſought which I call - - -	1	a
To which 1290, or b, being added we have - -	2	$a + b$
The ſquare Root of which Sum by Art. 34. is - -	3	$\sqrt{a + b}$

From

From which ſubſtracting 29, or d, we have	4	$\sqrt{a+b}:-d$
Which by the Queſtion is equal to 71, or x, hence	5	$\sqrt{a+b}:-d=x$
Now begin the Solution, with tranſpoſing d, it being a known Quantity, and not under the radical Sign, and then	6	$\sqrt{a+b}=x+d$
All the Quantities on one Side of the Equation being now under the radical Sign, to take away that, as the unknown Quantity is under it, ſquare both Sides of the Equation as before. Now the Square of $\sqrt{a+b}$ is $a+b$, by Art. 43. and the Square of $x+d$ is $xx+2xd+dd$, by Art. 32. and as the Squares of equal Numbers, or Quantities, muſt be equal to one another, hence	7	$a+b=xx+2xd+dd$
Now tranſpoſe b, it being a known Quantity, and then	8	$a=xx+2xd+dd-b$

From whence we may find the Value of a, in Numbers. Thus,

The Square of the Number repreſented by x is xx, or	5041
The Product of the two Numbers repreſented by x and d, is xd, or 2059, and twice that Product is $2xd$, or	4118
The Sum is $xx+2xd$, or	9159
The Square of the Number repreſented by d is dd, or	841
The Sum is $xx+2xd+dd$, or	10000
From which ſubſtracting the Number repreſented by b	1290
There remains 8710, or $xx+2xd+dd-b$, which is $=a$, or the Number ſought	8710

Which is proved thus, from the Conditions of the Queſtion.

I ſay

I ſay the Number ſought is	8710
For if to that we add	1290
The Sum is	10000
The ſquare Root of which is	100
From which ſubſtracting	29
There remains what the Queſtion requires	71.

Queſtion 24. *A Perſon being asked his Age, replied, that if from my Age you ſubſtract* 11, *and extract the ſquare Root of the Remainder, to which Root adding* 13, *this Sum will be equal to* 20. *What was the Age of the Perſon?*

Let $a=$ the Number of Years, or Age of the Perſon, $b=11$, $m=13$, $d=20$.

Now the Age of the Perſon is	1	a
From which if we ſubſtract 11, or *b*, we have	2	$a-b$
The ſquare Root of which by Art. 34. is	3	$\sqrt{a-b}:$
To which adding 13, or *m*, we have	4	$\sqrt{a-b}:+m$
Which by the Queſtion, is equal to 20, or *d*, hence we have	5	$\sqrt{a-b}:+m=d$
The Queſtion being thus expreſſed in *Algebra*, and *m* not being under the radical Sign, therefore tranſpoſe *m*, then	6	$\sqrt{a-b}=d-m$
Now ſquare both Sides of the Equation, to clear it of the Surd, as in the former Queſtions. But the Square of $\sqrt{a-b}$, is $a-b$, by Art. 43. and the Square of $d-m$ by Art. 32. is $dd-2dm+mm$, then as the Square of equal Quantities are equal, we have	7	$a-b=dd-2dm+mm$
And by tranſpoſing *b*, we have	8	$a=dd-2dm+mm+b$

By which we may find what *a* is in Numbers. Thus,

The Square of the Number repreſented by d is dd, or	400
The Product of the two Numbers repreſented by d and m, is dm, or 260, and twice that Product is $2dm$, or	520
Which 520 being ſubſtracted from 400, leaves $dd-2dm$, or -120, ſee Queſtion 6. - -	-120
The Square of the Number repreſented by m is mm, or	169
Which 169 being added to -120, makes $dd-2dm+mm$, or $+49$, ſee Queſtion 6. - - - -	49
To which adding the Number repreſented by b - -	11
The Sum is 60, which I ſay is $=a$, or the Age of the Perſon	60

And is proved from the Conditions of the Queſtion, Thus,

I ſay the Perſon was - - -	60 Years old
For if from that you ſubſtract - -	11
There remains - - -	49
The Square Root of which is - -	7
To which adding - - -	13
The Sum is what the Queſtion requires -	20

To reduce an Equation by Evolution.

50. This is done by the Extraction of Roots, for if after all the known Quantities have been carried to the other Side of the Equation from the unknown Quantity, and it appears that one Side of the Equation is the *Square*, *Cube*, or any Power of the unknown Quantity, then you muſt extract ſuch Root of both Sides of the Equation as will depreſs or lower this Power of the unknown Quantity to the *firſt* Power, that is, if one Side of the Equation is the *Square* of the unknown Quantity, then the *Square Root* muſt be extracted, and if it is the *Cube* of the unknown Quantity, then the *Cube Root* muſt be extracted, and ſo on, which depreſſing the unknown Quantity to the firſt Power, the Queſtion is anſwered.

Queſtion. 25. *What is that Number, if to the Square of which there is* 51 *added, the Sum may be* 100.

Let a = the Number ſought, $b=51$, $m=100$.

Now there is a Number ſought which I call	1	a
The Square of which by Art. 31. is	2	aa
To which 51, or *b*, being added we have	3	$aa+b$
And this $aa+b$ is by the Queſtion, to be equal to 100, or *m*, hence	4	$aa+b=m$
The Queſtion being expreſſed in *Algebra*, begin and tranſpoſe *b*, then	5	$aa=m-b$
The known Quantities being now all on one Side of the Equation, and the other Side being aa, or the Square of *a*, therefore by the Rule, extract the ſquare Root of both Sides of the Equation. Now the Square Root of aa is *a*, by Art. 33. and the Square Root of $m-b$ is $\sqrt{m-b}$, by Art. 34. and as the ſquare Root of equal Quantities muſt be equal, therefore	6	$a=\sqrt{m-b}$

Hence *a*, or the Number ſought is equal to the Number repreſented by *m*, ſubſtracting from it the Number repreſented by *b*, and extracting the ſquare Root of the Remainder.

The Number repreſented by *m*, is	100
From which ſubſtracting *b*, or	51
There remains $m-b$, or	49
The ſquare Root of which is $\sqrt{m-b}$, or 7, and is equal to *a*, or the Number ſought	7

And is thus proved.

I ſay the Number ſought is	7
The Square of which is	49
To which adding	51
The Sum is what the Queſtion requires	100

Queſtion 26. *A Merchant had gained ſo many Pounds, that if from the Square of his Gains there is ſubſtracted* 101, *and to this Remainder adding* 500, *this Sum is* 3000 *Pounds. What had the Merchant gained?*

Let $a =$ the Gain of the Merchant, $b = 101$, $m = 500$, $p = 3000$.

Then a Merchant had gained a certain Number of Pounds	1	a
The Square of which is, by Art. 31.	2	aa
From which ſubſtracting 101, or b, we have	3	$aa-b$
To which adding 500, or m, we have	4	$aa-b+m$
Which by the Queſtion, is to be equal to 3000, or p, hence	5	$aa-b+m=p$
By tranſpoſing m we have	6	$aa-b=p-m$
By tranſpoſing b it is	7	$aa=p-m+b$
By extracting the ſquare Roots, as at the ſixth Step of the laſt Example, then	8	$a=\sqrt{p-m+b}$

That is, a is equal to the Number repreſented by p, ſubſtracting from it the Number repreſented by m, and adding to this Remainder the Number repreſented by b, and extracting the ſquare Root of this Sum.

The Number repreſented by p, is	3000
From which ſubſtracting m, or	500
There remains $p-m$, or	2500
To which adding b, or	101
The Sum is $p-m+b$, or	2601
The ſquare Root of which is $\sqrt{p-m+b}$, or 51, and is equal to a or the Number ſought	51

And is thus proved, from the Conditions of the Queſtion.

I ſay the Merchant gained	51 Pounds
For the Square of this is	2601
From which ſubſtracting	101
There remains	2500
To which adding	500
The Sum is what the Queſtion requires	3000

Queſtion 27. *If to the Square of the Number of Miles a Perſon had travelled there is added* 97, *ſubſtracting from this Sum* 251, *and adding to this Remainder* 160, *this Sum will be* 10006. *How many Miles had he travelled?*

Let $a=$ the Number of Miles he had travelled, $b=97$, $m=251$, $x=160$, $z=10006$.

Then a Perſon had travelled a certain Number of Miles	1	a
The Square of which is, by Art. 31.	2	aa
To which adding 97, or *b*, it is	3	$aa+b$
From which ſubſtracting 251, or *m*, gives	4	$aa+b-m$
To which adding 160, or *x*, it is	5	$aa+b-m+x$
Which by the Queſtion, is to be equal to 10006, or *z*, whence	6	$aa+b-m+x=z$
By tranſpoſing *x* it is	7	$aa+b-m=z-x$
By tranſpoſing *m* we have	8	$aa+b=z-x+m$
By tranſpoſing *b* then	9	$aa=z-x+m-b$
By extracting the ſquare Root as at the eighth Step of the laſt Example, or at the ſixth Step of Queſtion 25, we have	10	$a=\sqrt{z-x+m-b}$

That is, from the Number repreſented by *z*, ſubſtract the Number repreſented by *x*, to the Remainder add the Number repreſented by *m*, from which Sum ſubſtract the Number repreſented by *b*, extract the ſquare Root of the Remainder and it will be the Number ſought.

The Number repreſented by *z*, is	10006
From which ſubſtracting *x*, or	160
There remains $z-x$, or	9846
To which adding *m*, or	251
The Sum is $z-x+m$, or	10097
From which ſubſtracting *b*, or	97
There remains $z-x+m-b$, or	10000
The ſquare Root of which, or $\sqrt{z-x+m-b}$, is the Number ſought	100

PROOF.

I ſay the Perſon had travelled - -	100 Miles
For the Square of that is - -	10000
To which adding - - -	97
The Sum is - - -	10097
From which ſubſtracting - -	251
There remains - - -	9846
To which adding - - -	160
The Sum is what the Queſtion requires -	10006

Queſtion 28. *A General, upon numbering his Army, found, that if from the Square of the Number of Men in his Army there was ſubſtracted* 3196, *and to this Remainder adding* 2721, *from which Sum ſubſtracting* 1711 *there would remain* 99997814. *To find the Number of Men in the Army?*

Let $a =$ the Number of Men in the Army, $b = 3196$, $m = 2721$, $x = 1711$, $z = 99997814$.

The Number of Men in the Army was - - - -	1	a
The Square of which is, by Art. 31. - - -	2	aa
From which ſubſtracting 3196, or b, it is - - -	3	$aa - b$
To which adding 2721, or m, gives	4	$aa - b + m$
From which ſubſtracting 1711, or x, we have - -	5	$aa - b + m - x$
Which by the Queſtion, is equal to 99997814, or z, hence -	6	$aa - b + m - x = z$
Firſt, by tranſpoſing x - -	7	$aa - b + m = z + x$
By tranſpoſing m - - -	8	$aa - b = z + x - m$
By tranſpoſing b - -	9	$aa = z + x - m + b$
By extracting the ſquare Root, as in the former Examples -	10	$a = \sqrt{z + x - m + b}$

By Numbers thus:

z is

z is in Numbers - - -	99997814
To which adding x, or - - -	1711
The Sum is $z+x$, or - - -	99999525
From which ſubſtracting m, or - -	2721
There remains $z+x-m$, or - - -	99996804
To which adding b, or - - -	3196
The Sum is $z+x-m+b$, or - -	100000000
The ſquare Root of which is a, or the Number ſought - - - - -	10000

Which is thus proved.

I ſay the Number of Men in the Army was -	10000
For the Square of that is - -	100000000
From which ſubſtracting - - -	3196
There remains - - -	99996804
To which adding - - -	2721
The Sum is - - -	99999525
From which ſubſtracting - - -	1711
There remains what the Queſtion requires -	99997814

51. Theſe being the particular Methods by which Equations are reduced, or Queſtions anſwered, we ſhall now add ſome Examples where all theſe Methods are promiſcuouſly uſed.

Queſtion 29. *A Merchant broke for ſo many Pounds, that if they were multiplied by* 4, *and this Product divided by* 6, *and extracting the ſquare Root of the Quotient, from which ſubſtracting* 60, *there remains* 40. *What was the Sum for which the Merchant broke?*

Let $a=$ the Number of Pounds ſought, $b=4$, $d=6$, $m=60$, $p=40$.

Then the Merchant broke for a Number of Pounds	1	a
Which being multiplied by 4, or b, we have -	2	ba
This being divided by 6, or d, we have - -	3	$\frac{ba}{d}$

The

The ſquare Root of which is, by Art. 33. -	4	$\sqrt{\frac{ba}{d}}$
From this ſubſtracting 60, or *m*, we have -	5	$\sqrt{\frac{ba}{d}} - m$
Which by the Queſtion, is equal to 40, or *p*, hence	6	$\sqrt{\frac{ba}{d}} - m = p$
Becauſe *m* is not under the radical Sign therefore tranſpoſe it, by Art. 49.	7	$\sqrt{\frac{ba}{d}} = p + m$
Now ſquaring both Sides of the Equation by Art. 49.	8	$\frac{ba}{d} = pp + 2pm + mm$
And multiplying by *d*, by Art. 47, then -	9	$\frac{dba}{d} = dpp + 2dpm + dmm$
Rejecting *d* from $\frac{dba}{d}$ and putting down the other Quantities without any alteration, as at Art. 47, we have -	10	$ba = dpp + 2dpm + dmm$
Dividing by *b*, by Art. 48, then	11	$\frac{ba}{b} = \frac{dpp + 2dpm + dmm}{b}$
Rejecting *b* from $\frac{ba}{b}$, and puting down the other Quantities without any alteration, as at Art. 47, or 48, we have -	12	$a = \frac{dpp + 2dpm + dmm}{b}$

In Numbers thus:

$$\begin{array}{rl} dpp = & 9600 \\ +\,2dpm = & 28800 \\ +\,dmm = & 21600 \\ \hline \end{array}$$

Sum 60000 or $dpp + 2dpm + dmm$

Now dividing 60000, or $dpp + 2dpm + dmm$ by 4, or *b*, we have $\frac{dpp + 2dpm + dmm}{b}$, or 60000, divided by $4 = 15000$ which is equal to *a*, or the Number of Pounds for which the Merchant broke.

PROOF.

PROOF.

$$\begin{array}{r} 15000 \\ 4 \\ \hline 6)\,60000 \\ \hline \end{array}$$

10000 (100 the ſquare Root of 10000
60

40 as the Queſtion requires.

Queſtion 30. *A Gentleman having bought a Houſe, and being diſpoſed to try the Knowledge of his Son in* Algebra, *told him, if the Number of Pounds the Houſe coſt, was divided by* 8, *and that Quotient being multiplied by* 50, *and extracting the ſquare Root of this Product, to which adding* 10, *this Sum would be* 60 *Pounds. What did the Houſe coſt?*

Let $a =$ the Price of the Houſe, $b = 8$, $d = 50$, $m = 10$, $p = 60$.

Now the Price of the Houſe is - -	1	a
Which being divided by 8, or b, it is -	2	$\frac{a}{b}$
This being multiplied by 50, or d, we have -	3	$\frac{da}{b}$
The ſquare Root of which is, by Art. 33. -	4	$\surd\frac{dd}{b}$
To which adding 10, or m	5	$\surd\frac{da}{b} + m$
Which, by the Queſtion, is equal to 66, or p, hence	6	$\surd\frac{da}{b} + m = p$
The Queſtion being now expreſſed in *Algebra*, and m not being under the radical Sign, tranſpoſe it by Art. 49. then -	7	$\surd\frac{da}{b} = p - m$
Now ſquaring both Sides of the Equation, by Art. 49.	8	$\frac{da}{b} = pp - 2pm + mm$
And multiplying by b, by Art. 47. - -	9	$\frac{bda}{b} = bpp - 2bpm + bmm$

Rejecting b from $\frac{bda}{b}$, and putting down the reſt as at the twelfth Step, of the laſt Queſtion, then	10	$da = bpp - 2bpm + bmm$
Dividing by d, by Art. 48. and	11	$\frac{da}{d} = \frac{bpp - 2bpm + bmm}{d}$
Rejecting d from $\frac{da}{d}$, and putting down the reſt as at Art. 47, or 48. and	12	$a = \frac{bpp - 2bpm + bmm}{d}$

In Numbers,

$bpp = 28800$
$-2bpm = -\ 9600$
19200
$+bmm = 800$
$d = 5|0)\ 2000|0$
$400 = a$, the Price of the Houſe, that is, the Houſe coſt 400 *l.*

PROOF.

8) 400
50
50
2500 (50 the ſquare Root of 2500
10
60 as the Queſtion requires.

CONSECTARY.

If the Reader compares the eighth, ninth, and tenth Steps of the laſt Work, he will find that to multiply any Fraction by its Denominator, or any Dividend by its Diviſor, is only to reject the Denominator, or Diviſor, from that Quantity, and multiply it into all the other Quantities, thus; the Equation at the eighth Step is $\frac{da}{b} = pp - 2pm + mm$, which being multiplied by its Denominator b, we have at the tenth Step $da=bpp-2bpm+bmm$; the ninth Step, or $\frac{bda}{b}=bpp-2bpm+bmm$, being only a more particular Illuſtraion of the Work.

And by comparing the tenth eleventh and twelfth Steps he will find, that to divide any Quantity, by any Letter in that Quantity, is only to reject that Letter from the Quantity, and placing it as a Diviſor to the other Quantities; thus, at the tenth Step, the Equation is $da=bpp-2bpm+bmm$, which being divided by d, gives us at the twelfth Step $a=\frac{bpp-2bpm+bmm}{d}$, the eleventh Step, or $\frac{da}{d}=\frac{bpp-2bpm+bmm}{d}$, being only a more particular Illuſtration of the Work.

Therefore for the future we ſhall leave out ſuch Steps as the ninth, and eleventh, the Learner being now a little acquainted with the Science, I did not chooſe to do it at firſt, my Deſign being to make this curious Science as eaſy as poſſible.

Queſtion 31. *A Running-Footman being ſent of an Errand was told, that if he ſquared the diſtance he was to run, and multiplied that by* 4, *and divided this Product by* 40, *to this Quotient adding* 500, *from which Sum ſubſtracting* 1400, *and extracting the ſquare Root of the Remainder it would be* 10. *How many Miles was the Footman to run?*

Let $a=$ the Number of Miles the Footman was to run, $b=4$, $d=40$, $m=500$, $x=1400$, $p=10$.

The Number of Miles the Footman was to run let be -	1	a
Which being ſquared is, by Art. 31. - -	2	aa
This being multiplied by 4, or b, we have - -	3	baa
This being divided by 40, or d, it is	4	$\frac{baa}{d}$
To which adding 500, or m, gives	5	$\frac{baa}{d}+m$

From which ſubſtracting 1400, or x, we have	6	$\frac{baa}{d}+m-x$
The ſquare Root of which is, by Art. 34.	7	$\sqrt{\frac{baa}{d}+m-x}$
Which by the Queſtion, is equal to 10, or p, therefore	8	$\sqrt{\frac{baa}{d}+m-x}=p$
Now ſquare both Sides of the Equation, by Art. 49. and	9	$\frac{baa}{d}+m-x=pp$
By tranſpoſing x, we have	10	$\frac{baa}{d}+m=pp+x$
By tranſpoſing m, we have	11	$\frac{baa}{d}=pp+x-m$
By multiplying by d by the *Conſectary*, Page 130.	12	$baa=dpp+dx-dm$
And dividing by b by the *Conſectary*, Page 130.	13	$aa=\frac{dpp+dx-dm}{b}$
Now extracting the ſquare Root, by Art. 50.	14	$a=\sqrt{\frac{dpp+dx-dm}{b}}$

In Numbers.

$$\begin{array}{rl} dpp = & 4000 \\ +dx = & 56000 \\ \hline & 60000 \\ -dm = & -20000 \\ \hline b = 4) & 40000 \\ \hline & 10000 \end{array}$$

10000 (100 the ſquare Root of 10000, hence the Footman was to run 100 Miles.

PROOF.

$$\sqrt{\frac{4aa}{40}+500-1400}=10$$

I have not drawn out the Proof of the laſt Queſtion into particulars but only expreſſed it at once, that is, four times the Square of a (which is found to be 100) being divided by 40, if to this Quotient we add 500, and from this Sum ſubſtract 1400; the ſquare Root of this Remainder will be equal to 10. And now I ſhall expreſs all the Conditions of the Queſtion at the firſt

Equation, that the Learner may form ſome little Judgment in what Manner to ſhorten his Work; and if he conceives how the Proof of the laſt Queſtion is expreſſed, it will eaſily lead him to the Knowledge of expreſſing the Conditions of the Queſtion, or raiſe the Equations as ariſe from the Queſtion without particularizing every Condition. But if the Learner finds any Difficulty in this he may proceed as before.

Queſtion 32. *A Gentleman who had been at the Gaming-Tables, and loſing, ſome of his Acquaintance laughing at him for his Folly, asked how much he had loſt; to which he anſwered, if you ſquare the Number of Pounds I have loſt, and divide that by* 4, *multiplying this Quotient by* 10, *to which Product add* 3900, *then extracting the ſquare Root of this Sum, from which ſubſtracting* 80, *the Remainder will be equal to* 90. *How much had he loſt?*

Let $a =$ the Money loſt, $b = 4$, $d = 10$, $m = 3900$, $p = 80$, $z = 90$.

Then by the Queſtion	1	$\sqrt{\frac{daa}{b}+m} : -p = z$
By tranſpoſing p it not being under the radical Sign, by Art. 49. we have	2	$\sqrt{\frac{daa}{b}+m} = z+p$
By ſquaring both Sides of the Equation, by Art. 49. then -	3	$\frac{daa}{b}+m = zz+2zp+pp$
By tranſpoſing m, it is	4	$\frac{daa}{b} = zz+2zp+pp-m$
Multiplying by b by the Corollary, Queſtion 30.	5	$daa = bzz+2bzp+bpp-bm$
Dividing by d by the ſame -	6	$aa = \frac{bzz+2bzp+bpp-bm}{d}$
Extracting the ſquare Root, by Art. 50.	7	$a = \sqrt{\frac{bzz+2bzp+bpp-bm}{d}}$

In Numbers.

bzz

$$\begin{array}{rl}
bzz = & 32400 \\
2bzp = & 57600 \\
bpp = & \underline{25600} \\
 & 115600 \\
-bm = & -\ \underline{15600} \\
d = 1|0) & \underline{10000|0} \\
 & 10000(100 = a \text{ the Sum of Money loſt.} \\
 & \underline{1} \\
 & 0
\end{array}$$

PROOF.

$$\sqrt{\frac{10aa}{4}+3900}:-80=90$$

To reduce an Equation when the unknown Quantity is in ſeveral Terms.

52. When the unknown Quantity is in more Terms than one, bring all thoſe Terms which have the unknown Quantity to one Side of the Equation, taking Care that the *greateſt Co-efficient* of the unknown Quantity has at laſt the *affirmative* Sign, and carrying all the Quantities that are known on the other Side of the Equation; then divide both Sides of the Equation by *all the Co-efficients* of the unknown Quantity, connected with the ſame Signs of + and − as they then happen to have, which will reduce the Equation as in the following Examples.

Queſtion 33. *There is a certain Number which being multiplied by* 10, *if this Product is divided by* 2, *to this Quotient adding* 19, *and ſubſtracting* 99 *from that Sum, the Remainder will be equal to the Number ſought.*

Let a = the Number ſought, $b = 10$, $d = 2$, $m = 19$, $z = 99$.

By the Queſtion	1	$\frac{ba}{d} + m - z = a$
By tranſpoſing z	2	$\frac{ba}{d} + m = a + z$
By tranſpoſing m	3	$\frac{ba}{d} = a + z - m$
By multiplying by d by the *Conſectary*, Page 130.	4	$ba = da + dz - dm$
Becauſe d is leſs than b, tranſpoſe da, that both the Terms which have the unknown Quantity may be on the ſame Side of the Equation, then	5	$ba - da = dz - dm$
And dividing according to the Rule by $b - d$, the two Coefficients of a, we have	6	$a = \frac{dz - dm}{b - d} = 20$ the (Number ſought.

PROOF.

$$\frac{10a}{2} + 19 - 99 = a$$

The Diviſion at the ſixth Step, *viz.* that $ba - da$ being divided by $b - d$, ſhould leave only a, may perhaps a little perplex the Learner, and if it does, I adviſe him to examine *Art.* 16. where he may obſerve that in multiplying any compound Quantity by any ſingle Letter, that Letter goes into every Term of the Product, therefore the Multiplier is not ſo many Times that Letter as the Number of Terms are in which that Letter is found, but only the ſingle Letter multiplied ſucceſſively into all the other Quantities; hence if this product is to be divided by all thoſe Quantities, the Quotient will be the ſingle Letter, and not ſo many Times that Letter as the Number of Terms are in which it is found. See further Queſtion 38.

Queſtion 34. *A Gentleman bought an Eſtate for ſo many Pounds, that if they were multiplied by* 4, *and this Product divided by* 5, *from which Quotient ſubſtracting* 600, *and adding to this Remainder* 6 *Times what the Eſtate coſt, this Sum will be equal to* 6200 *Pounds. How much did the Eſtate coſt?*

Let $a =$ what the Eſtate coſt, $b = 4$, $d = 5$, $m = 600$, $p = 6$, $x = 6200$.

Then by the Queſtion -	1	$\frac{ba}{d} - m + pa = x$
By tranſpoſing m, we have -	2	$\frac{ba}{d} + pa = x + m$
Multiplying by d by the *Conſectary*, Page 130. -	3	$ba + dpa = dx + dm$
Dividing by $b + dp$ the Co-efficients of a, as in the laſt Queſtion, and we have -	4	$a = \frac{dx + dm}{b + dp} = 1000$

Therefore the Eſtate coſt 1000 Pounds.

PROOF.

$$\frac{4a}{5} - 600 + 6a = 6200$$

Queſtion 35. *A Perſon had a certain Number of Shillings, which being multiplied by* 4, *this Product being divided by* 11, *to this Quotient adding* 90, *and from this Sum taking away* 30, *the ſquare Root of this Remainder will be equal to the ſquare Root of the Number of Shillings ſought, when diminiſhed by* 10.

Let $a =$ the Number of Shillings ſought, $b = 4$, $d = 11$, $x = 90$, $p = 30$, $z = 10$.

Then by the Queſtion -	1	$\sqrt{\frac{ba}{d} + x - p} = \sqrt{a - z}$
Becauſe there is no Quantity on each Side of the Equation but what is under the radical Sign, therefore ſquare both Sides of the Equation, by Art. 49.	2	$\frac{ba}{d} + x - p = a - z$
Multiplying by d by the *Conſectary*, Page 130. -	3	$ba + dx - dp = da - dz$
Becauſe d one Co-efficient of a is greater then b the other Co-efficient of a, tranſpoſe ba, then - - -	4	$dx - dp = da - dz - ba$

Tranſpoſing

Tranſpoſing dz - -	5	$dz+dx-dp=da-ba$
Dividing by $d-b$ the two Co-efficients of a, as at Queſtion 33. Step 6, we have - -	6	$a=\frac{dz+dx-dp}{d-b}=110$ the (Number ſought.

PROOF.

If $a=110$, then $\sqrt{\frac{4a}{11}+90}-30=\sqrt{a-10}$

Queſtion 36. *A Running-Footman, forward to ſhew his Learning, being in Company, ſaid, if the Number of Miles he had run was multiplied by* 7, *to which Product adding* 550, *and ſubſtracting* 20 *from this Sum, and dividing this Remainder by* 10, *the ſquare Root of this Quotient will be the ſame as if you had added* 14 *Miles to thoſe he had run, and extracted the ſquare Root of that Sum.*

Let $a=$ the Number of Miles he had run, $b=7$, $d=550$, $m=20$, $p=10$, $x=14$.

Then by the Queſtion -	1	$\sqrt{\frac{ba+d-m}{p}}=\sqrt{a+x}$
There being no Quantity but what is under the radical Sign, therefore ſquare both Sides of the Equation as at the ſecond Step of the laſt Queſtion	2	$\frac{ba+d-m}{p}=a+x$
Multiplying by p by the *Conſectary*, Page 130.	3	$ba+d-m=pa+px$
Becauſe p one Co-efficient of a is greater then b the other Co-efficient of a, therefore tranſpoſe ba	4	$d-m=pa+px-ba$
By tranſpoſing px - -	5	$d-m-px=pa-ba$
Dividing by $p-b$ the two Co-efficients of a, as at Queſtion 33. Step 6, we have - - -	6	$a=\frac{d-m-px}{p-b}=130$ the Number of Miles the Footman had run.

PROOF.

$$\sqrt{\frac{7a+550-20}{10}} = \sqrt{a+14}$$

To reduce an Equation when the ſame Quantity, either known or unknown, is in every Term of the Equation.

53. In any Algebraic Operation if the ſame Quantity either known or unknown is in every Term of any Equation, *then divide every Term of the Equation by that Quantity which will reduce the Equation to more ſimple Terms,* as in the following Queſtions.

Queſtion 37. *To find a Number which multiplied by 4, and the Product added to the Quotient of the ſame Number, multiplied by 56 and divided by 7, this Sum will be equal to the ſquare of the Number ſought.*

Let $a =$ the Number ſought, $b = 4$, $d = 56$, $m = 7$.

Then by the Queſtion -	1	$ba + \frac{da}{m} = aa$
Multiplying by *m*, by the *Conſectary*, Page 130.	2	$mba + da = maa$
Becauſe *a* is in every Term of the Equation, divide by *a*, then - -	3	$mb + d = ma$
Dividing by *m* the Coefficient of *a* by the *Conſectary*, Page 130. -	4	$a = \frac{mb+d}{m} = 12$ the Number ſought.

PROOF.

$$4a + \frac{56a}{7} = aa$$

Queſtion 38. *There are two Towns at ſuch a Diſtance, that if the Number of Miles between them is multiplied by 79, and this Product added to their Diſtance, the ſquare Root of this Sum will be equal to the Diſtance of the two Towns multiplied by 2.*

Let $a =$ the Diſtance of the Towns, $b = 79$, $m = 2$.

Then by the Queſtion - - -	1	$\sqrt{ba + a} = ma$
There being no rational Quantities on the ſame Side of the Equation where the radical Sign is, ſquare both Sides of the Equation by Art. 49. -	2	$ba + a = mmaa$
Dividing by *a*, it being in every Term of the Equation, and - - -	3	$b + 1 = mma$
Dividing by *m m* the Co-efficient of *a* by the *Conſectary*, Page 136. -	4	$a = \frac{b+1}{mm} = 20$

Hence the Diſtance between the two Towns is 20 Miles.

PROOF.

$$\sqrt{79a + a} = 2a$$

If the Reader does not eaſily conceive that dividing $ba + a$, or $ba + 1a$, at the ſecond Step, by a, gives $b + 1$, as at the third Step, I would adviſe him to conſider what is ſaid at Queſtion 33; to which may be added, that $\overline{b + 1} \times a = ba + a$, whereas $\overline{b + 1} \times 2a = 2ba + 2a$, a Product very different from $ba + a$. Or it may be explained thus, $\frac{ba + 1a}{a} = b + 1$, the a being rejected by Art. 22, and 26.

The Manner of regiſtering the Steps of an Algebraic *Operation explained.*

54. Having, in this copious Manner, explained to the young *Analyſt*, the different Methods of managing Equations, to ſave the Trouble of uſing ſo many Words; I ſhall now ſhew him the Method of *regiſtering the Steps*, introduced by the ingenious Dr *John Pell.*

To *regiſter the Steps* of an Analytic Operation is only to expreſs in the Margin of the Work, by *Symbols* inſtead of Words, what has been done; and to render it as eaſy as may be to the Learner, we ſhall reſume the Work of one of the former Queſtions, and expreſs by Words what is done in one Column, in another Column expreſs the ſame thing by *Symbols*, or Characters, and in the third Column place the Work itſelf, that by comparing the Operation with the Obſervations that follow it, the Reader may the more eaſily underſtand the Manner of *regiſtering the Steps.*

Queſtion 38. *A Running Footman being ſent of an Errand was told, that if he ſquared the Diſtance he was to run, and multiplied that by* 4, *and divided this Product by* 40, *to this Quotient adding* 500, *from which Sum ſubſtracting* 1400, *and extracting the ſquare Root of the Remainder it would be* 10. *How many Miles was the Footman to run?* (this is Queſtion 31.)

Let $a =$ the Number of Miles the Footman was to run. $b = 4$, $d = 40$, $m = 500$, $x = 1400$, $p = 10$.

	Regiſter		
Then, by the Queſtion, we have the ſame Equation as at the eighth Step, Queſtion 31.		1	$\sqrt{\frac{baa}{d} + m - x} = p$
Squaring both Sides of the Equation, or involving them to the ſecond Power, by Art. 49.	1 ⊛ 2	2	$\frac{baa}{d} + m - x = pp$
By tranſpoſing x at the ſecond Equation	$2 + x$	3	$\frac{baa}{d} + m = pp + x$
By tranſpoſing m at the third Equation	$3 - m$	4	$\frac{baa}{d} = pp + x - m$
Multiplying the fourth Equation by d	$4 \times d$	5	$baa = dpp + dx - dm$
Dividing the fifth Equation by b	$5 \div b$	6	$aa = \frac{dpp + dx - dm}{b}$
Extracting the ſquare Root of the ſixth Equation, by Art. 50.	6 ɯ 2	7	$a = \sqrt{\frac{dpp+dx-dm}{b}} = 10$

For another Inſtance let us take Queſtion 33.

Queſtion 39. *There is a certain Number, which being multiplied by* 10, *if this Product is divided by* 2, *to this Quotient adding* 19, *and ſubſtracting* 99 *from that Sum, the Remainder will be equal to the Number ſought.*

Let $a =$ the Number ſought, $b = 10$, $d = 2$, $m = 19$, $x = 99$.

Then

	Regiſter		
Then by the Queſtion - -		1	$\frac{ba}{d}+m-z=a$
By tranſpoſing *z* from the firſt Equation -	1 + *z*	2	$\frac{ba}{d}+m=a+z$
By tranſpoſing *m* from the ſecond Equation -	2 — *m*	3	$\frac{ba}{d}=a+z-m$
Multiplying the third Equation by *d* -	3 × *d*	4	$ba=da+dz-dm$
By tranſpoſing *d a* from the fourth Equation	4 — *d a*	5	$ba-da=dz-dm$
Dividing the fifth Equation by *b* — *d* the two Co-efficients of *a*, by Art. 52. - -	5 ÷ *b* — *d*	6	$a=\frac{dz-dm}{b-d}=20$

From theſe two Examples we may obſerve, that to *regiſter* any Operation is only to put down the Figure which ſtands in the Column againſt that Equation, from which we intend to raiſe the next Equation, and after that the *Sign* of either *Addition*, *Subſtraction*, *Multiplication*, *Diviſion*, *Involution* and *Evolution*, according as the Caſe requires, and after that the Quantity which ſuffers the Alteration.

Thus at Queſtion 38, the firſt Equation being raiſed or involved to the ſecond Power produces the ſecond Equation, therefore, I ſay in the *Regiſter* 1 ⊚ 2, that is, the firſt Equation involved to the ſecond Power gives the ſecond Equation, and in the ſame Operation.

Becauſe the fourth Equation is produced from the third by tranſpoſing *m* with the Sign —, therefore in the *Regiſter* I ſay 3 — *m*, that is, the third Equation — *m*, produces the fourth Equation. And,

As the fifth Equation is produced from the fourth by multiplying by *d*, therefore I ſay in the *Regiſter* 4 × *d*, that is, the fourth Equation, multiplied by *d*, produces the fifth Equation. And,

As the ſixth Equation is produced from the fifth by dividing by *b*, therefore, I ſay in the *Regiſter* 5 ÷ *b*, that is, the fifth Equation, divided by *b*, produces the ſixth Equation. And,

As the ſeventh Equation is produced from the ſixth by extracting the ſquare Root, I ſay in the *Regiſter* 6 *ɯ* 2, that is, the ſixth Equation, having the ſquare Root extracted, produces the ſeventh Equation.

That

Whence, as I ſaid above, to regiſter any Operation is only to put down, whether it is the firſt, ſecond, third, fourth, or any other Equation, which ſuffers the Alteration, and from which the new Equation is raiſed; and after that Figure to expreſs in Characters, or Signs, the Alteration that is then made to gain the new Equation.

The Method of reſolving Queſtions that contain two Equations, and two unknown Quantities.

55. THE foregoing Queſtions requiring only one unknown Number to be found, their Conditions were all expreſſed in one Equation, which Equation being reduced by the Rules already delivered, the Queſtion was anſwered.

But if the Queſtion requires two unknown Quantities to be found, then there are generally raiſed two Equations from the Queſtion, each of them including both the unknown Quantities; whereas all the former Queſtions were expreſſed by Equations that contained only one unknown Quantity, and their Conditions were likewiſe expreſſed by one Equation.

And when any Queſtion is propoſed, which being *Algebraically* expreſſed, if it is found to contain two Equations, and two unknown Quantities, ſuch Queſtions may be reſolved by this

RULE.

Find what the ſame unknown Quantity is *equal* to in each of the two Equations which ariſe from the Conditions of the Queſtion, then make theſe two Equations *equal* to one another, and in this Equation there will be but *one unknown Quantity*, conſequently if this Equation is reduced by the Rules already given at Art. 46 to 53. we ſhall find what this unknown Quantity is.

To find the Value of the ſame unknown Quantity in each of the given Equations, and making theſe two Equations equal to one another, which clears the Work of that unknown Quantity whoſe Value was found, is called the *exterminating an unknown Quantity*.

And finding the Value of the unknown Quantity in any Equation, is only to find to what it is equal, therefore all the other Quantities, whether known or unknown, muſt be carried to the other Side of the Equation by the Directions at Art. 46 to 53, and then it will appear to what this unknown Quantity is equal, as this makes one Side of the Equation, the other Side of the Equation being known Quantities, with the other unknown Number or Quantity ſought.

Queſtion 39. *To find two Numbers that the greater being added to the leſſer, the Sum may be* 262.

But if from the greater you ſubſtract the leſſer, the Remainder may be equal to 144.

Let $a =$ the greater Number, and $e =$ the leſſer Number ſought, $b = 262$, $x = 144$.

Now the Sum of the two Numbers in *Algebra* is $a + e$, which is to be equal to 262, or b, hence we have - -	1	$a + e = b$	By the Queſtion
And the leſſer Number being ſubſtracted from the greater is $a - e$, which is to be equal to 144, or x, hence we have	2	$a - e = x$	

The Conditions of the Queſtion being now expreſſed there appears in it to be the above two Equations with two unknown Quantities a and e, therefore according to the Rule find what a is equal to in the firſt Equation, by tranſpoſing e.

	$1 - e$	3	$a = b - e$
Now find what a is equal to in the ſecond Equation, by tranſpoſing e. - - -			
	$2 + e$	4	$a = x + e$

Therefore make the third and fourth Equations equal to one another, for they are both equal to the ſame Quantity a, which exterminates that unknown Quantity, this Step is *regiſtered* by placing the 3 and 4 with a Point between them as in the Work, which expreſſes that the fifth Equation is from comparing the third and fourth Equation.

3 · 4	5	$x + e = b - e$

The unknown Quantity e being on both Sides of the Equation, bring it on one Side of the Equation, by Art. 52.

$5+e$	6	$x+2e=b$
$6-x$	7	$2e=b-x$
$7\div\bar{2}$	8	$e=\frac{b-x}{2}$

Here it appears that e, or the leſſer Number ſought, is equal to b, or 262, ſubſtracting from it x, or 144, and dividing the Remainder by 2.

When any Equation is divided by an abſolute Number, as the ſeventh Equation is divided by 2, place them in the *Regiſter* as uſual, but draw a Line over the 2 to diſtinguiſh that it is an abſolute Number by which you divide, and not by the ſecond Equation in the Work.

Now $b=262$
$-x=144$
2) 118
$59=e$, the leſſer of the two Numbers ſought.

It being now known what e is in Numbers, we may find a by the third or fourth Equation, that is, by the third Equation we have $a=b-e$.

But $b=262$
$-e=\ \ 59$
$203=a$, the greater of the two Numbers ſought.

Whence 203 and 59 are the two Numbers required in the Queſtion, and is thus proved from its Conditions.

The greater Number is	203	- -	203
The leſſer Number is	59	- -	59
	262 Sum		144 Remains

Queſtion 40. *Two Men diſcourſing of their Money found, that if the Shillings each had were added together the Sum would be* 38. *But if from him that had the greater Number of Shillings, there be ſubſtracted twice the Number of Shillings the other Perſon had, there would remain* 5. *How many had each Man?*

Let $a=$ the greater Number of Shillings, $e=$ the leſſer Number of Shillings, $b=38$, $x=5$.

And becauſe the Sum of their Shillings or $a+e$ was 38, or b, hence - - -	1	$a+e=b$	By the Queſtion
And twice the leſſer Number being taken from the greater, or $a-2e$; was equal to 5, or x, hence - - -	2	$a-2e=x$	
Now to find the Value of a in the firſt Equation, tranſpoſe e. $1-e$	3	$a=b-e$	
And to find the Value of a in the ſecond Equation tranſpoſe $2e$. $2+2e$	4	$a=x+2e$	

Make the third and fourth Equations equal to one another; becauſe they are both equal to the ſame Quantity a, and *regiſter* it as directed in the laſt Queſtion; and this exterminates that unknown Quantity.

3 · 4	5	$x+2e=b-e$

The unknown Quantity e being on both Sides of the Equation, bring it on one Side of the Equation, by Art. 52.

$5+e$	6	$x+3e=b$
$6-x$	7	$3e=b-x$
$7\div3$	8	$e=\frac{b-x}{3}$

Hence the Queſtion is anſwered for $b=38$

$-x=-5$

3) 33

$11=e$, the leſſer Number of Shillings.

And as e is now known, we may find what a is by the third or fourth Equation; taking the fourth Equation, we have

$$\begin{array}{r} x = 5 \\ 2e = 22 \\ \hline 27 = a, \end{array}$$ the greater Number of Shillings.

PROOF.

The greater Number of Shillings	27	- - -	27
The leſſer Number of Shillings	11	Twice the leſſer Number of Shillings	22
	Sum 38	Remains	5

Queſtion 41. *Two Men laying a Wager concerning two Droves of Sheep, as they could not decide it, appealed to a third Perſon, who told them that if* 31 *was added to the Number of Sheep in the greateſt Drove, that Sum would be equal to twice the Number of Sheep in the leaſt Drove.*

And that if they added 44 *to the Number of Sheep in the leaſt Drove, that Sum would be as many as were in the greateſt Drove, and deſired they would now find the Number of Sheep in each Drove.*

Let $a =$ the Number of Sheep in the greateſt Drove, $e =$ the Number of Sheep in the leaſt Drove, $x = 31$, $d = 44$.

Now the Number of Sheep in the greateſt Drove being added to 31, which being equal to twice the Number of Sheep in the leſſer Drove, we have	1	$a + x = 2e$	By the Queſtion
And the Number of Sheep in the leaſt Drove when added to 44, and this Sum being equal to the Number of Sheep in the greateſt Drove, we have	2	$e + d = a$	
$1 - x$	3	$a = 2e - x$	

Now by the third Equation, a is equal to $2e - x$, and by the ſecond Equation, a is equal to $e + d$, therefore make theſe Equations equal to one another, for they are both equal to the ſame Quantity a, which exterminates a, as before.

$2 \cdot 3$	4	$2e - x = e + d$
$4 - e$	5	$e - x = d$
$5 + x$	6	$e = d + x$

$d = 44$
$x = 31$

$75 = e$, the Number of Sheep in the leaſt Drove.

Then having found e, we may find a by the ſecond Equation.

$e = 75$
$d = 44$

$119 = a$, the Number of Sheep in the greateſt Drove.

PROOF.

119
31

150 which is twice the Number of Sheep in the leaſt Drove.

75
44

119 which is the Number of Sheep in the greateſt Drove.

Queſtion 42. *Two Gentlemen who had ſold their Eſtates, by comparing what each Eſtate was ſold for, found, that twice the Sum of what both the Eſtates were ſold for was* 11468 *Pounds:*

And if what the leaſt Eſtate was ſold for be ſubſtracted from what the greateſt Eſtate was ſold for, there will remain 1408 *Pounds. For how much was each Eſtate ſold?*

Let $a =$ what the greateſt Eſtate was ſold for, $e =$ what the leaſt Eſtate was ſold for, $b = 11468$, $x = 1408$.

By the firſt Condition - -	1	$2a + 2e = b$
By the ſecond Condition - -	2	$a - e = x$
To find the Value of a, in the firſt Equation.		
$1 - 2e$	3	$2a = b - 2e$
$3 \div 2$	4	$a = \frac{b - 2e}{2}$

Now find the Value of a, in the ſecond Equation.

$2 + e$ | 5 | $a = x + e$

And by making the fourth, and fifth, Equations equal to one another, becauſe they are both equal to the ſame Quantity a, and therefore muſt be equal to one another, by which a will be exterminated.

$4 \cdot 5$ | 6 | $x + e = \frac{b - 2e}{2}$ *

6×2 | 7 | $2x + 2e = b - 2e$

$7 + 2e$ | 8 | $2x + 4e = b$

$8 - 2x$ | 9 | $4e = b - 2x$

$9 \div 4$ | 10 | $e = \frac{b - 2x}{4} = 2163$, the Sum for which the leaſt Eſtate was ſold; and e being now known, then

* Here we have only to find what e is by the Rules already delivered, at Art. 46 to 53.

By the fifth Step | 11 | $a = x + e = 3571$, the Sum for which the greateſt Eſtate was ſold.

PROOF.

Now if $a = 3571$, and $e = 2163$, then $2a + 2e = 11468$, and $a - e = 1408$.

Queſtion 43. *Two Gameſters,* A *and* B, *found, that if twice the Number of Pounds won by* A *was added to what had been won by* B, *the Sum was* 48 *Pounds:*

And if what had been won by A *was added to three times what had been won by* B, *the Sum was* 39 *Pounds. What was the Sum won by each Gameſter?*

Let $a =$ the Money won by A, $e =$ the Money won by B, $b = 48$, $x = 39$.

By the firſt Condition } | 1 | $2a + e = b$

By the ſecond Condition } | 2 | $a + 3e = x$

To find the Value of a, from the firſt Equation.

$1 - e$	3	$2a = b - e$
$3 \div \overline{2}$	4	$a = \dfrac{b-e}{2}$

And to find the Value of a, from the ſecond Equation,

$2 - 3e$	5	$a = x - 3e$

And making the fourth and fifth Equations equal to one another, becauſe they are each equal to the ſame Quantity, which Equation will exterminate a.

$4 \cdot 5$	6	$\dfrac{b-e}{2} = x - 3e$ *
$6 \times \overline{2}$	7	$b - e = 2x - 6e$
$7 + 6e$	8	$b + 5e = 2x$
$8 - b$	9	$5e = 2x - b$
$9 \div \overline{5}$	10	$e = \dfrac{2x-b}{5} = 6$ Pounds, the Money won by B.
Then from the fifth Equation	11	$a = x - 3e = 21$ Pounds, the Money won by A.

* Here we have only to find e by the Rules already delivered, at Art. 46 to 53.

PROOF.

$$2a + e = 48$$
$$a + 3e = 39$$

Queſtion 44. *What are thoſe two Numbers that twice the greater being added to three times the leſſer, the Sum is* 29*:*

And three times the greater being ſubſtracted from five times the leſſer, the Remainder is 4.

Let $a =$ the greater Number, $e =$ the leſſer Number, $b = 29$, $m = 4$.

By the firſt Condition.	1	$2a + 3e = b$
By the ſecond Condition.	2	$5e - 3a = m$

To find the Value of *a*, from the firſt Equation.

$1 - 3e$	3	$2a = b - 3e$
$3 \div 2$	4	$a = \frac{b - 3e}{2}$

Now to find the Value of *a*, from the ſecond Equation, tranſpoſe $3a$ becauſe it has the *negative* Sign.

$2 + 3a$	5	$5e = m + 3a$
$5 - m$	6	$5e - m = 3a$
Or	7	$3a = 5e - m$
$7 \div \overline{3}$	8	$a = \frac{5e - m}{3}$

By making the fourth and eighth Equations equal to one another, for they are each equal to the ſame Quantity *a*, and this unknown Quantity will be exterminated.

$4 \cdot 8$	9	$\frac{5e - m}{3} = \frac{b - 3e}{2}$
$9 \times \overline{2}$	10	$\frac{10e - 2m}{3} = b - 3e$
$10 \times \overline{3}$	11	$10e - 2m = 3b - 9e$
$11 + 9e$	12	$19e - 2m = 3b$
$12 + 2m$	13	$19e = 3b + 2m$
$13 \div \overline{19}$	14	$e = \frac{3b + 2m}{19} = 5$, the leſſer Number.
By the fourth Equation.	15	$a = \frac{b - 3e}{2} = 7$, the greater Number.

PROOF.

$$2a + 3e = 29$$
$$5e - 3a = 4$$

Queſtion 45. *Two Travellers,* A *and* B, *meeting on the Road, found, that if the Number of Miles travelled by* A *was divided by five, adding to this Quotient three times the Diſtance travelled by* B, *the Sum was* 249:

But if twice the Diſtance travelled by A *was added to four times the Diſtance travelled by* B, *the Sum was* 540. *How many Miles had each travelled?*

Let $a =$ the Number of Miles travelled by A, $e =$ the Number of Miles travelled by B, $x = 249$, $z = 540$.

By the firſt Condition.	1	$\frac{a}{5} + 3e = x$
By the ſecond Condition.	2	$2a + 4e = z$
$1 \times \overline{5}$	3	$a + 15e = 5x$
$3 - 15e$	4	$a = 5x - 15e$

The Value of a being now found by the firſt Equation, find its Value from the ſecond Equation.

$2 - 4e$	5	$2a = z - 4e$
$5 \div \overline{2}$	6	$a = \frac{z - 4e}{2}$

Now make the fourth and ſixth Equations equal to one another as before, which exterminates a.

$4 . 6$	7	$\frac{z - 4e}{2} = 5x - 15e$
$7 \times \overline{2}$	8	$z - 4e = 10x - 30e$
$8 + 30e$	9	$z + 26e = 10x$
$9 - z$	10	$26e = 10x - z$
$10 \div \overline{26}$	11	$e = \frac{10x - z}{26} = 75$, the Miles travelled by B.
Then by the 4th Equation.	12	$a = 5x - 15e = 120$, the Miles travelled by A.

PROOF.

$$\frac{a}{5} + 3e = 249$$

$$2a + 4e = 540$$

The Learner being now a little converſant with theſe kind of Queſtions, let the laſt be repeated, and put Letters for all the Numbers both known and unknown, and if he finds any Difficulty in ſolving it by comparing the two Operations, the former may in ſome Manner explain this to him; and to illuſtrate it the more I have placed the Equations in the laſt Work, againſt their correſpondent Equations in the next Operation.

Queſtion 46. *Two Travellers,* A *and* B, *meeting on the Road, found, that if the Number of Miles travelled by* A *was divided by* 5, *and adding to the Quotient* 3 *times the Miles travelled by* B, *the Sum was* 249:

But the Diſtance travelled by A *being multiplied by* 2, *and added to* 4 *times the Diſtance travelled by* B, *the Sum was* 540. *How many Miles had each travelled?*

Let $a =$ the Number of Miles travelled by A, $e =$ the Number of Miles travelled by B, $x = 249$, $z = 540$, as before, but now put $d = 5$, $m = 3$, $q = 2$, $p = 4$.

By the firſt Condition.	1	$\frac{a}{d}+me=x$, that is, $\frac{a}{5}+3e=x$
By the ſecond Condition.	2	$qa+pe=z$, that is, $2a+4e=z$
$1 \times d$	3	$a+dme=dx$, that is, $a+15e=5x$
$3-dme$	4	$a=dx-dme$, that is, $a=5x-15e$

Having found the Value of a, from the firſt Equation, find its Value from the ſecond Equation.

$2-pe$	5	$qa=z-pe$, that is, $2a=z-4e$
$5 \div q$	6	$a=\frac{z-pe}{q}$, that is, $a=\frac{z-4e}{2}$

Now make the fourth and ſixth Equations equal to one another, for they are both equal to the ſame Quantity a, which exterminates that unknown Quantity.

$4 . 6$	7	$\frac{z-pe}{q}=dx-dme$, that is, $\frac{z-4e}{2}=5x-15e$
$7 \times q$	8	$z-pe=dqx-dmeq$, that is, $z-4e=10x-30e$

8 + $dmeq$

$8 + dmeq$	9	$dmeq + z - pe = dqx$, that is, $26e + z = 10x$
$9 - z$	10	$dmeq - pe = dqx - z$, that is, $26e = 10x - z$

The unknown Quantity e being in two Terms, therefore divide by both the Co-efficients of e, as at Art. 52.

$10 \div dmq - p$	11	$e = \frac{dqx - z}{dmq - p} = 75$, that is, $e = \frac{10x - z}{26} = 75$, the Miles travelled by B.

And it being found that e is 75, we may find a by the fourth, or ſixth, Equation to be 120.

Queſtion 47. *There are two Armies ready to engage, and if the Number of Soldiers in both Armies are added together, and that Sum multiplied by 4, the Product is 84440:*

But if the Number of Men in the greateſt Army be multiplied by 2, and added to the Product of the Number of Men in the leſſer Army multiplied by 3, the Sum is 52219. To find the Number of Men in each Army?

Let $a =$ the Number of Men in the greateſt Army, $e =$ the Number of Men in the leſſer Army, $d = 4$, $m = 84440$, $z = 2$, $x = 3$, $b = 52219$.

By the firſt Condition.	1	$da + de = m$
By the ſecond Condition.	2	$za + xe = b$

To find the Value of a, in the firſt Equation.

$1 - de$	3	$da = m - de$
$3 \div d$	4	$a = \frac{m - de}{d}$

Now find the Value of a, from the ſecond Equation.

$2 - xe$	5	$za = b - xe$
$5 \div z$	6	$a = \frac{b - xe}{z}$

Then make the fourth and ſixth Equations equal to one another, to exterminate *a*.

$4 \cdot 6$	7	$\frac{m - de}{d} = \frac{b - xe}{z}$
$7 \times d$	8	$m - de = \frac{db - dxe}{z}$
$8 \times z$	9	$zm - zde = db - dxe$

Now in this Equation *e* being on both Sides of the Equation, find which of its *Co-efficients* dx, or zd, is the greateſt. Now zd is 8, but dx is 12, therefore tranſpoſe dxe, that the unknown Quantity, with the greateſt Co-efficient, may have the affirmative Sign, as at Art. 52.

$9 + dxe$	10	$dxe + zm - zde = db$
$11 - zm$	11	$dxe - zde = db - zm$
$11 \div dx - zd$	12	$e = \frac{db - zm}{dx - zd} = 9999$
By the ſixth Equation.	13	$a = \frac{b - xe}{z} = 11111$, the Number of Men in the greateſt Army.

Dividing the eleventh Equation by $dx - zd$, the two Co-efficients of *e*, as at Art. 52. gives the twelfth Equation.

PROOF.

$$4a + 4e = 84440$$
$$2a + 3e = 52219$$

Queſtion 48. *A Gentleman bought a pair of Horſes for his Coach, his Son having learnt* Algebra, *the Father propoſed for him to determine the price of each Horſe from ſaying,*

That if the Sum of what both the Horſes coſt was multiplied by 4, *and this Product divided by* 8, *the Quotient would be* 20 *Pounds:*

But if what the beſt Horſe coſt was multiplied by 3, and this Product added to 5 times what the worſt Horſe coſt, this Sum would be 158 Pounds. Now what was the price of each Horſe?

Let $a =$ what the beſt Horſe coſt, $e =$ what the worſt Horſe coſt, $b = 4$, $d = 8$, $m = 20$, $p = 3$, $x = 5$, $z = 158$.

By the firſt Condition.	1	$\frac{ba + be}{d} = m$
By the ſecond Condition.	2	$pa + xe = z$
$1 \times d$	3	$ba + be = dm$
$3 - be$	4	$ba = dm - be$
$4 \div b$	5	$a = \frac{dm - be}{b}$
$2 - xe$	6	$pa = z - xe$
$6 \div p$	7	$a = \frac{z - xe}{p}$
		To exterminate a
$5 \cdot 7$	8	$\frac{dm - be}{b} = \frac{z - xe}{p}$
$8 \times p$	9	$\frac{pdm - pbe}{b} = z - xe$
$9 \times b$	10	$pdm - pbe = bz - bxe$
$10 + bxe$	11	$bxe + pdm - pbe = bz$
$11 - pdm$	12	$bxe - pbe = bz - pdm$
$12 \div bx - pb$	13	$e = \frac{bz - pdm}{bx - pb} =$ 19 Pounds, the price of the worſt Horſe.
By the ſeventh Equation.	14	$a = \frac{z - xe}{p} =$ 21 Pounds, the price (of the beſt Horſe.

PROOF.

$$\frac{4a + 4e}{8} = 20.$$

$$3a + 5e = 158.$$

Queſtion 49. *Two young Gentlemen, who had ſtudied Numbers, not agreeing about their Age, referred the diſpute to their Father, who ſmiling told them, that if the Age of the eldeſt was divided* *by 2, to which Quotient adding 4 times the Age of the youngeſt, and extracting the ſquare Root of this Sum, it would be* 10:

But if the Age of the eldeſt was multiplied by 3, *and added to the Age of the youngeſt multiplied by* 5, *this Sum would be* 201. *To find the Age of each Perſon?*

Let $a =$ the Age of the elder, $e =$ the Age of the younger, $b = 2$, $d = 4$, $m = 10$, $p = 3$, $h = 5$, $r = 201$.

By the firſt Condition.	1	$\sqrt{\frac{a}{b} + de} = m$
By the ſecond Condition.	2	$pa + he = r$

Becauſe in the firſt Equation a the unknown Quantity is under the radical Sign, therefore ſquare both Sides of the Equation, as at Art. 49. The 1 ⊚ 2 in the *Regiſter* ſignifies that the firſt Equation being involved or raiſed to the ſecond Power or Square makes the third Equation, for ⊚ is the Sign of Involution.

1 ⊚ 2	3	$\frac{a}{b} + de = mm$
$3 - de$	4	$\frac{a}{b} = mm - de$
$4 \times b$	5	$a = bmm - bde$
$2 - he$	6	$pa = r - he$
$6 \div p$	7	$a = \frac{r - he}{p}$
		Now to exterminate a
$5 \cdot 7$	8	$\frac{r - he}{p} = bmm - bde$
$8 \times p$	9	$r - he = pbmm - pbde$
$9 + pbde$	10	$pbde + r - he = pbmm$
$10 - r$	11	$pbde - he = pbmm - r$
$11 \div pbd - h$	12	$e = \frac{pbmm - r}{pbd - h} = 21$, the Age of the youngeſt.
By the ſeventh Step.	13	$a = \frac{r - he}{p} = 32$, the Age of the eldeſt.

PROOF.

$$\sqrt{\frac{a}{2}} + 4e = 10.$$

$$3a + 5e = 201.$$

Queſtion 50. *Two Tradeſmen, who had been at a Fair, upon comparing their gains, found, that if the Profits of him who had gained the moſt was multiplied by* 2, *to which adding* 3 *times the Profit of him that gained the leaſt, the ſquare Root of this Sum would be* 11 *Pounds:*

But that if 6 *times the Profit of him that gained the leaſt, was added to the Quotient of his Profit that gained the moſt when divided by* 10, *this Sum would be* 47 *Pounds. To find the Profit of each Tradeſman?*

Let $a =$ the greateſt Profit, $e =$ the leaſt Profit, $b = 2$, $d = 3$, $n = 11$, $p = 6$, $z = 10$, $x = 47$.

By the firſt Condition.	1	$\sqrt{ba + de} = n$
By the ſecond Condition.	2	$pe + \frac{a}{z} = x$

In the firſt Equation the unknown Quantity a being under the radical Sign, ſquare both Sides of the Equation as in the laſt Queſtion.

1 ⊙ 2	3	$ba + de = nn$
$3 - de$	4	$ba = nn - de$
$4 \div b$	5	$a = \frac{nn - de}{b}$
$2 \times z$	6	$zpe + a = zx$
$6 - zpe$	7	$a = zx - zpe$
$5 \cdot 7$	8	$\frac{nn - de}{b} = zx - zpe$
$8 \times b$	9	$nn - de = bzx - bzpe$
$9 + bzpe$	10	$bzpe + nn - de = bzx$
$10 - nn$	11	$bzpe - de = bzx - nn$

$11 \div bzp-d$	12	$e=\frac{bzx-nn}{bzp-d}=7$ Pounds, the leaſt Profit.
By the ſeventh Step.	13	$a=zx-zpe=50$ Pounds, the greateſt Profit.

PROOF.

$$\sqrt{2a+3e}=11.$$

$$6e+\frac{a}{10}=47.$$

Queſtion 51. *Two Perſons,* A *and* B, *owe each a Sum of Money, that if what* A *owes is divided by* 5, *to which Quotient adding* 4 *times the Debt of* B, *and extract the ſquare Root of this Sum, it will be* 6 *Pounds :*

But if from 3 *times the Debt of* A, *there is ſubſtracted* 50 *times the Debt of* B, *and extract the ſquare Root of this Remainder it will be* 10 *Pounds. What did each Perſon owe ?*

Let $a=$ the Debt of A, $e=$ the Debt of B, $m=5$, $n=4$, $d=6$, $p=3$, $x=50$, $z=10$.

By the firſt Condition.	1	$\sqrt{\frac{a}{m}+ne}=d$
By the ſecond Condition.	2	$\sqrt{pa-xe}=z$

To find the Value of a in the firſt Equation, raiſe it to the ſecond Power as before.

$1 \odot 2$	3	$\frac{a}{m}+ne=dd$
$3-ne$	4	$\frac{a}{m}=dd-ne$
$4\times m$	5	$a=mdd-mne$

To find the Value of a in the ſecond Equation, raiſe it to the ſecond Power as before.

$2 @ 2$	6	$pa - xe = zz$
$6 + xe$	7	$pa = zz + xe$
$7 \div p$	8	$a = \frac{zz + xe}{p}$

Now make the fifth and eighth Equations equal to one another, to exterminate *a*.

$5 \cdot 8$	9	$\frac{zz + xe}{p} = mdd - mne$
$9 \times p$	10	$zz + xe = pmdd - pmne$
$10 + pmne$	11	$pmne + zz + xe = pmdd$
$11 - zz$	12	$pmne + xe = pmdd - zz$
$12 \div pmn + x$	13	$e = \frac{pmdd - zz}{pmn + x} = 4$ Pounds, the Debt (of B.
Then by the eighth Step.	14	$a = \frac{zz + xe}{p} = 100$ Pounds, the (Debt of A.

PROOF.

$$\sqrt{\frac{a}{5} + 4e} = 6.$$

$$\sqrt{3a - 50e} = 10.$$

Queſtion 52. *Two Men,* A *and* B, *going to Market with Eggs, if the Number of Eggs that* A *had was multiplied by* 6, *to which adding* 100, *and dividing this Sum by the Number of Eggs that* B *had, the Quotient would be* 16 :

And if from 9 *times the Number of Eggs* A *had, there be ſubſtracted* 4 *times the Number of Eggs* B *had, there would remain* 350. *How many Eggs had each Perſon?*

Let $a =$ the Number of Eggs A had, $e =$ the Number of Eggs B had, $d = 6$, $m = 100$, $p = 16$, $b = 9$, $x = 4$, $z = 350$.

	1	$\frac{da + m}{e} = p$	By the Queſtion.
	2	$ba - xe = z$	
$1 \times e$	3	$da + m = pe$	

3 — *m*

$3 - m$	4	$da = pe - m$
$4 \div d$	5	$a = \frac{pe - m}{d}$
$2 + xe$	6	$ba = z + xe$
$6 \div b$	7	$a = \frac{z + xe}{b}$

Make the fifth and ſeventh Equations equal to one another, to exterminate *a*.

$5 \cdot 7$	8	$\frac{pe - m}{d} = \frac{z + xe}{b}$
$8 \times d$	9	$pe - m = \frac{dz + dxe}{b}$
$9 \times b$	10	$bpe - bm = dz + dxe$
$10 - dxe$	11	$bpe - dxe - bm = dz$
$11 + bm$	12	$bpe - dxe = dz + bm$
$12 \div bp - dx$	13	$e = \frac{dz + bm}{bp - dx} = 25$, the Number of (Eggs B had.
By the ſeventh Step.	14	$a = \frac{z + xe}{b} = 50$, the Number of (Eggs A had.

PROOF.

$$\frac{6a + 100}{e} = 25.$$

$$9a - 4b = 350.$$

Queſtion 53. *Two Perſons,* A *and* B, *loſing at the Gameing-Table, were asked how much they loſt, to which* A *replied, that if what I loſt be multiplied by* 3, *and adding* 100 *to this Product, if this Sum is divided by what* B *loſt, the ſquare Root of this Quotient will be* 10 *Pounds:*

But if what B *loſt is multiplied by* 250, *from which Product ſubſtracting* 600, *and dividing this Remainder by what* A *loſt, the ſquare Root of this Quotient will be* 2 *Pounds. How much had each Perſon loſt?*

Let $a =$ what A loſt, $e =$ what B loſt; $d = 3$, $m = 100$; $n = 10$, $x = 250$, $z = 600$, $b = 2$.

$$\left.\begin{array}{c|l} 1 & \sqrt{\dfrac{da+m}{e}} = n \\ 2 & \sqrt{\dfrac{xe-z}{a}} = b \end{array}\right\}\text{By the Queſtion.}$$

To find the Value of a in the ſecond Equation, raiſe it to the ſecond Power by Art. 49.

$$\begin{array}{r|c|l} 1 \odot 2 & 3 & \dfrac{da+m}{e} = nn \\ 3 \times e & 4 & da+m = enn \\ 4 - m & 5 & da = enn - m \\ 5 \div d & 6 & a = \dfrac{enn-m}{d} \end{array}$$

To find the Value of a in the ſecond Equation, raiſe it to the ſecond Power, by Art. 49.

$$\begin{array}{r|c|l} 2 \odot 2 & 7 & \dfrac{xe-z}{a} = bb \\ 7 \times a & 8 & xe - z = abb \\ 8 \div bb & 9 & \dfrac{xe-z}{bb} = a \end{array}$$

Make the ſixth and ninth Equations equal to one another, to exterminate a.

$$\begin{array}{r|c|l} 6 \,.\, 9 & 10 & \dfrac{enn-m}{d} = \dfrac{xe-z}{bb} \\ 10 \times d & 11 & enn - m = \dfrac{dxe-dz}{bb} \\ 11 \times bb & 12 & bbnne - bbm = dxe - dz \end{array}$$

Becauſe dx one Co-efficient of e is greater than $bbnn$ the other Co-efficient of e, therefore tranſpoſe $bbnne$, by Art. 52.

$12 - bbnne$	13	$-bbm = dxe - dz - bbnne$
Or	14	$dxe - dz - bbnne = -bbm$
$14 + dz$	15	$dxe - bbnne = dz - bbm$
$15 \div dx - bbnn$	16	$e = \frac{dz - bbm}{dx - bbnn} = 4$, the Sum that (B loſt.
By the ninth Step	17	$a = \frac{xe - z}{bb} = 100$, what A loſt.

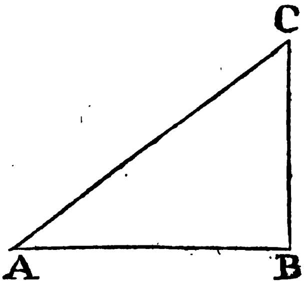
PROOF.

$$\sqrt{\frac{3a + 100}{e}} = 10.$$

$$\sqrt{\frac{250e - 600}{a}} = 2.$$

Queſtion 54. *In the right angled Triangle* A B C, *there is given the Baſe* A B $= 4$, *and the difference between the Hypothenuſe* A C *and Perpendicular* B C $= 2$. *To find the Hypothenuſe* A C *and Perpendicular* B C?

Let A C $= a$, B C $= e$, A B $= b = 4$, $m = 2$.



Having put Letters for the three Sides of the Triangle, and amongſt theſe there being two unknown Quantities a and e, therefore we muſt raiſe two Equations either from the Properties of the Figure, or from the Conditions of the Queſtion. And in the Solution of the Geometrical Queſtions, I would recommend it to the Learner that after all the Parts of the Figure which are neceſſary to the Solution of the Queſtion are expreſſed by Letters, to obſerve how many of them are unknown, for generally ſo many different Equations are raiſed from the Properties of the Figure, or the Conditions of the Queſtion; afterwards the Work is regulated by the Rules already given.

Now from the Property of the Figure, the Square of the Hypothenuſe A C, or aa, is equal to the Square of the Baſe A B, or bb, added to the ſquare of the Perpendicular B C, or ee, by 47 *e* 1.

That

That is	1	$aa=bb+ee$ from the Property of the Figure by 47 *e* 1.

Becauſe by the Queſtion the difference between the Hypothenuſe A C, or a, and Perpendicular B C, or e, is $=2$, or m.

Hence	2	$a-e=m$ by the Conditions of the Queſtion.

Having raiſed the two Equations, proceed as in the former Examples, that is, firſt find the Value of a in the firſt Equation, by the Extraction of Roots, as at Art. 50.

1 ɯ 2	3	$a=\sqrt{bb+ee}$

Now find the Value of a, in the ſecond Equation.

$2+e$	4	$a=m+e$

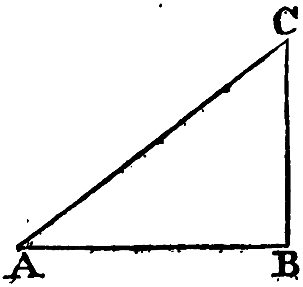
By making the third and fourth Equations equal to one another, we exterminate a.

3 . 4	5	$m+e=\sqrt{bb+ee}$

Becauſe e the unknown Quantity is under the radical Sign, and there being no other Quantities on the ſame Side of the Equation with it, that are not under the radical Sign, therefore ſquare both Sides of the Equation as at Art. 49.

5 ⊛ 2	6	$mm+2me+ee=bb+ee$
$6-ee$	7	$mm+2me=bb$
$7-mm$	8	$2me=bb-mm$
$8\div 2m$	9	$e=\dfrac{bb-mm}{2m}=3$, the Perpendicular B C.
By the fourth Step	10	$a=m+e=5$, the Hypothenuſe A C.

That theſe are the three Sides of the right angled Triangle is proved, by ſquaring the Hypothenuſe 5, and ſee if that is equal to the Square of the Baſe 4, added to the Square of the Perpendicular 3; for this is the celebrated Property of the right angled Triangle to have the Square of the Hypothenuſe equal to the Sum of the Squares of the Baſe and Perpendicular.



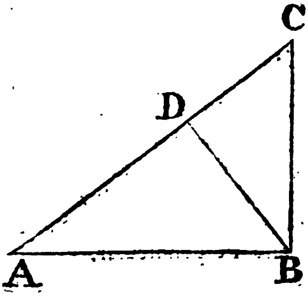
Queſtion 55. *In the right angled Triangle* A B C, *given the Perpendicular* B C = 3, *and the Difference between the Hypothenuſe* A C, *and Baſe* A B = 1. *To find the Hypothenuſe* A C, *and Baſe* B A?

Let A C = a, B C = 3 = b, A B = e, x = 1.

Then	1	$aa = bb + ee$ by the Property of the Figure, as in the laſt Queſtion.
And	2	$a - e = x$ by the Queſtion

There being as many Equations raiſed from the Property of the Figure, and the Conditions of the Queſtion as there are unknown Quantities, the Work proceeds upon the ſame general Rules, thus

1 ɯ 2	3	$a = \sqrt{bb + ee}$
2 + e	4	$a = x + e$
3 · 4	5	$x + e = \sqrt{bb + ee}$
5 @ 2	6	$xx + 2xe + ee = bb + ee$
6 − ee	7	$xx + 2xe = bb$
7 − xx	8	$2xe = bb - xx$
8 ÷ $2x$	9	$e = \frac{bb - xx}{2x} = 4$, the Baſe A B.
By the fourth Step	10	$a = x + e = 5$, the Hypothenuſe A C.



Queſtion 56. *In the right-angled Triangle* A B C, *there is given the Hypothenuſe* A C = 5, *the Baſe* A B = 4, *and the Perpendicular* B C = 3, *to find the Perpendicular* B D, *let fall from the Angle* B, *upon the Hypothenuſe* A C.

Let A C = b = 5, A B = m = 4, B C = x = 3, D C = a, A D = e.

The Queſtion requiring that we find B D, if we find C D we can anſwer the Queſtion, for the Triangle B D C being a right-angled Triangle, B D being Perpendicular to A C, conſequently B C being known, and by finding D C, we ſhall afterwards eaſily find D B, by the common Property of the Triangle.

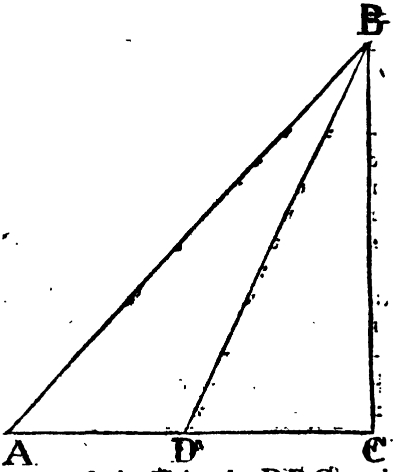
It is exactly the ſame, if we find A D, for the Triangle A D B is right-angled, and A B is given by the Queſtion.

Now B D being a Perpendicular, common to the two Triangles A B D, and B D C, let B D $= p$, then by the Triangle A B D, we have $mm - ee = pp$, and by the Triangle C B D, we have $xx - aa = pp$, from the ſame reaſoning as in the two laſt Queſtions.

Conſequently	1	$mm - ee = xx - aa$ for both $mm - ee$ and $xx - aa$, are each equal to the ſame Quantity pp, and therefore equal to another.
And	2	$a + e = b$, that is, $AD + DC = AC$ by the Figure.
		To find the Value of a in the firſt Equation.
$1 + aa$	3	$aa + mm - ee = xx$
$3 + ee$	4	$aa + mm = xx + ee$
$4 - mm$	5	$aa = xx + ee - mm$
5 ɯ 2	6	$a = \sqrt{xx + ee - mm}$
		Now find the Value of a in the ſecond Equation.
$2 - e$	7	$a = b - e$
6 . 7	8	$\sqrt{xx + ee - mm} = b - e$
8 @ 2	9	$xx + ee - mm = bb - 2be + ee$
$9 - ee$	10	$xx - mm = bb - 2be$
$10 + 2be$	11	$2be + xx - mm = bb$
$11 + mm$	12	$2be + xx = bb + mm$
$12 - xx$	13	$2be = bb + mm - xx$
$13 \div 2b$	14	$e = \frac{bb + mm - xx}{2b} = 3.2 = AD.$

Having found A D to be 3 . 2 it will be eaſy to find D B by what was ſaid above. Thus,

16 the Square of A B.
— 10.24 the Square of A D.
5.76 (2.4 = D B, the Perpendicular required.
4
44) 176
176
0



Queſtion 57. *In the oblique Triangle* A D B, *there is given the Side* A B = 15, *the Side* B D = 12, *and the Side* A D = 6, *to find the Perpendicular* B C *falling without the Triangle from the Angle* B, *on the Side* A D, *continued.*

This Queſtion will be anſwered from finding D C, for the Triangle B D C being right-angled, and D B being known from finding D C, we may then find B C from the common Property of the Triangle D B C, as in the laſt Queſtion.

Let $AB = b = 15$, $AD = m = 6$, $DB = x = 12$, $DC = a$, then $AC = AD + DC = m + a$; $BC = e$.

Becauſe the Triangle A B C is right-angled, therefore if from the Square of A B, or bb we ſubſtract the Square of A C, or $mm + 2ma + aa$, the Remainder is equal to the Square of C B, or ee.

Therefore | 1 | $bb - mm - 2ma - aa = ee$.

Becauſe the Triangle D B C is right-angled, by the ſame reaſoning we have

Again | 2 | $xx - aa = ee$.

And as the firſt and ſecond Equations are each $=ee$, therefore make them equal to one another, which exterminates every Power of e in thoſe Equations.

$1 \,.\, 2$	3	$xx - aa = bb - mm - 2ma - aa$
$3 + aa$	4	$xx = bb - mm - 2ma$
$4 + 2ma$	5	$2ma + xx = bb - mm$
$5 - xx$	6	$2ma = bb - mm - xx$
$6 \div 2m$	7	$a = \frac{bb - mm - xx}{2m} = 3.75 = DC.$

And from hence we may find B C as was ſaid above, thus

DB, or $x = 12$
12
144

DC, or $a = 3.75$
3.75
1875
2625
1125
140625

144.
14.0625
129.9375 (11.39 = BC, the Perpendicular required.
1
21) 29
21
223) 893
669
2269) 22475
20421
2054

Of

Of Quadratic EQUATIONS.

56. WHEN all the known Quantities are on one Side of the Equation, and thoſe Quantities only on the other Side which have ſome Power of the unknown Quantity; then if the unknown Quantity appears to be to the *ſecond Power or Square* in one Term, and to the *firſt Power* only in another Term; or if in one Term its Power or Heighth is *double* its Power or Heighth in another Term, and there is no *other* Power of the unknown Quantity in the Equation, theſe Equations are called *Quadratic*, as in the following Queſtions.

Queſtion 58. *Two Men had a Number of Shillings, that the leſſer being ſubſtracted from the greater there remains* 10:

But the Number of Shillings one Man had being multiplied by the Number of Shillings the other Man had, the Product is 75. *To find each Man's Number of Shillings?*

Let $a =$ the greater Number of Shillings one of the Men had, $e =$ the leſſer Number of Shillings the other Man had, $b = 10$, $m = 75$.

Then	1	$a - e = b$	By the Queſtion.
And	2	$ae = m$	
$1 + e$	3	$a = b + e$	
$2 \div e$	4	$a = \frac{m}{e}$	
$3 \cdot 4$	5	$e + b = \frac{m}{e}$	
$5 \times e$	6	$ee + be = m$	

From comparing the ſixth Equation, with what is ſaid above, it appears to be *Quadratic*, for one Quantity is ee, or e to the ſecond Power, and in the other Quantity it is only e, or e to the firſt Power.

And to reſolve this Equation, take the Co-efficient of e to the firſt Power, which is b, divide this Co-efficient by 2 and the Quotient is $\frac{b}{2}$, which ſquare, or multiply, by itſelf, and the Product is $\frac{bb}{4}$, which add to both Sides of the Equation, thus

$$6\,c\,\square \;\Big|\; 7 \;\Big|\; ee+be+\frac{bb}{4}=m+\frac{bb}{4}$$

The $c\,\square$ in the *Regiſter* ſignifies, that the ſixth Step is made a Square at the ſeventh Step, or the *Square is compleated.*

Now if we compare the Side of the Equation $ee+be+\frac{bb}{4}$, with ſome of the Examples at Art. 34. we ſhall find this Side of the Equation to be a *rational* Quantity, or a Square, that extracting the ſquare Root of both Sides of the Equation;

$$\begin{array}{r|c|l} 7\;\text{ɯ}\;2 & 8 & e+\frac{b}{2}=\sqrt{m+\frac{bb}{4}} \\ 8-\frac{b}{2} & 9 & e=\sqrt{m+\frac{bb}{4}}:-\frac{b}{2} \end{array}$$

In Numbers.

$$\begin{array}{l} m=75 \\ \frac{bb}{4}=25 \\ \hline 100 \end{array}$$

100 the ſquare Root of which is 10

$$\begin{array}{l} -5=-\frac{b}{2} \\ \hline 5=e, \end{array}$$

$5=e$, the Number of Shillings one of the Men had.

Then by the fourth Step $a=\frac{m}{e}=15$, the Number of Shillings the other Man had.

PROOF.

$$a - e = 10$$
$$ae = 75$$

Queſtion 59. *There are two Numbers, that if the Square of the leſſer is taken from the greater, there remains* 36 :

But the greater being added to 6 *times the leſſer, the Sum is* 148. *What are the two Numbers?*

Let $a =$ the greater Number, $e =$ the leſſer Number, $b = 36$, $m = 6$, $x = 148$.

Then	1	$a - ee = b$	By the Queſtion.
And	2	$a + me = x$	
$1 + ee$	3	$a = b + ee$	
$2 - me$	4	$a = x - me$	
$3 \cdot 4$	5	$b + ee = x - me$	
$5 - b$	6	$ee = x - me - b$	
$6 + me$	7	$ee + me = x - b$	

The unknown Quantities being brought on one Side of the Equation, the Equation appears to be *Quadratic*, by Art. 56.

Now the Co-efficient of e at the firſt Power is m, which being divided by 2 is $\frac{m}{2}$, this being ſquared is $\frac{mm}{4}$, and adding $\frac{mm}{4}$, to both Sides of the Equation as in the laſt Queſtion, we have

7 ¢ □	8	$ee + me + \frac{mm}{4} = x - b + \frac{mm}{4}$

The 7 ¢ □ ſignifies that the ſeventh Equation is made a compleat Square, at the eighth Step.

And extracting the Roots of both Sides of the Equation, as in the laſt Queſtion.

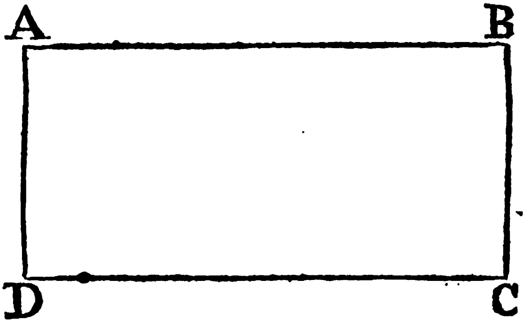
8 ɯ 2	9	$e + \frac{m}{2} = \sqrt{x - b + \frac{mm}{4}}$
$9 - \frac{m}{2}$	10	$e = \sqrt{x - b + \frac{mm}{4}} : - \frac{m}{2} = 8$, the (leſſer Number.
By the fourth Step	11	$a = x - me = 100$, the greater Number.

PROOF.

$$a - ee = 36$$
$$a + 6e = 148$$

Queſtion 60. *In the Parallellogram* ABCD, *if from the longeſt Side* AB *multiplied by* 3, *there is ſubſtracted the Square of the leſſer Side* BC, *the Remainder will be* 5:

But if the greater Side AB *is added to* 4 *times the leſſer Side* BC, *the Sum is* 30. *To find the Sides of the Parallellogram* AB, *and* BC?



Let $a = AB$, $BC = e$, $d = 3$, $m = 5$, $z = 4$, $x = 30$.

	1	$da - ee = m$	By the Queſtion.
	2	$a + ze = x$	
$1 + ee$	3	$da = m + ee$	
$3 \div d$	4	$a = \frac{m + ee}{d}$	
$2 - ze$	5	$a = x - ze$	

4 · 5	6	$\frac{m+ee}{d}=x-ze$
$6\times d$	7	$m+ee=dx-dze$
$7-m$	8	$ee=dx-dze-m$
$8+dze$	9	$ee+dze=dx-m$

Now the Equation appears to be *Quadratic*, by Art. 56. and the Co-efficient of e is dz, which being divided by 2, is $\frac{dz}{2}$, this being ſquared is $\frac{ddzz}{4}$, which added to both Sides of the Equation, as in the two laſt Examples, we have

9 c □	10	$ee+dze+\frac{ddzz}{4}=dx-m+\frac{ddzz}{4}$

And extracting the Roots of both Sides of the Equation, as in the two laſt Queſtions.

10 ɯ 2	11	$e+\frac{dz}{2}=\sqrt{dx-m+\frac{ddzz}{4}}$
$11-\frac{dz}{2}$	12	$e=\sqrt{dx-m+\frac{ddzz}{4}}:-\frac{dz}{2}=5$ (= B C,
From the fifth Step	13	$a=x-ze=10=$ A B.

PROOF.

$3a-ee=5$
$a+4e=30$

Queſtion 61. *Two Gentlemen having had their Parks ſurveyed, had loſt the Account, but they remembered, that if the Number of Acres in* A's *Park, was added to the Number of Acres in* B's *Park, the Sum was* 110:

But if the Number of Acres in B's *Park was multiplied by* 80, *from which Product ſubſtracting the Square of the Number of Acres in* A's *Park, there remain'd* 1400. *How many Acres was there in each Park?*

Let $a =$ the Number of Acres in A's Park, $e =$ the Number of Acres in B's Park, $b = 110$, $m = 80$, $x = 1400$.

	1	$a+e=b$	By the Queſtion.
	2	$me-aa=x$	
$1-a$	3	$e=b-a$	
$2+aa$	4	$me=x+aa$	
$4 \div m$	5	$e=\frac{x+aa}{m}$	
$3 \cdot 5$	6	$\frac{x+aa}{m}=b-a$	
$6 \times m$	7	$x+aa=mb-ma$	
$7-x$	8	$aa=mb-ma-x$	
$8+ma$	9	$aa+ma=mb-x$	

Here the Equation appears *Quadratic*, and compleating the Square as in the former Examples, we have

9 c □	10	$aa+ma+\frac{mm}{4}=mb-x+\frac{mm}{4}$

And extracting the ſquare Roots of both Sides of the Equation, as in the laſt Examples.

10 ш 2	11	$a+\frac{m}{2}=\sqrt{mb-x+\frac{mm}{4}}$
$11-\frac{m}{2}$	12.	$a=\sqrt{mb-x+\frac{mm}{4}} : -\frac{m}{2}=60$. (the Number of Acres in A's Park.
From the third Step.	13	$e=b-a=50$, the Number of Acres (in B's Park.

PROOF.

$a+e=110$

$80e-aa=400$

The Manner of ſubſtituting one Quantity for ſeveral others, explained.

57. But if, after the Work is prepared for having the *Square compleated*, it appears that the firſt Power of the unknown Quantity is in more Terms than one, it will be more commodious to *ſubſtitute* ſome other Letter for the Co-efficients of the firſt Power of the unknown Quantity, as in the following Examples.

Queſtion 62. *A Gentleman propoſed to give his two Sons,* A *and* B, *each an Eſtate, on the Condition, they could tell him what were their Rents by knowing, that if the Square of the Rent of the Eſtate he intended to give* A *was added to the ſame Rent multiplied by* 7, *and this added to the Rent of the Eſtate he intended to give* B, *when multiplied by* 4, *this Sum would be* 4220 *Pounds:*

But if the Sum of the Rents of the two Eſtates was divided by 10, *the Quotient would be* 11 *Pounds. What was the Rent of each Eſtate?*

Let $a =$ the Rent of the Eſtate which A was to have, $e =$ the Rent of the Eſtate B was to have, $b = 7$, $m = 4$, $d = 4220$, $p = 10$, $x = 11$.

$$\left.\begin{array}{c|l} 1 & aa + ba + me = d \\ 2 & \dfrac{a + e}{p} = x \end{array}\right\} \text{By the Queſtion.}$$

Theſe being the two Equations which ariſe from the Queſtion, and becauſe the Terms are more ſimple that have the unknown Quantity e, than thoſe that have the unknown Quantity a, it may be more commodious to find the Value of e, in each of the two given Equations. This Caution the Learner may obſerve for the future, to find the Value of that unknown Quantity whoſe Terms are the moſt ſimple in the given Equations; and they may be taken for the more ſimple, whoſe Powers are the loweſt in both the Equations that ariſe from the Queſtion; thus, if one of the unknown Quantities is only to the *firſt* Power in both the given Equations, when the other unknown Quantity is to the *ſecond* Power in one of the given Equations, the Terms of the former may be ſaid to be more ſimple, and therefore beſt to find the Value of that unknown Quantity: the Reader will find this Method obſerved in the following Queſtions, and comparing their Work with what is ſaid may make this Direction more intelligible.

$1-ba$	3	$aa+me=d-ba$
$3-aa$	4	$me=d-ba-aa$
$4\div m$	5	$e=\frac{d-ba-aa}{m}$
$2\times p$	6	$a+e=px$
$6-a$	7	$e=px-a$
$5\cdot 7$	8	$px-a=\frac{d-ba-aa}{m}$
$8\times m$	9	$mpx-ma=d-ba-aa$
$9+aa$	10	$aa+mpx-ma=d-ba$
$10+ba$	11	$aa+ba+mpx-ma=d$
$11-mpx$	12	$aa+ba-ma=d-mpx$

Here the Equation appears to be Quadratic, and the firſt Power of a is in two Terms, *viz.* ba and ma, the two Co-efficients being b and m, which are connected by the Sign $-$.

But b and m, being known Quantities, therefore $b-m=7-4=3$, now *ſubſtitute*, or put $z=3$, or $z=b-m$, then the laſt Equation is,

By Subſtitution | 13 | $aa+za=d-mpx$; for by Subſtitution $za=ba-ma$ and therefore in the Room of $ba-ma$, we uſe only za. Now taking z for the Co-efficient of a, and compleating the Square as before.

13 c □	14	$aa+za+\frac{zz}{4}=d-mpx+\frac{zz}{4}$
14 ш 2	15	$a+\frac{z}{2}=\sqrt{d-mpx+\frac{zz}{4}}$
$15-\frac{z}{2}$	16	$a=\sqrt{d-mpx+\frac{zz}{4}}:-\frac{z}{2}=60$, the Rent of the Eſtate which A was to have.
By the ſeventh Step.	17	$e=px-a=50$, the Rent of the (Eſtate which B was to have.

PROOF.

PROOF.

$$aa + 7a + 4e = 4220.$$

$$\frac{a+e}{10} = 11.$$

It may be juſt obſerved to the Learner, that the Method of *Subſtitution* is only to ſave Trouble and Labour, for after the twelfth Step, if we had not ſubſtituted $b - m = z$, then to have *compleated the Square*, we muſt have divided $b - m$, the two Co-efficients of a by 2, the Quotient of which is $\frac{b-m}{2}$, which ſquared is $\frac{bb - 2bm + mm}{4}$, and this muſt have been added to both Sides of the Equation, whereas by *ſubſtituting* $b - m = z$, the Quantity to be added on both Sides of the Equation is only $\frac{zz}{4}$.

Queſtion 63. *A Draper bought a Parcel of Linnen, and a Parcel of woollen Cloth, if the Square of the Pounds he gave for the linnen Cloth, be divided by* 4, *and to this Quotient there is added what each ſort coſt, the Sum is* 1000 *Pounds:*
But if what the Linnen coſt is added to the Quotient of what the Woollen coſt, when divided by 8, *the Sum is* 65 *Pounds. How much was given for each Sort?*

Let $a =$ the price of the Linnen, $e =$ the price of the Woollen, $b = 4$, $d = 1000$, $m = 8$, $x = 65$.

	1	$\frac{aa}{b} + a + e = d$	By the Queſtion.
	2	$a + \frac{e}{m} = x$	
$1 - a$	3	$\frac{aa}{b} + e = d - a$	
$3 - \frac{aa}{b}$	4	$e = d - a - \frac{aa}{b}$	

$2 \times m$	5	$ma + e = mx$
$5 - ma$	6	$e = mx - ma$
$4 \,.\, 6$	7	$mx - ma = d - a - \frac{aa}{b}$
$7 + \frac{aa}{b}$	8	$\frac{aa}{b} + mx - ma = d - a$
$3 + a$	9	$\frac{aa}{b} + a + mx - ma = d$
$9 - mx$	10	$\frac{aa}{b} + a - ma = d - mx$
$10 \times b$	11	$aa + ba - bma = bd - bmx$

Here the Equation appears *Quadratic*, and the firſt Power of the unknown Quantity a has two Co-efficients, b and bm, both which are known, but $b - bm = 4 - 32 = -28$, therefore as -28 is a *negative* Quantity, ſubſtitute $-z = -28$, or $-z = b - bm$, then the laſt Equation becomes,

By Subſtitution | 12 | $aa - za = bd - bmx$, for $ba - bma$ is a *negative* Quantity, bm being greater than b: And compleating the Square as before.

$12 \; c \; \square$	13	$aa - za + \frac{zz}{4} = bd - bmx + \frac{zz}{4}$, for $-\frac{z}{2} \times -\frac{z}{2} = +\frac{zz}{4}$, by Art. 9.

And extracting the ſquare Root as in the former Queſtions.

$13 \; \omega \; 2$	14	$a - \frac{z}{2} = \sqrt{bd - bmx + \frac{zz}{4}}$
$14 + \frac{z}{2}$	15	$a = \sqrt{bd - bmx + \frac{zz}{4}} : + \frac{z}{2} = 60$ (Pounds, what the Linnen coſt.
By the ſixth Equation.	16	$e = mx - ma = 40$ Pounds, what the (Woollen coſt.

PROOF.

$$\frac{aa}{4}+a+e=1000.$$

$$a+\frac{e}{8}=65.$$

To reſolve a Quadratic Equation when the Square of the unknown Quantity has a Co-efficient.

58. But if the Square of the unknown Quantity has any Co-efficient, then before you begin to *compleat the Square*, divide every Term in the Equation by that Co-efficient, after which compleat the Square, and proceed as before.

Queſtion 64. *To find two Numbers, that the Square of the greater being multiplied by 4, if this Product is added to 3 times the leſſer, the Sum may be 1606:*

But if 5 times the greater is added to 6 times the leſſer, the Sum may be 112.

Let a = the greater Number, e = the leſſer Number, $b=4$, $d=3$, $m=1606$, $p=5$, $x=6$, $z=112$.

	1	$baa+de=m$	By the Queſtion.
	2	$pa+xe=z$	
$1-baa$	3	$de=m-baa$	
$3\div d$	4	$e=\frac{m-baa}{d}$	
$2-pa$	5	$xe=z-pa$	
$5\div x$	6	$e=\frac{z-pa}{x}$	
$4 . 6$	7	$\frac{z-pa}{x}=\frac{m-baa}{d}$	
$7\times x$	8	$z-pa=\frac{xm-xbaa}{d}$	

8 × d

$8 \times d$	9	$dz-dpa=xm-xbaa$
$9+xbaa$	10	$xbaa+dz-dpa=xm$
$10-dz$	11	$xbaa-dpa=xm-dz$

The Equation appearing to be *Quadratic*, and all the known Quantities, except thoſe which contain the unknown one being on one Side of the Equation, divide by the *Co-efficient* of the higheſt Power of the unknown Quantity.

$11 \div xb$	12	$aa-\frac{dpa}{xb}=\frac{xm-dz}{xb}$

To avoid the Trouble of dividing $\frac{dp}{xb}$, the Co-efficient of a by 2, and ſquaring the Quotient, and adding it to both Sides of the Equation that the Square may be compleated, as in the former Queſtions, ſubſtitute $-r=-\frac{dp}{xb}=-$, 625 then,

By Subſtitution	13	$aa-ra=\frac{xm-dz}{xb}$
13 c □	14	$aa-ra+\frac{rr}{4}=\frac{xm-dz}{xb}+\frac{rr}{4}$

Now extracting the ſquare Root as in the laſt Queſtion.

$14 \omega 2$	15	$a-\frac{r}{2}=\sqrt{\frac{xm-dz}{xb}+\frac{rr}{4}}$
$15+\frac{r}{2}$	16	$a=\sqrt{\frac{xm-dz}{xb}+\frac{rr}{4}}:+\frac{r}{2}$ $(=20$, the greater Number.
By the ſixth Step	17	$e=\frac{z-pa}{x}=2$, the leſſer Number.

PROOF.

$$4aa+3e=1606$$
$$5a+6e=112.$$

Queſtion 65. *Two Gameſters, A and B, loſing at the Gaming-Tables, upon comparing their Loſſes found, that if the Square of what A loſt was multiplied by 5, and this Product added to 6 times what B loſt, the Sum was 548 Pounds:*

But if what A loſt was multiplied by 3, and to this Product adding what B loſt, multiplied by 2, the Sum was 46 Pounds. To find the loſs of each?

Let $a=$ the loſs of A, $e=$ the loſs of B, $x=5$, $m=6$, $d=548$, $b=3$, $z=2$, $r=46$.

	1	$xaa+me=d$	By the Queſtion.
	2	$ba+ze=r$	
$1-xaa$	3	$me=d-xaa$	
$3\div m$	4	$e=\frac{d-xaa}{m}$	
$2-ba$	5	$ze=r-ba$	
$5\div z$	6	$e=\frac{r-ba}{z}$	
$4\cdot 6$	7	$\frac{r-ba}{z}=\frac{d-xaa}{m}$	
$7\times m$	8	$\frac{rm-mba}{z}=d-xaa$	
$8\times z$	9	$rm-mba=zd-zxaa$	
$9+zxaa$	10	$zxaa+rm-mba=zd$	
$10-rm$	11	$zxaa-mba=zd-rm$	

The Equation being *Quadratic*, and all thoſe Terms which contain any Power of *a* being on one Side of the Equation, divide by the Co-efficient of the higheſt Power of the unknown Quantity.

$11\div zx$	12	$aa-\frac{mba}{zx}=\frac{zd-rm}{zx}$
		Subſtitute $-p=-\frac{mb}{zx}=-1.8$
By Subſtitution	13	$aa-pa=\frac{zd-rm}{zx}$
$13\ c\ \square$	14	$aa-pa+\frac{pp}{4}=\frac{zd-rm}{zx}+\frac{pp}{4}$

$14 \omega 2$	15	$a - \frac{p}{2} = \sqrt{\frac{zd - rm}{zx} + \frac{pp}{4}}$
$15 + \frac{p}{2}$	16	$a = \sqrt{\frac{zd - rm}{zx} + \frac{pp}{4}} : + \frac{p}{2} = 10$ (Pounds, the Sum loſt by A.
By the ſixth Step	17	$e = \frac{r - ba}{z} = 8$ Pounds, the Sum (loſt by B.

PROOF.

$$5aa + 6e = 548$$
$$3a + 2e = 46$$

Queſtion 66. *Two Brothers,* A *and* B, *trying each other's Skill in Algebra, ſays the eldeſt Brother, the Sum of our Ages is* 45 : *But ſays the youngeſt, if they are multiplied together, the Product is* 500. *What is the Age of each of them?*

Let $a =$ the Age of the eldeſt, $e =$ the Age of the youngeſt, $s = 45$, $p = 500$.

	1	$a + e = s$ } By the Queſtion.
	2	$ae = p$
$1 - e$	3	$a = s - e$
$2 \div e$	4	$a = \frac{p}{e}$
$3 \cdot 4$	5	$\frac{p}{e} = s - e$
$5 \times e$	6	$p = se - ee$

Becauſe the Square of the unknown Quantity has the Sign —, therefore tranſpoſe it, that the higheſt Power of the unknown Quantity may have the *affirmative* Sign,

$6 + ee$	7	$ee + p = se$
$7 - p$	8	$ee = se - p$
$8 - se$	9	$ee - se = -p$
$9\ c\ \square$	10	$ee - se + \frac{ss}{4} = \frac{ss}{4} - p$

$10 \omega 2$	11	$e - \frac{s}{2} = \sqrt{\frac{ss}{4} - p}$
$11 + \frac{s}{2}$	12	$e = \frac{s}{2} + \sqrt{\frac{ss}{4} - p} = 25$, *the Age of* (*the youngeſt.*
By the third Step	13	$a = s - e = 20$, *the Age of the eldeſt.*

This Anſwer to the Queſtion contains an *Abſurdity*, for e that is put for the Age of the *youngeſt* Brother is 25, when a that is put for the Age of the *eldeſt* Brother is only 20.

The two Roots of Quadratic Equations explained.

59. And now we ſhall explain to the young *Analyſt* that every *Quadratic* Equation has *two* Values for the unknown Quantity, and that of theſe two Values, or Roots, ſometimes one is *affirmative*, and the other is *negative*, and ſometimes both the Values are *affirmative*.

There are *three Forms* of Quadratic Equations.

The firſt is the ſixth Step of Queſtion 58, where we have $ee + be = m$.

And of this *Form* are the Equations at Queſtion 59, Step 7. Queſtion 60, Step 9. Queſtion 61, Step 9. Queſtion 62, Step 12.

The ſecond *Form* is the twelfth Step of Queſtion 63, where we have $aa - za = bd - bmx$.

And of this *Form* are the Equations at Queſtion 64, Step. 11. Queſtion 65, Step 13.

The Difference between theſe two *Forms* of Quadratic Equations is only in the loweſt Power of the unknown Quantity, having the Sign + or −, for in the firſt *Form* it has the Sign + it being be, but in the ſecond *Form* it has the Sign −, for it is $-za$. And if the loweſt Power of the unknown Quantity has ſeveral Co-efficients connected by the Signs + or −, as at Queſtion 62, Step 12. Queſtion 63, Step 11. Then if the Sum of the *poſitive*, or *affirmative* Co-efficients exceeds the Sum of the *negative* Co-efficients, the Equation is of the firſt *Form*: But, on the contrary, if the Sum of the *negative* Co-efficients exceeds the Sum of the *poſitive*, or *affirmative* Co-efficients, then the Equation is of the ſecond *Form*.

But the third *Form* is the ninth Step of the laſt Queſtion, where we have $ee - se = -p$, which differs from the other two *Forms* of Quadratic Equations, in this, that if the Side of the Equation which is known, conſiſts but of one Quantity as in the preſent Caſe, it has the Sign —, and if that Side of the Equation conſiſts of ſeveral known Quantities connected by the Signs + or —, that then the Sum of the *negative* Quantities are always greater than the Sum of the *affirmative* Quantities; but in the firſt and ſecond *Form*, if there is but one known Quantity which compoſes that Side of the Equation it will always have the *affirmative* Sign, and if there are ſeveral known Quantities connected by the Signs + or —, that then the Sum of the *affirmative* will always exceed the Sum of the *negative* Quantities.

Now of the two Values, or Roots of a in the firſt and ſecond *Form* of Quadratic Equations, one of the Values of a is *affirmative*, and the other is *negative*; and as the negative Value in theſe Equations does not come out in the Operation without a miſtake in the Work, therefore theſe two Forms of Quadratic Equations give the true Numbers required.

But the *two* Values of a in the third *Form*, are generally both *affirmative*, and the Anſwer ſometimes giving one, and ſometimes the other Number, and it being doubtful in many Caſes which of theſe two Values of a will anſwer the Conditions of the Queſtion; this *Form* of Quadratic Equations is therefore called the *Ambiguous* Form.

The Nature of Diviſion explained, when the Quotient conſiſts of ſeveral Quantities connected by the Signs + or —.

60. Before we ſhow the Reaſon of theſe two Values of the unknown Quantity in Quadratic Equations, and how from having found one Number, or Value, the Learner may find the other Number; we muſt explain the Diviſion in *Algebra*, where the Quotient conſiſts of ſeveral Quantities connected by the Signs + and —.

To render this the more eaſy to the Learner, let us reſume Example 1, Article 22, where we are to divide $ab + am$ by a, which being placed as uſual in common Arithmetic, thus, Now the Number of times a may be had in ab is b, that is, b is the Quotient of ab divided by a; place b in the Quotient, and multiply it by a, and place the Product ab as in common Diviſion, and ſubſtracting it from $ab + am$ the Dividend, there remains am; then find how many times a will go in am, and it is m, that is, m is the Quotient of am divided by a, and becauſe the Signs of the Diviſor a, and Dividend am are alike, therefore it muſt be $+m$, which being placed in the Quotient and multiplied by a, the Product is am, which placing under am, and ſubſtracting it from am, there remains 0.

$$\begin{array}{l} a)\ ab + am\ (b + m \\ ab \\ \hline +am \\ +am \\ \hline 0 \end{array}$$

Hence the Quotient is $b + m$.

To divide $xx + xm + xab$ by x.

$$\begin{array}{l} x)\ xx + xm + xab\ (x + m + ab \\ xx \\ \hline xm + xab \\ xm \\ \hline xab \\ xab \\ \hline 0 \end{array}$$

Here dividing xx by x, the Quotient is x, which being placed in the Quotient, and multiplied by the Diviſor x, and placing the Product xx under the Dividend, from which it being ſubſtracted there remains $xm + xab$.

Then dividing xm by x, the Quotient is m, or $+m$, for the Signs of xm and x are alike, put $+m$ in the Quotient, by which multiply the Diviſor x, and put the Product xm under $xm + xab$, and ſubſtracting, there remains xab.

Then dividing xab by x, the Quotient is ab, or $+ab$, for the Signs of xab and x are alike, put $+ab$ in the Quotient, by which multiply the Diviſor x, and put the Product xab, under xab, and ſubſtracting there remains 0, hence the Quotient is $x + m + ab$.

To divide $xx+2xa+aa$ by $x+a$.

$$\begin{array}{l} x+a)\ xx+2xa+aa\ (x+a \\ \qquad\quad xx+xa \\ \hline \qquad\qquad\quad xa+aa \\ \qquad\qquad\quad xa+aa \\ \hline \qquad\qquad\qquad\quad 0 \end{array}$$

Dividing xx by x, the Quotient is x, by which multiplying the Diviſor $x+a$, the Product is $xx+xa$, which being placed under the Dividend and ſubſtracted, there remains $xa+aa$.

Then dividing xa by x, the Quotient is a, or $+a$, for the Signs of xa and a are alike, put $+a$ in the Quotient, multiplying it by the Diviſor $x+a$, the Product is $xa+aa$, which put under the Remainder, or Dividend $xa+aa$, and ſubſtracting there remains 0, hence the Quotient is $x+a$.

To divide $aa-bb$ by $a+b$.

$$\begin{array}{l} a+b)\ aa-bb\ (a-b \\ \qquad\quad aa+ab \\ \hline \qquad\quad -ab-bb \\ \qquad\quad -ab-bb \\ \hline \qquad\qquad\quad 0 \end{array}$$

Dividing aa by a, the Quotient is a, which place in the Quotient, and multiply the Diviſor by a, which gives $aa+ab$, this ſubſtracted from the Dividend leaves $-ab-bb$; for here the Quantity ab, which is to be ſubſtracted, is by the Rule for Subſtraction to have its Sign changed and then added, hence $+ab$ becomes in the Remainder $-ab$.

Then dividing $-ab$ by a, the Quotient is $-b$, for the Signs of ab and a are now unlike; multiplying the Diviſor $a+b$, by $-b$, and ſubſtracting the Product $-ab-bb$, from the Remainder, or Dividend $-ab-bb$, there remains 0, hence the Quotient is $a-b$.

To divide $aaa-3aax+3axx-xxx$ by $a-x$.

$$\begin{array}{l} a-x)\ aaa-3aax+3axx-xxx\ (aa-2ax+xx \\ \qquad\quad aaa-aax \\ \hline \qquad\qquad -2aax+3axx-xxx \\ \qquad\qquad -2aax+2axx \\ \hline \qquad\qquad\qquad\qquad axx-xxx \\ \qquad\qquad\qquad\qquad axx-xxx \\ \hline \qquad\qquad\qquad\qquad\quad 0 \end{array}$$

In theſe Diviſions we may at Pleaſure take any Term in the Dividend, we have a Mind to uſe at firſt, and find how many times any Term in the Diviſor can be had in it, and when the Diviſor is multiplied by the Quotient Quantity, we ſubſtract it from the whole Dividend, that is, take any Term in the Product, from any Term in the Dividend, without regarding whether they ſtand immediately over one another or no.

And to diſcover how many times any one Quantity can be had in another, we are only to conſider into what Quantities we muſt multiply that Term in the Diviſor, to make it the ſame with the Term in the Dividend, at which we aſk the Queſtion. Or, it is no more than to find the Quotient which ariſes from dividing that particular Quantity in the Dividend, by the Quantity in the Diviſor, which is done by the Rules in Diviſion. Let us take the laſt Example and change the Poſition of the Quantities,

$$\begin{array}{rrrrl}
-x+a)\ -xxx & +aaa & +3axx & -3aax & (xx+aa-2ax \\
-xxx & & +axx & & \\
\hline
& aaa & +2axx & -3aax & \\
& aaa & & -aax & \\
\hline
& & 2axx & -2aax & \\
& & 2axx & -2aax & \\
\hline
& & & 0 &
\end{array}$$

where we have the ſame Quotient as before.

The Truth of theſe Operations is proved as in Diviſion of common Numbers, for if the Work is true the Quotient being multiplied by the Diviſor, the Product will be the given Dividend; thus in the laſt Example,

$xx+aa-2ax$ is the Quotient.
$-x+a$

$-xxx-aax+2axx$ the Product, from multiplying $xx+aa-2ax$, by $-x$.
$axx+aaa-2aax$ the Product, from multiplying $xx+aa-2ax$, by a.

$-xxx-3aax+3axx+aaa$ the ſame with the given Dividend, for though they do not ſtand in the ſame Poſition as in the Example, yet as the Quantities in each Term are alike, and they have the ſame Co-efficients, and connected by the ſame Signs, their whole Value, or Amount, muſt be the ſame.

The

The Manner of finding the two Roots, or Values, of the unknown Quantity, in Quadratic Equations, reſumed.

61. And to find the other Value of a, in the *Ambiguous* Quadratic Equation, Queſtion 66.

Take the Work at the Step immediately before you begin to *compleat the Square*, which is at the ninth Step, where the Equation is - - - - $ee - se = -p$

Make this Equation equal to *nothing*, by tranſpoſing p . - - } $ee - se + p = 0$

Then put it in Numbers, and it is $ee - 45e + 500 = 0$

By the Work we found - - $e = 25$

Make this Equation equal to *nothing*, by tranſpoſing the 25, thus, - - - } $e - 25 = 0$

Then divide $ee - 45e + 500$ by $e - 25$, thus,

$$
\begin{array}{r|l}
e - 25) & ee - 45e + 500 \; (e - 20 \\
 & \underline{ee - 25e } \\
 & -20e + 500 \\
 & \underline{-20e + 500} \\
 & 0
\end{array}
$$

Hence the Quotient is $e - 20$, but as the Dividend is *nothing*, for $ee - 45e + 500 = 0$ as above; and as the Diviſor $e - 25$ is *nothing*, for $e - 25 = 0$ as above, it follows that the Quotient muſt be *nothing*, or equal to *nothing*, that is, $e - 20 = 0$; then tranſpoſing 20, we have $e = 20$, which is the other Value of e, in this Quadratic *Ambiguous* Equation, therefore, I ſay the *youngeſt* Brother was but 20 Years of Age.

And upon this Value of e, if we take the third Step of the Work to the Queſtion, that is, $a = s - e$, we ſhall find $a = 25$, whence the *eldeſt* Brother was 25 Years of Age, and theſe are the true Ages of the two Brothers; for their Ages anſwer the Conditions of the Queſtion, and it is a poſſible Caſe, whereas though the other Numbers anſwered the Conditions of the Queſtions, yet it was impoſſible for the *youngeſt* Brother to be 25, when the *eldeſt* was but 20 Years old.

Now in the firſt Caſe, or by the Work of the Queſtion we found - - - - - } $e = 25$

But now we have found - - - - $e = 20$

The Sum of theſe two Values of e, is - - - - 45

But obſerving where we put this Quadratic Equation into Numbers, and made it equal to *nothing*, we ſhall find the Co-efficient of the firſt Power of e to be -45, but the Sum of the two Values of e is $+45$, as above, and concerning theſe *Quadratic* Equations, *Algebraiſts* give us this

SCHOLIUM.

62. That in Quadratic Equations *the Sum of both the Roots, or Values of the unknown Quantity, is equal to the Co-efficient of the loweſt Power of the unknown Quantity; But the Sum of theſe two Values of the unknown Quantity will have the contrary Sign to the Co-efficient of the loweſt Power of the unknown Quantity, at the Step immediately preceeding the compleating the Square*; that is, if the Co-efficient of the loweſt Power of the unknown Quantity has the Sign +, the Sum of both the Roots will be the ſame as the Co-efficient, but will have the Sign —.

And if the Co-efficient of the loweſt Power of the unknown Quantity has the Sign —, that then the Sum of both the Roots, or Values, will be the ſame as the Co-efficient, but will have the Sign +.

Therefore having found any one Root, the other is eaſily found.

63. *To find the other Value of the unknown Quantity in the firſt Form of Quadratic Equations, or where the Co-efficient of the loweſt Power of the unknown Quantity has the Sign* +, it is done *by adding the Value of the unknown Quantity, found from the Operation to the Co-efficient of the loweſt Power of the unknown Quantity, and to their Sum prefix the Sign* —.

Thus, at Queſtion 58, Step 6, the Co-efficient of e, is b, or - - - - - - } 10

To which adding the Value of e, as found by that Operation - - - - - - - - } 5

The Sum is - - - - - - - 15

And prefixing to this 15 the Sign $-$, then the other Value of e is -15, that is, $e=-15$, which is an *imaginary* Value of e, it being abſurd for a *poſitive* Quantity to be equal to a *negative* one.

However we ſhall find this *imaginary* Value of e, if we proceed by Diviſion according to the Directions at Art. 61.

For the ſixth Step, Queſtion 58, is that which immediately preceeds the compleating the Square, where the Equation is - - - - - - $ee+be=m$

Which is in Numbers - - - $ee+10e=75$

Tranſpoſe 75, to make the Equation equal to *nothing* - - - $ee+10e-75=0$

By the Work we found - - - - - $e=5$

Tranſpoſe 5, to make this Equation equal to *nothing* $e-5=0$

Then dividing $ee+10e-75$ by $e-5$

$$\begin{array}{l} e-5)\;ee+10e-75\;(e+15 \\ \quad\;\; \underline{ee-5e\qquad\quad} \\ \qquad\qquad 15e-75 \\ \qquad\qquad \underline{15e-75} \\ \qquad\qquad\quad\; 0 \end{array}$$

Hence the Quotient is $e+15$, but as the Dividend is equal to *nothing*, for $ee+10e-75=0$, and as the Diviſor $e-5$ is equal to *nothing*, for $e-5=0$ as above, conſequently the Quotient muſt be equal to *nothing*, that is, $e+15=0$, by tranſpoſing the 15, we have $e=-15$ as before.

For another Example of this kind take Queſtion 59, where the ſeventh Step is that which immediately preceeds the compleating the Square, where the Equation is $ee+me=x-b$, which being put in Numbers is - - $ee+6e=112$

By tranſpoſing 112, to make the Equation equal to *nothing*, we have - - $ee+6e-112=0$

By the Work it was found - - - - $e=8$

Tranſpoſing 8, to make the Equation equal to *nothing*, we have - - - - - $e-8=0$

And dividing to find the other Root of e, as before.

$e-8$

$$\begin{array}{r} e-8)\ ee+6e-112\ (e+14 \\ \underline{ee-8e} \\ 14e-112 \\ \underline{14e-112} \\ 0 \end{array}$$

Hence the Quotient is $e+14$, which for the ſame Reaſon as before, it is $e+14=0$, hence $e=-14$, for the other Value of e.

And this Value of e will be found by the Rule Art. 62.

Thus at Queſtion 59, Step 7, the Co-efficient of e is m, or - - - - - - -	6
To which adding the Value of e, found at the Operation - - - - - - - -	8
The Sum is - - - - - - -	14

Then by the Rule prefixing the Sign — to 14, we have —14 for the other, or *imaginary* Value of e, the ſame as before.

And if we add theſe two Values of e together, we ſhall find their Sum anſwer to the *Scholium*, Art. 62.

The firſt Value of e is - - - -	8
The ſecond Value of e is - - - -	—14
	—6

Hence their Sum is the ſame with the Co-efficient of e, but has the contrary Sign.

If the Reader has a Mind to perſue this Speculation any further, he may try Queſtion 60, Step 9. Queſtion 61, Step. 9. Queſtion 62, Step 12, or 13, which are Equations of this firſt Form, as well as ſome that follow them.

64. *To find the other Value of the unknown Quantity in the Second Form of Quadratic Equations;*

Which is when the Co-efficient of the loweſt Power of the unknown Quantity has the Sign —; in this Caſe *ſubſtract the Co-efficient of the loweſt Power of the unknown Quantity in the given Equation, at the Step immediately preceeding the compleating the Square, from the Value of the unknown Quantity found by the Work,*

Work, to the Remainder prefix the Sign —, and it will be the other Value of the unknown Quantity. Or place down the Co-efficient with its Sign —, to which add the Value of the unknown Quantity found by the Work, and to this Sum prefix the Sign — and it will be the other Value, or Root of the unknown Quantity.

An Equation of this ſecond *Form* is Step 12, Queſtion 63, where we have $aa - za = bd - bmx$.

Here the Co-efficient of *a*, is $-z$, or - - - -28
And the Value of *a* found in that Equation is - $+60$

The Sum is 32, but to it prefix the Sign —, and it is -32, the other Value of *a*, which is *imaginary* as it has the Sign —. - - - - - } 32

And if we proceed by Diviſion according to the Directions at Art. 61. we ſhall find this *imaginary* Value of *a*.

Thus if we take the twelfth Step of Queſtion 63, which immediately preceeds compleating the Square, we have this Equation - - - $aa - za = bd - bmx$
Which being put in Numbers is - $aa - 28a = 1920$
Tranſpoſing 1920 to make the Equation equal to *nothing* - - } $aa - 28a - 1920 = 0$

By the Work it was found - - - $a = 60$
Tranſpoſe 60 to make the Equation equal to *nothing* - - - - - - } $a - 60 = 0$

And dividing to find the other Root of *a* as before.

$$\begin{array}{l} a - 60)\; aa - 28a - 1920 \;(a + 32 \\ \qquad\quad\; \underline{aa - 60a \qquad\qquad} \\ \qquad\qquad\quad 32a - 1920 \\ \qquad\qquad\quad \underline{32a - 1920} \\ \qquad\qquad\qquad\quad 0 \end{array}$$

Hence the Quotient is $a + 32$, which becauſe the Dividend and Diviſor are each equal to *nothing*, conſequently the Quotient muſt be equal to *nothing*, hence $a + 32 = 0$
By tranſpoſing 32, we have - - - $a = -32$, the ſame *imaginary* Value of *a*, as was found before.

And if we add theſe two Values of a together, we ſhall find their Sum agree with the *Scholium* Art. 62.

The Value of a found by the Operation, Queſtion 63, is 60
The Value of a now found is - - - - -32
Sum is 28, or $+28$, the ſame Number as the Co-efficient of a, but with a contrary Sign - - } 28

Another Equation of this ſecond *Form* is Queſtion 65, Step 13, where the Equation is $aa - pa = \frac{zd - rm}{zx}$.

Which being put into Numbers is - $aa - 1.8a = 82$
Tranſpoſing 82 to make the Equation equal to *nothing* - - - - } $aa - 1.8a - 82 = 0$

By the Work it was found - - - $a = 10$
Tranſpoſing 10 to make the Equation equal to *nothing* - - - - - - - } $a - 10 = 0$

And dividing to find the other Value of a, as before,

$$\begin{array}{r}
a - 10)\ aa - 1.8a - 82\ (a + 8.2 \\
\underline{aa - 10.a\quad\quad} \\
8.2a - 82 \\
\underline{8.2a - 82} \\
0
\end{array}$$

Hence the Quotient is $a + 8.2$ which muſt be equal to *nothing*, for the Dividend and Diviſor are each equal to *nothing*, but if $a + 8.2 = 0$.

By tranſpoſing 8.2, we have $a = -8.2$ which is the other Value of a, and it is *imaginary* becauſe it has the Sign —.

The ſame *imaginary* Value of a may be found by Art. 64, thus,

The Co-efficient of a, is - - - -1.8
The Value of a found by the Queſtion, is - - $10.$
The Sum is - - - 8.2

Now to this 8.2 prefix the Sign —, and we have -8.2 for the *imaginary* Value of a, the ſame as before.

And if theſe two Values of a are added together, this Sum will agree with the *Scholium*, Art. 62.

The firſt Value of a, is - - -	10.
The ſecond Value of a, is - -	-8.2
Sum - - - - - -	1.8

But the Co-efficient of a, is -1.8

65. But in the *ambiguous*, or,

Third *Form* of Quadratic Equations,

If the Value of the unknown Quantity, found by the Operation, is ſubſtracted from the Co-efficient of its loweſt Power, at the Step immediately before the Square is compleated, the Co-efficient being ſuppoſed affirmative, the Remainder is its other Value.

At Queſtion 66, Step 9, the Co-efficient of e is s, or	45
The Value of e, found by the Operation, is -	25
The Remainder is the other Value of e - -	20

And it is this ſecond Value of e that is the true anſwer to the Queſtion, as was obſerved before Page 187; and here the Learner may again obſerve that both the Values in this Caſe are *affirmative*, which makes this be called the *ambiguous* Caſe, but in the other two preceeding Caſes, or in the four former Examples, the other Value of the unknown Quantity was *negative*, which is only an *imaginary* Value, it being impoſſible for an *affirmative*, or *poſitive* Quantity, which the Queſtion requires, to be a *negative*, or equal to a *negative* Quantity.

But we may find the other Value of e, in this *ambiguous* Caſe, by Diviſion, as in the former Inſtances, thus,

The Equation Queſtion 66, Step 9, immediately before the Square was compleated, is - - - - - $ee - se = -p$

Which being put in Numbers is - $ee - 45e = -500$

Tranſpoſing the 500, to make the Equation equal to *nothing* - $ee - 45e + 500 = 0$

By the Work it was found - - $e = 25$

Tranſpoſing 25, to make the Equation equal to *nothing* - - - - - $e - 25 = 0$

And dividing to find the other Value, or Root of e, as before.

$$\begin{array}{rl} e-25)\; & ee-45e+500\;(e-20 \\ & ee-25e \\ \hline & -20e+500 \\ & -20e+500 \\ \hline & 0 \end{array}$$

Hence the Quotient is $e-20$, which muſt be equal to *nothing*, for the reaſon in the former Caſes, but if - $e-20=0$
Tranſpoſing 20, we have - - $e=20$ the other Value of e, the ſame as before.

And in this *ambiguous* Caſe, if we add the two Values of e together, we ſhall find them agree with what is ſaid at the *Scholium*, Art. 62.

The firſt Value of e, is - - - 25
The ſecond Value of e, is - - - 20

45

But the Co-efficient of e is -45.

The Manner of expreſſing the Roots of an ambiguous Quadratic Equation explained.

66. Now to explain the uſual Manner in which *Algebraiſts* expreſs the Value of the unknown Quantity, in the *ambiguous* Quadratic Equation; let us reſume the ſolution of Queſtion 66, at the eighth Step, where there is this

Equation	1	$ee=se-p$
$1-se$	2	$ee-se=-p$
2 c □	3	$ee-se+\frac{ss}{4}=\frac{ss}{4}-p$
3 ɯ 2	4	$e-\frac{s}{2}=\sqrt{\frac{ss}{4}-p}$
$4+\frac{s}{2}$	5	$e=\frac{s}{2}\pm\sqrt{\frac{ss}{4}-p}$

That is, *prefix both the Signs + and —, to the Quantity under the radical Sign, for that being added to* $\frac{s}{2}$, *or the rational Quantities on that Side of the Equation, gives* one *of the Values of e but if it is ſubſtracted from* $\frac{s}{2}$, *or the rational Quantities on that Side of the Equation, then it gives the* other *Value of e, thus,*

$$\begin{array}{l} s=45 \\ s=45 \\ \hline \quad 225 \\ \quad 180 \\ \hline 4)\ 2025=ss \\ \hline \qquad 506.25=\frac{ss}{4} \\ -p=500. \\ \hline \qquad 6.25=\frac{ss}{4}-p\ (2.5=\sqrt{\frac{ss}{4}-p} \\ \qquad 4 \\ \hline 45)\ 225 \\ \qquad 225 \\ \hline \qquad 0 \end{array}$$

Then to find the two Values, or Roots of *e*.

$$\begin{array}{r} \frac{s}{2}=22.5 \\ +\sqrt{\frac{ss}{4}-p}=2.5 \\ \hline 25.0= \end{array}\text{ one of the Values of } e,$$

$$\begin{array}{r} \frac{s}{2}=22.5 \\ -\sqrt{\frac{ss}{4}-p}=-2.5 \\ \hline 20. \end{array}$$

20. the other Value of *e*, which two Values of *e* are the ſame as was found at Art. 61.

And this is the common Method in which *Algebraiſts* ſet down, or expreſs the Value of the unknown Quantity, in the *ambiguous* Quadratic Equation.

The reaſon of Quadratic Equations having two different Values of the ſame unknown Quantity, is becauſe the ſame Quadratic Equation can be formed from two different Suppoſitions, or Values of the unknown Quantity, or ſuppoſing the ſame unknown Quantity to be equal to two different Numbers.

For let us reſume the Equation $ee-se=-p$, or $ee-45e$ $=500$, in this *ambiguous* Equation we found the firſt Value of e, to be 25, by making e equal to 25, we have

	1	$e=25$
1 ⊙ 2	2	$ee=625$
Multiplying the firſt Equation by —45, the Co-efficient of e, in the given Equation. -	3	$-45e=-1125$
2 + 3	4	$ee-45e=-500$, the ſame with the given Quadratic Equation.

And if we take the other Value of e, *viz.* 20, we can form the given Equation, for

Let	1	$e=20$
1 ⊙ 2	2	$ee=400$
1 × —45 the Co-efficient of e, in the given Equation. - -	3	$-45e=-900$
2 + 3	4	$ee-45e=-500$, the ſame with the given Quadratic Equation.

Likewiſe if we take the firſt *Form* of Quadratic Equations, *viz.* $ee+be=m$, or $ee+10e=75$, ſee Queſtion 58, Step 6. Now the two Values of e in this Equation we found to be 5, and —15, and from either of theſe Values of e we can form the given Quadratic Equation, ſuppoſe

	1	$e=5$
1 ⊙ 2	2	$ee=25$

1 x

$1 \times \overline{10}$ the Co-efficient of e, in the given Equation. - - -	3	$10e = 50$
$2 + 3$	4	$ee + 10e = 75$, the ſame with the given Quadratic Equation.

Again, ſuppoſe	1	$e = -15$
1 ⊚ 2	2	$ee = 225$, for $-15 \times -15 = +225$, the Signs being alike.
$1 \times \overline{10}$ as above	3	$10e = -150$
$2 + 3$	4	$ee + 10e = 75$, the ſame Equation as before.

And if we take the ſecond *Form* of Quadratic Equations, *viz.* $aa - za = bd - bmx$, or $aa - 28a = 1920$, ſee Queſtion 63, Step 12. The two Values of a in this Equation we have found to be 60, and -32, from either of which we can form the given Equation, for

Suppoſe	1	$a = 60$
1 ⊚ 2	2	$aa = 3600$
$1 \times -\overline{28}$ the Co-efficient of a, in the given Equation. - -	3	$-28a = -1680$
$2 + 3$	4	$aa - 28a = 1920$, the ſame with the given Equation.

Again, if	1	$a = -32$
1 ⊚ 2	2	$aa = 1024$, for $-32 \times -32 = +1024$.
$1 \times -\overline{28}$ the Co-efficient of a, as above.	3	$-28a = 896$, for $-28 \times -32 = +896$.
$2 + 3$	4	$aa - 28a = 1920$, the ſame with the given Equation.

From this the Learner may obſerve, *that making the unknown Quantity equal to either of its Values, and raiſing this Equation to the Square, and adding it to the former Equation, after it has been multiplied by the Co-efficient of the loweſt Power of the unknown Quantity in the Quadratic Equation, this Sum will be the given Quadratic Equation.*

Queſtion 67. *Two Men,* A *and* B, *diſcourſing of their Shillings,* A *who had the greateſt Number ſaid if my Number of Shillings is divided by your's, and this Quotient is added to your Number of Shillings, the Sum will be* 15:

But if the Sum of both our Shillings is multiplied by 4, *and this Product divided by* 10, *the Quotient will be* 22. *How many Shillings had each Perſon?*

Let $a =$ the Number of Shillings A had, or the greateſt Number, $e =$ the Number of Shillings B had, $s = 15$, $m = 4$, $n = 10$, $d = 22$.

Then	1	$\frac{a}{e} + e = s$	By the Queſtion.
And	2	$\frac{ma + me}{n} = d$	
$1 \times e$	3	$a + ee = se$	
$3 - ee$	4	$a = se - ee$	
$2 \times n$	5	$ma + me = dn$	
$5 - me$	6	$ma = dn - me$	
$6 \div m$	7	$a = \frac{dn - me}{m}$	
$4 \cdot 7$	8	$\frac{dn - me}{m} = se - ee$	
$8 \times m$	9	$dn - me = mse - mee$	
$9 + mee$	10	$mee + dn - me = mse$	
$10 - dn$	11	$mee - me = mse - dn$	
$11 - mse$	12	$mee - me - mse = - dn$	

Here the Equation appears to be *Quadratic* and of the *ambiguous* kind; becauſe dn the known Side of the Equation has the *negative* Sign. Then by Art. 57, dividing by m, the Co-efficient of ee,

$12 \div m$	13	$ee - e - se = - \frac{dn}{m}$.

For m being in every Term on one Side of the Equation, dividing that part of the Equation by m, is only to caſt away m, out of every Term of that Side of the Equation, and place it as a Denominator to the other Side of the Equation, it not being in that Term. The Equation being now prepared for *compleating the Square*, and the firſt Power of e being in two Terms, *viz.* $- e - se$, whoſe Co-efficients are $- 1$, and $- s$,

Therefore

Therefore		Subſtitute $-z=-1-s$, then by
Subſtitution	14	$ee-ze=-\frac{dn}{m}$
$14\ e\ \square$	15	$ee-ze+\frac{zz}{4}=\frac{zz}{4}-\frac{dn}{m}$
$15\ w\ 2$	16	$e-\frac{z}{2}=\sqrt{\frac{zz}{4}-\frac{dn}{m}}$
$16+\frac{z}{2}$	17	$e=\frac{z}{2}\pm\sqrt{\frac{zz}{4}-\frac{dn}{m}}=8\pm3$,

(that is, e is either 5, or 11.

And if e is 5, by the fourth Step, we ſhall find $q=50$, which two Numbers of Shillings anſwers the Conditions of the Queſtion; or, if we ſuppoſe $e=11$, then by the fourth Step we ſhall find $a=44$, which two Numbers will likewiſe anſwer the Conditions of the Queſtions, and in theſe ambiguous Equations this kind of double Anſwer is owing to the Poſſibility of having the ſame Concluſion from different Numbers: Thus,

$$4+2=6 \qquad \text{And } 1+5=6$$

$$\frac{10\times4}{8}=5 \qquad \frac{25\times4}{20}=5$$

where the Learner may obſerve that the ſame Concluſions follow though the Numbers differ: But ſometimes one of the Numbers, or Roots, of theſe ambiguous Equations will not anſwer all the Conditions of the Queſtion, as at Queſtion 72, and then the other Root muſt be found.

Queſtion 68. *Two Merchants,* A *and* B, *had gained in Trade, but* A, *who had gained the moſt, found, that if the Square of what he had gained was multiplied by* 2, *and this Product added to* 8 *times what* B *had gained, if this Sum was divided by* 4, *the Quotient was* 816 *Pounds:*

But if 3 *times what* A *had gained was added to* 10 *times what* B *had gained, and this Sum divided by* 40, *the Quotient was* 5 *Pounds. How many Pounds had each Man gained?*

Put $a=$ the Pounds gained by A, $e=$ the Pounds gained by B, $x=2$, $m=8$, $p=4$, $d=816$, $b=3$, $z=10$, $r=40$, $n=5$.

	1	$\frac{xaa+me}{p}=d$ } By the Queſtion.
	2	$\frac{ba+ze}{r}=n$
$1\times p$	3	$xaa+me=pd$
$3-xaa$	4	$me=pd-xaa$
$4\div m$	5	$e=\frac{pd-xaa}{m}$
$2\times r$	6	$ba+ze=rn$
$6-ba$	7	$ze=rn-ba$
$7\div z$	8	$e=\frac{rn-ba}{z}$
8.5	9	$\frac{rn-ba}{z}=\frac{pd-xaa}{m}$
$9\times z$	10	$rn-ba=\frac{zpd-zxaa}{m}$
$10\times m$	11	$mrn-mba=zpd-zxaa$

Tranſpoſe $zxaa$, that the higheſt Power of the unknown Quantity may have the Sign $+$.

$11+zxaa$	12	$zxaa+mrn-mba=zpd$
$12-mrn$	13	$zxaa-mba=zpd-mrn$

The Equation now appears to be quadratic, but to know if it is *ambiguous*, find which Quantity is greateſt zpd, or mrn, but zpd is 32640, and mrn is only 1600, hence $zpd-mrn=32640-1600=31040$, which having the Sign $+$, or being an *affirmative* Number, the Equation is not ambiguous, by Art. 59. And becauſe the Square of the unknown Quantity has a Co-efficient, therefore, by Art. 57,

$13\div zx$	14	$aa-\frac{mba}{zx}=\frac{zpd-mrn}{zx}$
		Subſtitute $-b=-\frac{mb}{zx}$ then by
Subſtitution	15	$aa-ba=\frac{zpd-mrn}{zx}$
15 c □	16	$aa-ba+\frac{bb}{4}=\frac{zpd-mrn}{zx}+\frac{bb}{4}$

16 w

16 ɯ 2	17	$a - \frac{b}{2} = \sqrt{\frac{zpd - mrn}{zx} + \frac{bb}{4}}$
$17 + \frac{b}{2}$	18	$a = \sqrt{\frac{zpd - mrn}{zx} + \frac{bb}{4}} : + \frac{b}{2}$ (= 40, the Pounds gained by A.
By the eighth Step	19	$e = \frac{rn - ba}{z} = 8$, the Pounds gained (by B.

Queſtion 69. *A Father by his Will, left his two Sons* A *and* B, *a Portion, whereof* A *had the greateſt Fortune; and if the Square of the Number of Pounds he was to have, be multiplied by* 2, *and to this Product there is added the Number of Pounds* B *was to have multiplied by* 35, *the Sum is* 6400 *Pounds:*

But if the Number of Pounds A *was to have be multiplied by* 20, *and this Product added to the Number of Pounds* B *was to have when multiplied by* 15, *the Sum is* 1600 *Pounds. To find the Fortune of each?*

Let $a =$ the Fortune of A, $e =$ the Fortune of B, $x = 2$, $m = 35$, $d = 6400$, $b = 20$, $z = 15$, $r = 1600$.

	1	$xaa + me = d$ } By the Queſtion.
	2	$ba + ze = r$
$1 - xaa$	3	$me = d - xaa$
$3 \div m$	4	$e = \frac{d - xaa}{m}$
$2 - ba$	5	$ze = r - ba$
$5 \div z$	6	$e = \frac{r - ba}{z}$
4 . 6	7	$\frac{r - ba}{z} = \frac{d - xaa}{m}$
$7 \times z$	8	$r - ba = \frac{zd - zxaa}{m}$
$8 \times m$	9	$mr - mba = zd - zxaa$

Tranſpoſe $zxaa$ becauſe the higheſt Power of a may be *affirmative.*

$9 + zxaa$	10	$zxaa + mr - mba = zd$
$10 - mr$	11	$zxaa - mba = zd - mr$

The Equation now appears to be *quadratic*, and to know if it is *ambiguous*, find what $z d$ and $m r$, are in Numbers. But $z d - m r = 96000 - 56000 = 40000$, a *poſitive* Quantity, whence the Equation is not *ambiguous* by Art. 59. And becauſe the Square of the unknown Quantity has a Co-efficient, therefore by Art. 58.

$11 \div z x$	12	$a a - \frac{m b a}{z x} = \frac{z d - m r}{z x}$
		Subſtitute $-b = -\frac{m b}{z x}$ then by
Subſtitution	13	$a a - b a = \frac{z d - m r}{z x}$
13 c □	14	$a a - b a + \frac{b b}{4} = \frac{z d - m r}{z x} + \frac{b b}{4}$
14 w 2	15	$a - \frac{b}{2} = \sqrt{\frac{z d - m r}{z x} + \frac{b b}{4}}$
$15 + \frac{b}{2}$	16	$a = \sqrt{\frac{z d - m r}{z x} + \frac{b b}{4}} : + \frac{b}{2}$

$= 49.9999$, &c. becauſe of the Imperfection of the Decimal Fraction. The true Number being 50, from which by

The ſixth Step	17	$e = \frac{r - b a}{z} = 40.$

Queſtion 70. *What are thoſe two Numbers, that the Quotient of the greater being divided by 5, and added to the leſſer, this Sum may be 12:*

But the Product of the two Numbers divided by 4, the Quotient is 40.

Put $a =$ the greater Number, $e =$ the leſſer Number, $m = 5$, $p = 12$, $d = 4$, $x = 40$.

	1	$\frac{a}{m} + e = p$	By the Queſtion.
	2	$\frac{a e}{d} = x$	
$1 \times m$	3	$a + m e = m p$	
$3 - m e$	4	$a = m p - m e$	
$2 \times d$	5	$a e = d x$	

5 ÷ 4

$5 \div e$	6	$a = \frac{dx}{e}$
$4 \cdot 6$	7	$\frac{dx}{e} = mp - me$
$7 \times e$	8	$dx = mpe - mee$
$8 + mee$	9	$mee + dx = mpe$
$9 - dx$	10	$mee = mpe - dx$
$10 - mpe$	11	$mee - mpe = -dx$

Here the Equation not only appears *quadratic* but *ambiguous*, for dx, the known Side of the Equation, is *negative*. Now by Art. 58.

$11 \div m$	12	$ee - pe = -\frac{dx}{m}$
12 c □	13	$ee - pe + \frac{pp}{4} = \frac{pp}{4} - \frac{dx}{m}$
13 ω 2	14	$e - \frac{p}{2} = \sqrt{\frac{pp}{4} - \frac{dx}{m}}$
$14 + \frac{p}{2}$	15	$e = \frac{p}{2} \pm \sqrt{\frac{pp}{4} - \frac{dx}{m}} = 6 \pm 2$, that is, e is either 8, or 4: But if $e = 8$, then by
The ſixth Step	16	$a = \frac{dx}{e} = 20$. Or if $e = 4$, then

$a = 40$, either of which anſwers the Queſtion, the Reaſon of which ſee Queſtion 67.

Queſtion 71. *Two young Gentlemen having been at the Gaming-Tables, and being asked by their Friend what they loſt, which being aſhamed to own, A ſaid if the Number of Pounds I loſt is divided by 4, and this added to the Number of Pounds B loſt divided by 2, the Sum is 9 Pounds:*

But if the Product of the Number of Pounds we both loſt is divided by 10, and extracting the ſquare Root of this Quotient it will be 4. How much did each Perſon loſe?

Let $a =$ the Number of Pounds loſt by A, $e =$ the Number of Pounds loſt by B, $b = 4$, $d = 2$, $m = 9$, $p = 10$, the Number 4 being in the firſt Part of the Queſtion, and being again repeated you need not put any new Letter for it.

	1	$\frac{a}{b} + \frac{e}{d} = m$	By the Queſtion.
	2	$\sqrt{\frac{ae}{p}} = b$	
$1 - \frac{e}{d}$	3	$\frac{a}{b} = m - \frac{e}{d}$	
$3 \times b$	4	$a = bm - \frac{be}{d}$	
2 ⊝ 2	5	$\frac{ae}{p} = bb$	
$5 \times p$	6	$ae = pbb$	
$6 \div e$	7	$a = \frac{pbb}{e}$	
4 · 7	8	$\frac{pbb}{e} = bm - \frac{be}{d}$	
$8 \times e$	9	$pbb = bme - \frac{bee}{d}$	
$9 \times d$	10	$dpbb = dbme - bee$	
		Dividing by b, by Art. 53.	
$10 \div b$	11	$dpb = dme - ee$	
		To have the higheſt Power of e *affirmative*, tranſpoſe ee.	
$11 + ee$	12	$ee + dpb = dme$	
$12 - dpb$	13	$ee = dme - dpb$	
$13 - dme$	14	$ee - dme = -dpb$	
		Here the Equation appears *quadratic*, and *ambiguous*.	
14 c □	15	$ee - dme + \frac{ddmm}{4} = \frac{mmdd}{4} - dpb$	
15 ɯ 2	16	$e - \frac{dm}{2} = \sqrt{\frac{mmdd}{4} - dpb}$	
$16 + \frac{dm}{2}$	17	$e = \frac{dm}{2} \pm \sqrt{\frac{mmdd}{4} - dpb} = 9 \pm 1$,	

that is, e is either 8, or 10, whence by the fourth, or ſeventh Steps, we ſhall find $a = 20$, or 16.

Queſtion 72. *In the Right-angled triangle* A B C *there is given the Hypothenuſe* A C = 10, *and the Sum of the Baſe* A B *and Perpendicular* B C = 14. *To find the Baſe* A B *and Perpendicular* B C? See Figure, Page 206.

Let $AC = b = 10$, $AB + BC = d = 14$, $AB = a$; and becauſe $AB + BC = d$, therefore from this ſubſtracting AB, or a, we have $BC = d - a$.

Having expreſſed all the Sides of the Figure in Symbols, and there being only one unknown Quantity, we are only to raiſe one Equation from the Property of the Figure; and the Triangle ABC being right-angled, we have by 47 *e* 1 the Square of the Hypothenuſe AC, or bb, equal to the Square of the Baſe AB, or aa, added to the Square of the Perpendicular BC, or $dd - 2da + aa$, that is,

In Symbols	1	$bb = dd - 2da + 2aa$
$1 - dd$	2	$2aa - 2da = bb - dd$

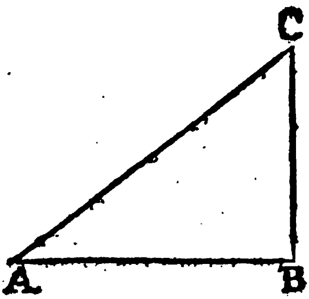
Here the Equation appears *quadratic*, and becauſe dd is greater then bb it is likewiſe *ambiguous*, for $bb - dd = 100 - 196 = -96$ a *negative* Quantity, but becauſe the Square of the unknown Quantity has a Co-efficient therefore divide by it by Art. 58.

$2 \div 2$	3	$aa - da = \frac{bb - dd}{2}$
$3 \; c \; \square$	4	$aa - da + \frac{dd}{4} = \frac{dd}{4} + \frac{bb - dd}{2}$
$4 \; w \; 2$	5	$a - \frac{d}{2} = \sqrt{\frac{dd}{4} + \frac{bb - dd}{2}}$
$5 + \frac{d}{2}$	6	$a = \frac{d}{2} \pm \sqrt{\frac{dd}{4} + \frac{bb - dd}{2}} = 7 \pm 1$

from whence the Baſe AB may be either 8, or 6; ſuppoſing the Baſe 8, then becauſe by the Queſtion the Sum of the Baſe and Perpendicular is 14, the Perpendicular BC will be 6, but if we ſuppoſe the Baſe to be 6, then from the ſame reaſoning the Perpendicular BC will be 8.

And the Queſtion not limiting which is longeſt, either the Baſe AB, or Perpendicular BC, we may take either 6, or 8, for the length of the Baſe AB, for either will anſwer the Conditions of the Queſtion.

But if the Queſtion had ſaid that the Baſe AB, is longer then the Perpendicular BC, then we muſt take $a = \frac{d}{2} + \sqrt{\frac{dd}{4} + \frac{bb - dd}{2}} = 8$, by which we ſhall find the Perpendicular BC $= 6$; for if we take $a = \frac{d}{2} - \sqrt{\frac{dd}{4} + \frac{bb-dd}{2}} = 6$, then we ſhall find the Perpendicular BC $= 8$, which cannot be, becauſe the Queſtion is ſuppoſed to determine the Baſe AB, to be longer than the Perpendicular BC.



Queſtion 73. *In the right-angled Triangle* ABC *there is given the Hypothenuſe* AC $= 10$, *the Perpendicular* BC *being ſhorter then the Baſe* AB, *by ſubſtracting the Perpendicular* BC *from the Baſe* AB, *and multiplying the Difference by* 20, *and dividing this Product by* 8 *the Quotient is* 5. *What is the length of the Baſe* AB, *and Perpendicular* BC?

Let $AC = b = 10$, $AB = a$, $BC = e$, $d = 20$, $m = 8$, $z = 5$.

	1	$aa + ee = bb$ by the Property of the Figure as in the laſt Queſtion.
	2	$\frac{da - de}{m} = z$ by the Queſtion.
$1 - ee$	3	$aa = bb - ee$
3 ω 2	4	$a = \sqrt{bb - ee}$
$2 \times m$	5	$da - de = zm$
$5 + de$	6	$da = zm + de$
$6 \div d$	7	$a = \frac{zm + de}{d}$
$4 \cdot 7$	8	$\frac{zm + de}{d} = \sqrt{bb - ee}$

Squaring both Sides of the Equation, becauſe the unknown Quantity is under the radical Sign.

8 ⊙ 2	9	$\frac{zzmm + 2zmde + ddee}{dd} = bb - ee$
$9 \times dd$	10	$zzmm + 2zmde + ddee = ddbb - ddee$
$10 + ddee$	11	$zzmm + 2zmde + 2ddee = bbdd$
$11 - zzmm$	12	$2ddee + 2zmde = bbdd - zzmm$

Dividing

		Dividing by the Co-efficient of ee by Art. 58.
$12 \div 2dd$	13	$ee + \frac{2zmde}{2dd} = \frac{bbdd - zzmm}{2dd}$
That is	14	$ee + \frac{zme}{d} = \frac{bbdd - zzmm}{2dd}$ for $\frac{2zmde}{2dd} = \frac{zme}{d}$, for $2d$ may be rejected as in Diviſion.

Here the Equation is *quadratic*, but it cannot be *ambiguous*, becauſe both the Quantities $ee + \frac{zme}{d}$ having the Sign +, the whole Side of the Equation muſt be *affirmative*, and conſequently the other Side of the Equation muſt be alſo *affirmative*, otherwiſe an *affirmative* Quantity would be equal to a *negative* Quantity, which is abſurd. Now,

		Subſtitute $x = \frac{zm}{d} = 2$, by Art. 57. then by
Subſtitution	15	$ee + xe = \frac{bbdd - zzmm}{2dd}$
15 c □	16	$ee + xe + \frac{xx}{4} = \frac{bbdd - zzmm}{2dd} + \frac{xx}{4}$
16 ω 2	17	$e + \frac{x}{2} = \sqrt{\frac{bbdd - zzmm}{2dd} + \frac{xx}{4}}$
$17 - \frac{x}{2}$	18	$e = \sqrt{\frac{bbdd - zzmm}{2dd} + \frac{xx}{4}} - \frac{x}{2} = 6 = BC.$
Then by Step 7th	19	$a = \frac{zm + de}{d} = 8 = AB.$

The ſame Queſtion done otherwiſe.

Let $AC = b = 10$, $AB = a$, then by 47 e 1, $BC = \sqrt{bb - aa}$; ſuppoſe $d = 20$, $m = 8$, $z = 5$, as before.

Now all the Sides of the Triangle being expreſſed in Symbols, and there being only one unknown Quantity, there is only one Equation required, which may be raiſed from the Conditions of the Queſtion, and theſe I ſhall particularly expreſs to prevent any Difficulty to the Learner.

Now | 1 | a, is the Baſe A B, which is longer than the Perpendicular B C, or $\sqrt{bb-aa}$, therefore connecting $\sqrt{bb-aa}$ to a, by the Sign —,

We have | 2 | $a-\sqrt{bb-aa}$ for the Difference between the Baſe and Perpendicular, which is to be multiplied by 20, or d, then

We have | 3 | $da-d\sqrt{bb-aa}$

But this Product is to be divided by 8, or m, then

We have | 4 | $\frac{da-d\sqrt{bb-aa}}{m}$ and this Quotient (is to be equal to 5, or z.

Whence | 5 | $\frac{da-d\sqrt{bb-aa}}{m}=z$, by the Queſtion.

Becauſe the unknown Quantity is divided by m, therefore by Art. 47.

$5\times m$ | 6 | $da-d\sqrt{bb-aa}=zm$

Becauſe the unknown Quantity is multiplied by d, therefore by Art. 48.

$6\div d$ | 7 | $a-\sqrt{bb-aa}=\frac{zm}{d}$

Now tranſpoſe the Surd becauſe it has the Sign —, the higheſt Power of the unknown Quantity being part of it.

$7+\sqrt{bb-aa}$ | 8 | $a=\frac{zm}{d}+\sqrt{bb-aa}$

$8-\frac{zm}{d}$ | 9 | $\sqrt{bb-aa}=a-\frac{zm}{d}$

There being no Quantities on the ſame Side of the Equation with the Surd, raiſe both Sides of the Equation to the ſecond Power to take away the radical Sign.

9 ⊙ 2

$9 \circledcirc 2$	10	$bb - aa = aa - \frac{2zma}{d} + \frac{zzmm}{dd}$
$10 + aa$	11	$2aa - \frac{2zma}{d} + \frac{zzmm}{dd} = bb$
$11 - \frac{zzmm}{dd}$	12	$2aa - \frac{2zma}{d} = bb - \frac{zzmm}{dd}$
$12 \div 2$	13	$aa - \frac{zma}{d} = \frac{bb}{2} - \frac{zzmm}{2dd}$

Here the Equation is *quadratic*, but because $\frac{bb}{2} - \frac{zzmm}{2dd} = 50 - 2 = 48$, a *poſitive* Quantity, the Equation is not *ambiguous*. Now by Art. 57, ſubſtitute $-x = -\frac{zm}{d} = -2$.

Then	14	$aa - xa = \frac{bb}{2} - \frac{zzmm}{2dd}$
14 c □	15	$aa - xa + \frac{xx}{4} = \frac{xx}{4} + \frac{bb}{2} - \frac{zzmm}{2dd}$
15 w 2	16	$a - \frac{x}{2} = \sqrt{\frac{xx}{4} + \frac{bb}{2} - \frac{zzmm}{2dd}}$
$16 + \frac{x}{2}$	17	$a = \frac{x}{2} + \sqrt{\frac{xx}{4} + \frac{bb}{2} - \frac{zzmm}{2dd}}$ (= 8 = the Baſe AB as before.

Hence in the right-angled Triangle ABC, becauſe we have given the Hypothenuſe AC which is 10, and having now found the Baſe AB to be 8, therefore the Perpendicular BC $= \sqrt{100 - 64} = 6$.

Queſtion 74. *Two Merchants,* A *and* B, *becoming Bankrupts, owe ſuch Sums of Money that if from the Number of Pounds* A *owes, we ſubſtract the Square of the Number of Pounds* B *owes, there will remain* 1900 *Pounds:*

But if the Square of the Number of Pounds B *owes, is multiplied by the Number of Pounds* A *owes, the Product is* 81000000 *Pounds. To find the Debt of each Merchant?*

Let a = the Money owed by A, e = the Money owed by B, b = 1900, d = 81000000.

	1	$a - ee = b$ } By the Queſtion.
	2	$aee = d$
$1 + ee$	3	$a = b + ee$
$2 \div ee$	4	$a = \frac{d}{ee}$
$3 \cdot 4$	5	$ee + b = \frac{d}{ee}$
$5 \times ee$	6	$eeee + bee = d$

Perhaps this Equation may appear new to the young *Analyſt*, but by turning to Art. 56. he will find it to be a *quadratic* Equation, for the unknown Quantity is only in two Terms, and in one of them its Power or Height, is *double* its Power and Height in the other, for it is $eeee$ and ee, therefore now take the Co-efficient of ee the loweſt Power of e in the preſent Caſe; divide the Co-efficient by 2, ſquare this Quotient, and add it to both Sides of the Equation as before, thus,

6 c □	7	$eeee + bee + \frac{bb}{4} = d + \frac{bb}{4}$
		Extracting the ſquare Root as uſual,
7 ɯ 2	8	$ee + \frac{b}{2} = \sqrt{d + \frac{bb}{4}}$
		Tranſpoſing $\frac{b}{2}$ becauſe it is a known Quantity,
$8 - \frac{b}{2}$	9	$ee = \sqrt{d + \frac{bb}{4}} : - \frac{b}{2}$
		Now extracting the ſquare Root to depreſs e to the firſt Power,
9 ɯ 2	10	$e = \sqrt{\sqrt{d + \frac{bb}{4}} : - \frac{b}{2}}$ = 90 Pounds, (the Money B owed.

To extract the ſquare Root of the Quantity $\sqrt{d + \frac{bb}{4}} : - \frac{b}{2}$, is only to place again the radical Sign before the ſame

Quantity, drawing it over the radical Sign already there; and the other Quantities without that Sign if there are any, for though theſe were not included in the firſt Root, yet as they were afterwards tranſpoſed to that Side of the Equation, and the Root is again required to be taken, they will now be included under the radical Sign by this ſecond Extraction.

Becauſe of the two radical Signs, I ſhall ſet down the Numerical Work, to make that Operation the plainer.

$$\begin{array}{rl} d= & 81000000 \\ \frac{bb}{4}= & 902500 \\ \hline & 81902500=d+\frac{bb}{4}\ (9050=\sqrt{d+\frac{bb}{4}} \\ & 81 \\ \hline 1805) & 9025 \\ & 9025 \\ \hline & 0 \end{array}$$

$$-950=-\frac{b}{2}$$

$$8100=\sqrt{d+\frac{bb}{4}}:-\frac{b}{2}$$

and the ſquare Root of 8100 is $90=\sqrt{\sqrt{d+\frac{bb}{4}}:-\frac{b}{2}}$.
Then by the third Step $a=b+ee$ $=$ 10000 Pounds, the Money owed by A.

The ſame Queſtion anſwered by exterminating the unknown Quantity e.

$$\begin{array}{r|c|l} & 1 & a-ee=b \\ & 2 & aee=d \end{array}\Big\}\text{By the Queſtion as before.}$$

$$\begin{array}{r|c|l} 1+ee & 3 & a=b+ee \\ 3-b & 4 & ee=a-b \\ 2\div a & 5 & ee=\frac{d}{a} \end{array}$$

Make the fourth and fifth Equations equal to one another, for each is equal to ee.

$$\begin{array}{r|c|l} 4\cdot 5 & 6 & a-b=\frac{d}{a} \\ 6\times a & 7 & aa-ba=d \end{array}$$

$7\ c\ \square$	8	$aa-ba+\frac{bb}{4}=d+\frac{bb}{4}$
$8\ \omega\ 2$	9	$a-\frac{b}{2}=\sqrt{d+\frac{bb}{4}}$
$9+\frac{b}{2}$	10	$a=\sqrt{d+\frac{bb}{4}}:+\frac{b}{2}=$10000 Pounds, (as before.

And by the fourth Step $e=\sqrt{a-b}$, or by the fifth Step $e=\sqrt{\frac{d}{a}}=90$ Pounds as before.

From hence the Learner may obſerve, there are different Methods of anſwering the ſame Queſtion, and that ſome are more elegant then others, as they give the Anſwer in more ſimple or leſs complicated Terms; and in this Part of the Science he is to exerciſe himſelf according to his own Prudence and Judgment, and ſome Meaſure in Proportion as he has acquired and clearly underſtands, or conceives the general and univerſal Methods by which Queſtions are anſwered; it being only my deſign to illuſtrate theſe by pertinent Examples, with ſuch Solutions as ariſe in an obvious Manner from the Directions, that theſe may be the more fully and compleatly underſtood.

Queſtion 73. *Two running Footmen,* A *and* B, *meeting on the Road, found, if the Number of Miles* A *had run was multiplied by* 5, *ſubſtracting from this Product the Square of the Miles run by* B, *there remained* 100:

But if the Square of the Miles run by B, *was multiplied by the Number of Miles run by* A, *and this Product multiplied by* 2, *the Product was* 80000. *How many Miles had each Perſon run?*

Let $a=$ the Number of Miles run by A, $e=$ the Number of Miles run by B, $m=5$, $x=100$, $d=2$, $b=80000$.

	1	$ma-ee=x$ } By the Queſtion.
	2	$daee=b$
$1+ee$	3	$ma=x+ee$
$3\div m$	4	$a=\frac{x+ee}{m}$
$2\div dee$	5	$a=\frac{b}{dee}$

4 · 5

4 · 5	6	$\frac{x+ee}{m} = \frac{b}{dee}$
6 × m	7	$ee + x = \frac{mb}{dee}$
7 × dee	8	$deeee + xdee = mb$

Here the Equation appears to be of the ſame kind with the laſt, that is quadratic, but not ambiguous. Now by Art. 58.

9 ÷ d	9	$eeee + xee = \frac{mb}{d}$ And compleating the Square as in the laſt Queſtion,
10 ç □	10	$eeee + xee + \frac{xx}{4} = \frac{mb}{d} + \frac{xx}{4}$
11 ɯ 2	11	$ee + \frac{x}{2} = \sqrt{\frac{mb}{d} + \frac{xx}{4}}$
12 − $\frac{x}{2}$	12	$ee = \sqrt{\frac{mb}{d} + \frac{xx}{4}} : - \frac{x}{2}$
13 ɯ 2	13	$e = \sqrt{\sqrt{\frac{mb}{d} + \frac{xx}{4}} : - \frac{x}{2}}$

2) $400000 = mb$

$200000 = \frac{mb}{d}$

$2500 = \frac{xx}{4}$

202500 $(450 = \sqrt{\frac{mb}{d} + \frac{xx}{4}}$

16

85) 425

425

0

$-50 = -\frac{x}{2}$

400 $(20 = \sqrt{\sqrt{\frac{mb}{d} + \frac{xx}{4}} : - \frac{x}{2}}$ = the Number of Miles run by B.

Then by the fourth Step $a = \frac{x+ee}{m} = 100$, the Number of Miles run by A.

This Queſtion as the laſt, may be reſolved in a more ſimple Manner, if we exterminate the unknown Quantity e inſtead of a, thus,

	1	$ma - ee = x$ } By the Queſtion as before.
	2	$daee = b$
$1 + ee$	3	$ma = x + ee$
$3 - x$	4	$ee = ma - x$
$2 \div da$	5	$ee = \frac{b}{da}$
$4 \cdot 5$	6	$ma - x = \frac{b}{da}$
$6 \times da$	7	$dmaa - dxa = b$

Dividing by dm the Co-efficient of aa, by Art. 58.

$7 \div dm$	8	$aa - \frac{xa}{m} = \frac{b}{dm}$ for $\frac{dxa}{dm} = \frac{xa}{m}$ the d being rejected as in Diviſion.

Now $\frac{x}{m}$ the Co-efficient of a being divided by 2, is $\frac{x}{2m}$. For making 2 an improper Fraction by the Rule in common Arithmetic, is $\frac{2}{1}$: But by the Rule for Diviſion of Vulgar Fractions in Arithmetic $\frac{x}{m} \div \frac{2}{1} = \frac{x}{2m}$. Now ſquaring $\frac{x}{2m}$ it is $\frac{xx}{4mm}$, and adding this to both Sides of the Equation the *Square is compleated*, by Art. 56.

8 c □	9	$aa - \frac{xa}{m} + \frac{xx}{4mm} = \frac{b}{dm} + \frac{xx}{4mm}$
9 ɯ 2	10	$a - \frac{x}{2m} = \sqrt{\frac{b}{dm} + \frac{xx}{4mm}}$
$10 + \frac{x}{2m}$	11	$a = \sqrt{\frac{b}{dm} + \frac{xx}{4mm}} : + \frac{x}{2m} = 100$ (as before.

Then by the fourth or fifth Step we ſhall find $e = 20$, as before.

Queſtion 76. *It is required to find two ſuch Numbers, that the greater being added to the Square of the leſſer, the Sum may be* 19: *But if the greater is multiplied into the Square of the leſſer, the Product may be* 90.

Let $a =$ the greater Number, $e =$ the leſſer Number, $s = 19$, $p = 90$.

	1	$a+ee=s$ } By the Queſtion.
	2	$aee=p$
$1 - ee$	3	$a=s-ee$
$2 \div ee$	4	$a=\frac{p}{ee}$
$3 \cdot 4$	5	$\frac{p}{ee}=s-ee$
$5 \times ee$	6	$p=see-eeee$
		Tranſpoſe $eeee$ to make it affirmative,
$6+eeee$	7	$eeee+p=see$
$7-see$	8	$eeee-see+p=0$
$8-p$	9	$eeee-see=-p$

Here the Equation not only appears to be *quadratic*; the Powers of the unknown Quantity e, being the ſame as in the two laſt Queſtions, but the Equation is likewiſe *ambiguous*, for that Side of the Equation which is known is *negative*, *viz.* $-p$.

$9 \; c \; \square$	10	$eeee-see+\frac{ss}{4}=\frac{ss}{4}-p$
10 ω 2	11	$ee-\frac{s}{2}=\sqrt{\frac{ss}{4}-p}$
$11+\frac{s}{2}$	12	$ee=\frac{s}{2}\pm\sqrt{\frac{ss}{4}-p}$, the Equation (being *ambiguous* as above.
12 ω 2	13	$e=\sqrt{\frac{s}{2}\pm\sqrt{\frac{ss}{4}-p}}$

That is, by reaſon of the *ambiguity* of *the Equation*, it may be

$e=\sqrt{\frac{s}{2}+\sqrt{\frac{ss}{4}-p}}$, or $e=\sqrt{\frac{s}{2}-\sqrt{\frac{ss}{4}-p}}$

Let us ſuppoſe $e = \sqrt{\frac{s}{2} + \sqrt{\frac{ss}{4} - p}}$

$$90.25 = \frac{ss}{4}$$
$$-90. = -p$$
$$\sqrt{.25} = .5 = \sqrt{\frac{ss}{4} - p}$$
$$9.5 = \frac{s}{2}$$
$$10. = \frac{s}{2} + \sqrt{\frac{ss}{4} - p}$$

$10\ (3.162$ neareſt $= \sqrt{\frac{s}{2} + \sqrt{\frac{ss}{4} - p}}$, the Value of e.

```
     9
61) 100
     61
626) 3900
     3756
6322) 14400
      12644
```

Then by the third Step $a = s - ee = 9$, if we take 10 for the Square of e, the ſquare Root of 10 being equal to e.

By trying theſe Numbers according to the Conditions of the Queſtion, we have

$a + ee = 19$
$a.ee = 90$ taking 10 for the Square of e as above.

But becauſe the Value of e is a Fraction which does not terminate, and therefore its exact Value cannot be found, let us try the other Root, *viz*, $e = \sqrt{\frac{s}{2} - \sqrt{\frac{ss}{4} - p}}$.

$$90.25 = \frac{ss}{4}$$
$$-90. = -p$$
$$\sqrt{.25} = .5 = \sqrt{\frac{ss}{4} - p}$$

$$\frac{s}{2} = 9.5$$

$$-\sqrt{\frac{ss}{4} - p} = -\ .5$$

$\frac{s}{2} - \sqrt{\frac{ss}{4} - p} = 9.$ extracting the ſquare Root of 9. we have

$\sqrt{\frac{s}{2} - \sqrt{\frac{ss}{4} - p}} = 3$, for the other Value of e exact.

Then by the third Step $a = s - ee = 10$.

And trying theſe two Numbers by the Conditions of the Queſtion, we have

$a + ee = 19$ } As the Queſtion requires, whence the two
$aee \ = 90$ } Numbers are 10 and 3.

I have been particular in the Arithmetical Work of this Queſtion, that the Learner may ſee the Method of finding both the Values of the unknown Quantity, in any *ambiguous* quadratic Equation, when the unknown Quantity is to the fourth Power.

But in this Queſtion, if we exterminate e inſtead of a, we ſhall have a more ſimple Solution.

	1	$a + ee = s$	By the Queſtion as before.
	2	$aee = p$	
$1 - a$	3	$ee = s - a$	
$2 \div a$	4	$ee = \frac{p}{a}$	
$4 \cdot 3$	5	$\frac{p}{a} = s - a$	
$5 \times a$	6	$p = sa - aa$	
$6 + aa$	7	$aa + p = sa$	
$7 - p$	8	$aa = sa - p$	
$8 - sa$	9	$aa - sa = -p$	

Here the Equation appears *quadratic* and *ambiguous*, as before.

9 c □	10	$aa - a + \frac{ss}{4} = \frac{ss}{4} - p$

$10 \,\omega\, 2$	11	$a - \frac{s}{2} = \sqrt{\frac{ss}{4} - p}$
$11 + \frac{s}{2}$	12	$a = \frac{s}{2} \pm \sqrt{\frac{ss}{4} - p}$

Let us firſt ſuppoſe the Root to be $\frac{s}{2} - \sqrt{\frac{ss}{4} - p}$

$$4)\ \underline{361 = ss}$$

$$90.25 = \frac{ss}{4}$$

$$\underline{-p = -90.}$$

$.25 = \frac{ss}{4} - p$, but the ſquare Root of .25 is .5 whence $.5 = \sqrt{\frac{ss}{4} - p}$.

Then $\frac{s}{2} = 9.5$

$$\underline{-\sqrt{\frac{ss}{4} - p} = -0.5}$$

$9. = a$, for one of the Roots of the ambiguous Equation, and from this Root, or Value of a, we ſhall find either from the third, or fourth Step, that e is equal to the ſquare Root of 10 as before; but this being a ſurd Number, and ſo its Root cannot be exactly extracted, find the other Root, or Value of a, then we have $a = \frac{s}{2} + \sqrt{\frac{ss}{4} - p}$.

$$\frac{s}{2} = 9.5$$

$$+\sqrt{\frac{ss}{4} - p} = .5$$

$10. = a$, the other Root of the ambiguous Equation; then by the third, or fourth Step, we ſhall find e to be equal to the ſquare Root of 9, which is 3; and theſe two Numbers 10, and 3, anſwers the Conditions of the Queſtions.

It may not be improper in this Place to add, that if the Learner meets with any Queſtions, where the Anſwers come out in Decimal Fractions, he is not from thence to conclude they are not the true Anſwers, or rather ſuch as the Queſtion will only admit, as theſe are very frequent and common: But if the Equation is *ambiguous*, it would not be improper to find the other Root, which may perhaps be free from Fractions; and if this Root anſwers the Conditions of the Queſtion, he has then found the Anſwer compleat: But if the Queſtion will not admit of ſuch an Anſwer, he can then only *approach* to the true Anſwer in continuing his Fractions at Pleaſure; but hitherto I have endeavoured to avoid theſe Circumſtances, as they only fatigue the Learner and perplex his Mind, inſtead of increaſing his Judgment, and advancing his Knowledge in this Science.

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66. The Method of reſolving Queſtions, that contain three Equations, and three unknown Quantities.

FIND the Value of one of the unknown Quantities, in one of the given Equations:

For the ſame unknown Quantity in the other two Equations, write, or put this Value, which exterminates *that unknown Quantity, from thoſe two Equations; and reduce the Queſtion to two Equations, and two unknown Quantities, which may be reſolved as the foregoing Queſtions,* by Art. 55. that is,

Find the Value of one of theſe two unknown Quantities, in theſe two Equations, and making theſe two Equations equal to one another, *exterminates* another unknown Quantity, for this laſt Equation will have only one unknown Quantity, which being reduced by the Directions already given, will give the Value of that unknown Quantity in Numbers, from which it will be eaſy to determine the other two.

To help the Learner in his Choice which to exterminate, if one of the three unknown Quantities is not multiplied, or divided by either of the other two, but theſe are multiplied, or divided by one another, then it will be eaſieſt to find the Value of that unknown Quantity, which is not multiplied, or divided by the others.

Or if one of the unknown Quantities, ſhould be to the firſt Power only in all the three given Equations, and the other two are raiſed to ſome higher Power, then it may be eaſieſt to exterminate the unknown Quantity, which is to the firſt Power only.

And if all the three unknown Quantities, are only to the firſt Power, and none of them are multiplied or divided by one another, then if one of them has no Co-efficient but *Unity*, and the other two have Co-efficients, it may be eaſieſt to exterminate that unknown Quantity, whoſe Co-efficient is *Unity.*

Theſe Directions may be of Uſe to the Learner, in aſſiſting his Choice which unknown Quantity to exterminate, and a little Care and Attention will help his Judgment in this Part of the Science; I ſhall only juſt mention, that if any particular Difficulties ariſe from the exterminating one unknown Quantity, it may not be improper to make an Eſſay how the Work will proceed, from exterminating ſome other unknown Quantity.

Queſtion 77. *There are three Numbers whoſe Sum is* 18:

The firſt being added to three times the ſecond, from which Sum ſubſtracting twice the third, the Remainder is 9:

But if the firſt is added to fourtimes the third, from which Sum there is ſubſtracted twice the ſecond, the Remainder is 21. *What are the three Numbers?*

Let $a=$ the firſt Number, $e=$ the ſecond Number, $y=$ the third Number, $b=18$, $m=9$, $p=21$.

	1	$a+e+y=b$	By the Queſtion.
	2	$a+3e-2y=m$	
	3	$a+4y-2e=p$	
$1-y$	4	$a+e=b-y$	
$4-e$	5	$a=b-y-e$	

Having found the Value of a in the firſt Equation, in the Room of a in the ſecond and third Equations, put its Value $b-y-e$, thus,

2 . 5	6	$b-y-e+3e-2y=m$	Here the Queſtion is reduced to two Equations, and two unknown Quantities, for a is exterminated.
3 . 5	7	$b-y-e+4y-2e=p$	
6 contracted	8	$b-3y+2e=m$	
7 contracted	9	$b+3y-3e=p$	

Now find the Value of either y, or e, in each of theſe two laſt Equations, and $3y$ being in each Equation, find what that is equal to.

$8+3y$	10	$b+2e=m+3y$
$10-m$	11	$3y=b+2e-m$
$9+3e$	12	$b+3y=p+3e$
$12-b$	13	$3y=p+3e-b$
$11\ .\ 13$	14	$p+3e-b=b+2e-m$

Here we have an Equation with one unknown Quantity only, which is reduced in the common Manner, thus,

$14-2e$	15	$p+e-b=b-m$
$15+b$	16	$p+e=2b-m$
$16-p$	17	$e=2b-m-p=6$, then
$13\div\overline{3}$	18	$y=\dfrac{p+3e-b}{3}=7$, and
By the fifth Step	19	$a=b-y-e=5$.

Hence the three Numbers ſought are 5 . 6 . and 7.

PROOF.

$$a+e+y=18$$
$$a+3e-2y=9$$
$$a+4y-2e=21.$$

Queſtion 78. *Three Men,* A, B, C, *diſcourſing of their Shillings, found, that if twice* A's *Shillings was added to* B's *Shillings, and from that Sum ſubſtracting* C's *Shillings, there remains* 15:
And if B's *Shillings was added to three times* C's *Shillings, and from that Sum ſubſtracting* A's *Shillings, there remains* 31:
But if ſix times A's *Shillings was added to four times* C's *Shillings, and this Sum added to* B's *Shillings, the Sum was* 97. *How many Shillings had each Perſon?*

Let $a=$ the Number of Shillings of A, $e=$ thoſe of B, and $y=$ thoſe of C; $b=15$, $d=31$, $m=97$.

$$\left.\begin{array}{c|l} 1 & 2a+e-y=b \\ 2 & e+3y-a=d \\ 3 & 6a+4y+e=m \end{array}\right\}\text{By the Queſtion.}$$

Becauſe e has no Co-efficient but *Unity*, begin with finding the Value of e as being the moſt ſimple.

$1 + y$	4	$2a + e = b + y$
$4 - 2a$	5	$e = b + y - 2a$

Now in the ſecond and third Equations, in the room of e put its Value, or $b + y - 2a$, as in the laſt Queſtion,

2 . 5	6	$b + y - 2a + 3y - a = d$
3 . 5	7	$6a + 4y + b + y - 2a = m$

Here the Queſtion is reduced to two Equations, and two unknown Quantities, e being exterminated, and therefore proceeding as in the laſt Queſtion,

6 contracted	8	$b + 4y - 3a = d$
7 contracted	9	$4a + 5y + b = m$

Then find the Value of either of the unknown Quantities in both theſe Equations, to find the Value of y.

$8 + 3a$	10	$b + 4y = d + 3a$
$10 - b$	11	$4y = d + 3a - b$
$11 \div \overline{4}$	12	$y = \dfrac{d + 3a - b}{4}$
$9 - b$	13	$4a + 5y = m - b$
$13 - 4a$	14	$5y = m - b - 4a$
$14 \div \overline{5}$	15	$y = \dfrac{m - b - 4a}{5}$
12 . 15	16	$\dfrac{d + 3a - b}{4} = \dfrac{m - b - 4a}{5}$

Here we have an Equation with only the unknown Quantity a.

$16 \times \overline{4}$	17	$d + 3a - b = \dfrac{4m - 4b - 16a}{5}$
$17 \times \overline{5}$	18	$5d + 15a - 5b = 4m - 4b - 16a$
$18 + 16a$	19	$5d + 31a - 5b = 4m - 4b$
$19 + 5b$	20	$5d + 31a = 4m + b$
$20 - 5d$	21	$31a = 4m + b - 5d$
$21 \div \overline{31}$	22	$a = \dfrac{4m + b - 5d}{31} = 8$, then

By

By the 12th Step	23	$y = \frac{d + 3a - b}{4} = 10$, and
By the fifth Step	24	$e = b + y - 2a = 9$

PROOF.

$$2a + e - y = 15$$
$$e + 3y - a = 31$$
$$6a + 4y + e = 97$$

I have done theſe two Queſtions without putting Letters for the given Numbers, it being more eaſy and familiar; but now to do the laſt univerſally, let us put Letters for the Numbers 2 . 3 . 6 and 4 which are given in the Queſtion, and comparing the two Operations that may render this the more eaſy; but if the Learner finds this too perplexing he may neglect it, and proceed to the next.

Let a . e and y be the three unknown Numbers as before, and $x = 2$, $z = 3$, $b = 6$, $p = 4$, then,

1	$xa + e - y = b$	By the Queſtion.
2	$e + zy - a = d$	
3	$ba + py + e = m$	

Becauſe e has no *ſpecious* Co-efficient in either of the given Equations, find the Value of e,

$1 + y$	4	$xa + e = b + y$
$4 - xa$	5	$e = b + y - xa$

Now in the ſecond and third Equations, in the room of e put its Value, or $b + y - xa$,

2 . 5	6	$b + y - xa + zy - a = d$
3 . 5	7	$ba + py + b + y - xa = m$

Here the Queſtion is reduced to two Equations, and two unknown Quantities, e being exterminated, but becauſe of the *ſpecious* Co-efficients we cannot contract them as before: Now find the Value of y, in both theſe Equations.

$6 + a$

$6+a$	8	$b+y-xa+zy=d+a$
$8+xa$	9	$b+y+zy=d+a+xa$
$9-b$	10	$zy+y=d+a+xa-b$
$10\div z+1$	11	$y=\frac{d+a+xa-b}{z+1}$ the Co-efficients of y being $z+1$
$7+xa$	12	$ba+py+b+y=m+xa$
$12-b$	13	$ba+py+y=m+xa-b$
$13-ba$	14	$py+y=m+xa-b-ba$
$14\div p+1$	15	$y=\frac{m+xa-b-ba}{p+1}$ the Co-efficients of y being $p+1$
$11\ .\ 15$	16	$\frac{d+a+xa-b}{z+1}=\frac{m+xa-b-ba}{p+1}$ an Equation with only the unknown Quantity a,
$16\times z+1$	17	$d+a+xa-b=\frac{zm+zxa-zb-zba+m+xa-b-ba}{p+1}$
$17\times p+1$	18	$pd+pa+pxa-pb+d+a+xa-b=zm+zxa-zb-zba+m+xa-b-ba$

The Learner may think theſe Multiplications diſcouraging, though perhaps they are not ſo perplexing as he may imagine, for at the ſeventeenth Step where $m+xa-b-ba$ is $\times z+1$, put down the Product of it by z firſt, which is $zm+zxa-zb-zba$, after which he need only write $m+xa-b-ba$, the next part of the Multiplier being *Unity*; or, if it had been another Letter, it had been no more then repeating the *Mitiplicand*, with the *multiplying Letter* joined to each of its Quantities, placing them one after another, taking due Care of the Signs by the Rules for Multiplication.

In the ſame Manner he will find the eighteenth Step multiplied, and a little Attention will familiarize the Operation; but if there is any Difficulty in multiplying theſe compound Quantities, the Learner may ſet them down one under the other, and multiply them in the uſual Manner.

$18-xa$	19	$pd+pa+pxa-pb+d+a-b=zm+zxa-zb-zba+m-b-ba$
$19+b$	20	$pd+pa+pxa-pb+d+a=zm+zxa-zb-zba+m-ba$

Now tranſpoſe all the unknown Quantities, to one Side of the Equation, and all the known ones to the other Side of the Equation.

$20 - zxa$	21	$pd + pa + pxa - pb + d + a - zxa = zm - zb - zba + m - ba$
$21 + zba$	22	$pd + pa + pxa - pb + d + a + zba - zxa = zm - zb + m - ba$
$22 + ba$	23	$pd + pa + pxa - pb + d + a + zba - zxa + ba = zm - zb + m$
$23 - pd$	24	$pa + pxa - pb + d + a + zba - zxa + ba = zm - zb + m - pd$
$24 + pb$	25	$pa + pxa + d + a + zba - zxa + ba = zm - zb + m - pd + pb$
$25 - d$	26	$pa + pxa + a + zba - zxa + ba = zm - zb + m - pd + pb - d$
$26 \div$	27	$a = \frac{zm - zb + m - pd + pb - d}{p + px + 1 + zb - zx + b} = 8,$ the Diviſor is the *Co-efficients* of *a*, connected by their Signs,
Then by 11th Step	28	$y = \frac{d + a + xa - b}{z + 1} = 10,$
And by 5th Step	29	$e = b + y - xa = 9$

Queſtion 78. *There are three Travellers,* A, B, C, *who have travelled in all* 62 *Miles:*

But if the Diſtance A *has travelled is multiplied by* 2, *and added to the Diſtance* B *has travelled multiplied by* 3, *this Sum is equal to the Diſtance* C *has travelled when multiplied by* 17:

And if 4 *times the Diſtance* C *has travelled, is added to the Diſtance* B *has travelled multiplied by* 2, *this Sum is equal to the Diſtance travelled by* A.

Let $a =$ the Diſtance travelled by A, $e =$ the Diſtance travelled by B, $y =$ the Diſtance travelled by C; $p = 62$, $b = 2$, $d = 3$, $m = 17$, $x = 4$ and 2 being in the Queſtion before, put no new Letter for it.

$$\left.\begin{array}{l|l} 1 & a + e + y = p \\ 2 & ba + de = my \\ 3 & xy + be = a \end{array}\right\} \text{By the Queſtion.}$$

Becauſe *a* ſeems to be in as ſimple Terms as any in the three given Equations, and having it's Value already by the third Equation, therefore for *a* in the firſt and ſecond Equation write its Value $xy + be$ at the third Equation, which exterminates *a*.

1 . 3	4	$xy+be+e+y=p$
2 . 3	5	$bxy+bbe+de=my$

Here the Queſtion is reduced to two Equations, and two unknown Quantities, then proceed as before, to find the Value of either y or e, in each of theſe Equations, as ſuppoſe y.

$4-be$	6	$xy+e+y=p-be$
$6-e$	7	$xy+y=p-be-e$
$7\div x+1$	8	$y=\frac{p-be-e}{x+1}$ the Co-efficients of y being $x+1$.
$5-bxy$	9	$my-bxy=bbe+de$
$9\div m-bx$	10	$y=\frac{bbe+de}{m-bx}$ the Co-efficients of y being $m-bx$.
8 . 10	11	$\frac{bbe+de}{m-bx}=\frac{p-be-e}{x+1}$ an Equation with only (one unknown Quantity.
$11\times m-bx$	12	$bbe+de=\frac{mp-mbe-me-pbx+bbxe+bxe}{x+1}$
$12\times x+1$	13	$xbbe+xde+bbe+de=mp-mbe-me$ $-pbx+bbxe+bxe$
$13-xbbe$	14	$xde+bbe+de=mp-mbe-me-pbx+bxe$ Now bring the Terms that have the unknown Quantity, to one Side of the Equation.
$14+mbe$	15	$xde+bbe+de+mbe=mp-me-pbx+bxe$
$15+me$	16	$xde+bbe+de+mbe+me=mp-pbx+bxe$
$16-bxe$	17	$xde+bbe+de+mbe+me-bxe=mp-pbx$
$17\div$	18	$e=\frac{mp-pbx}{xd+bb+d+mb+m-bx}=9$, the Diviſor is the Co-efficients of e, connected by their Signs.
By Step 8, or 10.	19	$y=7$
By Step 3.	20	$a=46$

Queſtion 79. *Three Men*, A, B, C, *diſcourſing of their Shillings, found, that* A's *Shillings added to* C's *Shillings, the Sum was double* B's *Shillings:*

And A's *Shillings added to three times* B's *Shillings, from which Sum ſubſtracting* C's *Shillings, there remained* 13 *Shillings:*

But if A's *Shillings was added to the Product of* B's *and* C's *Shillings, the Sum was* 34. *How many Shillings had each Perſon?*

Let a = A's Shillings, e = B's Shillings, y = C's Shillings, $b = 13$, $d = 34$.

	1	$a+y=2e$	By the Queſtion.
	2	$a+3e-y=b$	
	3	$a+ey=d$	
$1-y$	4	$a=2e-y$	
$2 \cdot 4$	5	$2e-y+3e-y=b$	Here the Queſtion is reduced to two Equations, and two unknown Quantities.
$2 \cdot 3$	6	$2e-y+ey=d$	
5 contracted	7	$5e-2y=b$	

Now find the Value of e, or y, in the ſixth and ſeventh Equations.

$6+y$	8	$2e+ey=d+y$
$8 \div \overline{2+y}$	9	$e=\frac{d+y}{2+y}$
$7+2y$	10	$5e=b+2y$
$10 \div \overline{5}$	11	$e=\frac{b+2y}{5}$
$9 \cdot 11$	12	$\frac{b+2y}{5}=\frac{d+y}{2+y}$, an Equation with only one unknown Quantity.
$12 \times \overline{2+y}$	13	$\frac{2b+4y+by+2yy}{5}=d+y$
$13 \times \overline{5}$	14	$2b+4y+by+2yy=5d+5y$

Now bring all the Quantities that have y, to one Side of the Equation.

$14-5y$	15	$2b-y+by+2yy=5d$
$15-2b$	16	$2yy+by-y=5d-2b$

Here the Equation appears *quadratic*, the unknown Quantity being to the ſecond and firſt Power only, then by Art. 58, divide by the Co-efficient of yy.

$16 \div \overline{2}$	17	$yy+\frac{by-y}{2}=\frac{5d-2b}{2}$

The Work being now prepared for *compleating the Square*, becauſe the Co-efficient of y is $\frac{b-1}{2}$, to avoid the Trouble of dividing this Fraction by 2, and ſquaring the Quotient, ſubſtitute $x=\frac{b-1}{2}$, by Art. 57.

Then	18	$yy+xy=\frac{5d-2b}{2}$
18 *c* □	19	$yy+xy+\frac{xx}{4}=\frac{5d-2b}{2}+\frac{xx}{4}$
19 ω 2	20	$y+\frac{x}{2}=\sqrt{\frac{5d-2b}{2}+\frac{xx}{4}}$
$20-\frac{x}{2}$	21	$y=\sqrt{\frac{5d-2b}{2}+\frac{xx}{4}}:-\frac{x}{2}=6$, (C's Shillings.
By the 9th, or 11th Steps	22	$e=5$, B's Shillings.
By the 4th Step	23	$a=4$, A's Shillings.

Queſtion 80. *Three young Gentlemen,* A, B, C, *having been at the Gaming-Tables, from comparing their Loſſes, found that twice what* A *loſt, when diminiſhed by what* B *loſt, was equal to the loſs of* C:

And that the Loſs of A, *added to the Loſs of* B, *and this Sum added to twice the Loſs of* C, *the Sum was* 19 *Pounds:*

But if to the Product of A's *and* C's *Loſs, there is added* B's *Loſs, the Sum is* 26 *Pounds. How much did each Perſon loſe?*

Let $a=$ A's Loſs, $e=$ B's Loſs, $y=$ C's Loſs, $d=19$, $b=26$.

1	$2a-e=y$	By the Queſtion.
2	$a+e+2y=d$	
3	$ay+e=b$	

Becauſe e ſeems to be in the moſt ſimple Terms, therefore find the Value of e.

$1+e$	4	$2a=y+e$
$4-y$	5	$e=2a-y$

2 · 3

2 . 5	6	$a+2a-y+2y=d$
3 . 5	7	$ay+2a-y=b$
6 contracted	8	$3a+y=d$

The Queſtion is here reduced to two Equations and two unknown Quantities, for *e* is exterminated.

Find the Value of *a*, or *y*, in the ſeventh and eighth Equations.

$7-2a$	9	$ay-y=b-2a$
$9\div a-1$	10	$y=\dfrac{b-2a}{a-1}$
$8-3a$	11	$y=d-3a$
10 . 11	12	$\dfrac{b-2a}{a-1}=d-3a$
$12\times a-1$	13	$b-2a=da-3aa-d+3a$

Now bring all the Quantities that have *a* on one Side of the Equation, obſerving to have the higheſt Power of *a* *affirmative*.

$13+3aa$	14	$3aa+b-2a=da-d+3a$
$14-3a$	15	$3aa+b-5a=da-d$
$15-da$	16	$3aa+b-5a-da=-d$
$16-b$	17	$3aa-5a-da=-d-b$

Here the Equation appears both *quadratic* and *ambiguous*, for the unknown Quantity is to the ſecond and firſt Power only, and it is *ambiguous*, becauſe $-d-b$ the Side of the Equation which is known, is *negative*; dividing by the Co-efficient of *a a*, as in the laſt Queſtion.

$17\div 3$	18	$aa-\dfrac{5a-da}{3}=\dfrac{-d-b}{3}$

The Work being now prepared for *compleating the Square*, ſubſtitute $-x=\dfrac{-5-d}{3}$ the Co-efficients of *a*, as in the laſt Example,

Then	19	$aa-xa=\dfrac{-d-b}{3}$
18 c □	20	$aa-xa+\dfrac{xx}{4}=\dfrac{-d-b}{3}+\dfrac{xx}{4}$

$19 \text{ w } 2$	21	$a - \frac{x}{2} = \pm\sqrt{\frac{-d-b}{3} + \frac{xx}{4}}$
$20 + \frac{x}{2}$	22	$a = \frac{x}{2} \pm \sqrt{\frac{-d-b}{3} + \frac{xx}{4}} = 4,$ $(\pm 1 = 3,$ or $5.$

For the Practice of the Learner, let us ſuppoſe $a = 3$
Then by the tenth, or eleventh Steps - $y = 10$
And by the fifth Step - - - - - $e = 6 - 10$ $= -4$, which is an Impoſſibility that e, an *affirmative* Quantity, can be equal to a *negative* 4.

Now let us ſuppoſe - - - - - $a = 5$
Then by the tenth, or eleventh Steps - $y = 4$
And by the fifth Step - - - - - $e = 6$

Then $2a - e = y$
$a + e + 2y = 19$
$ay + e = 26$

And as theſe three Numbers anſwers the Conditions of the Queſtion, they are the true Numbers ſought; from hence the young *Analyſt* may obſerve, that in the *quadratic ambiguous* Equation, if one of the Roots of the unknown Quantity does not anſwer the Conditions of the Queſtion, he ſhould find the other Root, and try that, before he concludes his Work is erroneous.

I ſhall now proceed to ſhow the Learner, the excellent Method of reſolving all Equations, be their Powers never ſo high, by the univerſal Method of *Converging Series.*

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67. The Reſolution of Affected Equations, by the univerſal Method of Converging Series.

CASE I.

Ex. I. SUPPOSE there was given $aaa + a = 9282$, to find a.

Then ſuppoſe, or imagine a to be - - - 20
Conſequently the Cube of a, or aaa is - - 8000

Theſe being added together, becauſe it is $aaa + a$ in the given Equation, the Sum is - - } 8020

Hence a is more then 20, for if that had been the true Root, then the Cube of 20 added to the firſt Power, or 20, this Sum muſt have been equal to 9282 the given Number, for theſe are the ſame Powers of a as in the given Equation; but that Sum being only 8020, which being leſs then 9282, the Value of a muſt be more than 20.

Now let $r = 20$, and for what 20 wants of the true Number or Root, put e:

Then will $r + e = a$, or the true Root of the Equation, hence by determining what e is, we find the Number that is to be added to r or 20, which Sum will be the Root of the given adfected Equation, to do which, put down,

| 1 | $r + e = a$

Now raiſe this Equation to the third Power, becauſe we have $a\,a\,a$ in the given Equation.

1 ⊛ 3 | 2 | $rrr + 3rre + 3ree + eee = aaa$

Then add theſe two Equations ~~together, because~~ in the given Equation it is $aaa + a$.

1 + 2	3	$rrr + 3rre + 3ree + eee + r + e = aaa + a$
		But from the given Equation
	4	$aaa + a = 9282$
3 · 4	5	$rrr + 3rre + 3ree + eee + r + e = 9282$
		Putting this Equation into Numbers, and rejecting the Powers of e above ee.
5 in Numbers	6	$8000 + 1200e + 60ee + 20 + e = 9282$
That is	7	$8020 + 1201e + 60ee = 9282$
7 — 8020	8	$1201e + 60ee = 1262$
		Divide by the Co-efficient of ee, and we have,
7 ÷ $\overline{60}$	9	$20.016e + ee + = 21.0333$ *ad infinitum.*
		Dividing by $20.016 + e$, that is, by the Co-efficient of e, *plus* e and we have,
10 ÷ 20.016 + e	10	$e = \dfrac{21.03333}{20.016 + e}$

In Numbers, thus.

$20.016)\ 21.0333\ (1 = e$

1

——— 21016

21.016 ———

17 the Remainder being very ſmall reject it,

The Reaſon of adding the Quotient Figure, or 1; to the Diviſor, ſee the next Article or Article 68.

By this it appears that $e = 1$, that is, 1 is to be added to the firſt ſuppoſed Number 20, which Sum is to be the Value of a, or the Root of the given adfected Equation.

$$\begin{array}{rl} \text{We aſſumed } r = & 20 \\ \text{And found } e = & 1 \\ \hline r + e = & 21 = a \end{array}$$

To try whether 21 is the true Root, raiſe it to the ſeveral Powers of a in the given Equation.

$$\begin{array}{r} a = 21 \\ a = 21 \\ \hline 21 \\ 42 \\ \hline aa = 441 \\ a = 21 \\ \hline 441 \\ 882 \\ \hline aaa = 9261 \\ a = 21 \\ \hline \end{array}$$

Then $aaa + a = 9282$, which being the ſame with the Number in the given Equation, it appears that $a = 21$.

68. By reviewing the Operation, the Learner may obſerve,

Firſt, That we ſuppoſed a Number for the true Root, which upon Trial was found leſs than the true Root.

Secondly, For that Deficiency or Want we put e, or any other Letter.

Thirdly, By putting $r =$ the Number firſt ſuppoſed to be the Root, and connecting e by the Sign $+$, we have $r+e$ for the true Root, r being a known Quantity, and e the unknown Quantity.

Fourthly, We raiſe $r+e$ to the ſeveral Powers of the unknown Quantity, that are in the given adfected Equation.

Fifthly, Then we *add* theſe ſeveral Equations together, rejecting all the Powers of e, or of the unknown Quantity above the *Square*, for in the Example all the Powers of the unknown Quantity have the Sign $+$, but when any of theſe have the Sign $-$, then their reſpective Equations muſt be ſubſtracted as at Step 5, Example 3, Page 238.

Sixthly, By which means we have an Equation in the Terms of r and e, equal to the given Equation.

Seventhly, Then this Equation is put in Numbers, r being a known Quantity, and the leſs abſolute Number is tranſpoſed to the Side of the Equation of the greater abſolute Number, and ſubſtracted from it.

Eighthly, After this the Equation is divided by the Co-efficient of the Square of e, or the unknown Quantity.

Ninthly, This laſt Equation is divided by the Co-efficient of e, *Plus* e, which leaves e on one Side of the Equation by itſelf.

Tenthly, In' the Arithmetical Work, becauſe the laſt Diviſor conſiſts of a Number *Plus* e, therefore, as the Quotient Figure is found, it is added to the Diviſor to make it compleat, which Figure is twice added, once before the Diviſion is made at that Figure, and once afterwards.

Eleventhly, The Quotient that is thus found being the Value of e, or the unknown Quantity, it is added to the Number firſt ſuppoſed to be the Root of the Equation, which is repreſented by r, and this Sum is the Root required.

This Operation to find e is the ſame as the common Method of finding the unknown Quantity, till we come to the tenth Step, where the unknown Quantity, making part of the Diviſor, it is carried to the other Side of the Equation, and the Diviſor being a known Number $+ e$, the Quotient as it is found is added to the Diviſor, to make it compleat as before mentioned.

But if this Number ſhould not be the true Root, the Operation muſt be repeated, making the Number thus found $= r$, and at the ſecond Operation the Work, in any common Caſe, will be ſufficiently exact: And from the repetition of the Operation, whereby we *approach* nearer and nearer to the true Root, this Method is called the Method of *Converging Series*, or of *Approximation.*

Example 2. Suppoſe $aaa + aa + a = 42997850$, to find a.

I imagine a to be - - - - - - -	300
Then the Cube of a, or aaa is - -	27000000
And the Square of a, or aa is - - -	90000
Theſe being added, becauſe it is $aaa + aa + a$ in the given Equation, the Sum is - -	27090300

But 42997850, the given Equation, is greater than 27090300, therefore the Root muſt be more than 300.

Now let $r=300$, and $e=$ what 300 wants of the true Root.

Then	1	$r+e=a$
1 ⊙ 3	2	$rrr+3rre+3ree+eee=aaa$ According to Particular 4, Art. 68.
1 ⊙ 2	3	$rr+2re+ee=aa$
1 + 2 + 3	4	$r+e+rrr+3rre+3ree+rr+2re+ee=aaa+aa+a$, by Particular 5 and 6, Art. 68.
But	5	$aaa+aa+a=42997850$, from the given Equation,
4 · 5	6	$r+e+rrr+3rre+3ree+rr+2re+ee=42997850$
6 in Numbers	7	$300+e+27000000+270000e+900e+90000+600e+ee=42997850$
That is	8	$27090300+270601e+901ee=42997850$
8 − 27090300	9	$270601e+901ee=15907550$
9 ÷ 901	10	$300.334e+ee=17655.43$, from Particular 8, Art. 68.
10 ÷ 300.334 + e	11	$e=\frac{17655.43}{300.334+e}$, from Particular 9, (Art. 68.

300.334)	17655.43 (50.34 $=e$
5	
Diviſor 350.334	1751670
50.3	
	1387300
Diviſor 400.664	1201982
.34	
	1853980
Diviſor 401.004	1604016
	249964

$(50.34=e$, the firſt Figure being in the Place of *Tens*, Place the 5 under the Place of Tens in the Diviſor, and this the Reader is to obſerve; to place the Quotient Figures he adds to the Diviſor, under thoſe of the ſame Denomination.

$r=300$
$e=\ 50.34$

$r+e=350.34=a$, hence I ſuppoſe the Root of the given Equation is 350.34 but to try it, raiſe 350.34 to the ſeveral Powers of a in the given Equation.

$a=$

$$\begin{array}{rr}
a = & 350.34 \\
a = & 350.34 \\
\hline
 & 140136 \\
 & 105102 \\
 & 1751700 \\
 & 105102 \\
\hline
aa = & 122738.1156 \\
a = & 350.34 \\
\hline
 & 4909524624 \\
 & 3682143468 \\
 & 61369057800 \\
 & 3682143468 \\
\hline
aaa = & 43000071.419304 \\
aa = & 122738.1156 \\
a = & 350.34 \\
\hline
\end{array}$$

Then $aaa + aa + a = 43123159.874904$, which being greater then 42997850 the Root cannot be ſo much as 350.34 and this leads us to explain the Method when the Number aſſumed is *greater* than the true Root, or,

CASE 2.

Let us take the laſt *Example*, *viz.* $aaa + aa + a = 42997850$. And ſuppoſe the Root to be 350.34 which we know is too much by the laſt Operation:

That now put $e =$ the Number to be ſubſtracted from 350.34 ſuppoſing $r = 350.34$ and to r connecting e by the Sign —, we have,

	1	$r - e = a$, or the true Root	By Particular 4. Art. 68.
1 & 3	2	$rrr - 3rre + 3ree - eee = aaa$	
1 & 2	3	$rr - 2re + ee = aa$	

Now collect theſe three Equations by Art. 68, Particular 5 and 6, and rejecting eee we have,

1 + 2 + 3	4	$r - e + rrr - 3rre + 3ree + rr - 2re + ee = aaa + aa + a$
But	5	$aaa + aa + a = 42997850$, from the given Equation.
4 . 5	6	$r - e + rrr - 3rre + 3ree + rr - 2re + ee = 42997850$

6 In Numbers	7	$350.34 - e + 43000071.42 - 3682143468e + 1051.02ee + 122738.1156 - 700.68e + ee = 42997850.$
That is	8	$43123159.8765 - 368916.0268e + 1052.02ee = 42997850$
		Now tranſpoſe 42997850, it being leſs then 43123159.8765
8 —	9	$125309.8765 - 368916.0268e + 1052.02ee = 0$, for one Side of the Equation being ſubſtracted from the other Side, the Remainder muſt be *nothing*, as both Sides of the Equation are equal. Then tranſpoſe the ſeveral Quantities which contain *e* to the other Side of the Equation.
9 +	10	$125309.8765 + 1052.02ee = 368916.0268e$
10 —	11	$368916.0268e - 1052.02ee = 125309.8765$
		Here dividing by the Co-efficient of *ee* as before.
11 ÷	12	$350.673e - ee = 119.1135$
		But now divide by the Co-efficient of *e*, *minus e*.
12 ÷	13	$e = \frac{119.1135}{350.673 - e}$

In Numbers.

350.673) 119.1135 (.34
— .3

Diviſor 350.373 1051068
— .34 1400670

Diviſor 350.033 1400132

538

Having thus determined *e* to be .34 it muſt now be ſubſtracted from *r*, becauſe it was aſſumed $r - e =$ the true Root.

But it was ſuppoſed $r = 350.34$
$-e$, which we have found $= .34$

Hence $r - e = 350. = a$, the Root of the given adfected Equation, which is proved by raiſing 350. to the ſeveral Powers of a in the given Equation,

$$\begin{aligned} \text{Thus, } aaa &= 42875000 \\ aa &= 122500 \\ a &= 350 \end{aligned}$$

Conſequently $aaa + aa + a = 42997850$ which being the ſame Number as in the given Equation, it ſhows that a is exactly equal to 350.

In the above Operation, at the thirteenth Step the Learner may obſerve, that the Diviſor is $350.673 - e$, therefore here, as the Quotient Figure is found, we *ſubſtract* it from the Part of the Diviſor 350.656 to have the Diviſor compleat, which is likewiſe done twice, once before the Diviſion is made at that Figure, and once afterwards: but in the firſt *Caſe* when r is aſſumed too little, then the Quotient Figure is added as at Particular 10, Art. 68. the Sign then being contrary to what it is now.

The Learner may further obſerve, that by this ſecond Operation, we have found the *true Root*, whereas by the firſt Operation it was .34 too much, and therefore if the true Root does not come out at the firſt Operation, make a ſecond Operation, ſuppoſing the Number found at the firſt Operation to be r, and call it $r + e$, or $r - e$, for the true Root as the occaſion requires, that is, as the Number is either greater or leſſer than the true Root; which ſecond Operation will give the true Root very near, and near enough for any common Caſe, though if the Arithmetical Diviſions were continued as they would not terminate, they do not give the true Root exactly, it being like the Diviſion of ſome Decimal Fractions which never terminates, and as in theſe Diviſions we leave off when the Quotient is to a ſufficient Degree of Exactneſs, ſo the ſame is done here when we are near enough the Truth; and in common Caſes, two or three Places of Decimal Fractions are ſufficient, and according as they happen to be choſe the true Root is ſometimes found; though in general you may continue the Diviſion, at the *ſecond Operation*, to as many Places of Decimal Fractions as there are Fractions in the Number found in the *firſt Operation*, which Number in the *ſecond Operation* is put $= r$: And after the Number found at the ſecond Operation is added to, or ſubſtracted from, the Number found at the firſt Operation, for that is now $= r$, if there is a very ſmall Fraction you may reject it; but if the Fraction ſhould be near *Unity*, then take 1 for it, which add to the Integers, and try whether the whole Number thus found is not the true Root. In Arithmetical Queſtions where Anſwers are often in whole Numbers, this Caution may help the Learner to chooſe the true Root exactly.

The reaſon why this Method does not *abſolutely* give the true Root is the *arbitrary* rejecting all the Powers of e above ee.

Example 3. Admit $aaa - aa + a = 46526760$, to find a.

Now ſuppoſe $a = 400$.

Then aaa is - - - - - -	64000000
And aa is 160000, which muſt be *ſubſtracted* becauſe it is $-aa$ in the given Equation. -	160000
	63840000
To which adding a, or 400, it being $+a$ in the given Equation - - - - -	400
Hence $aaa - aa + a$ is - - -	63840400

Which exceeding 46526760 the Number in the given Equation, a muſt be leſs than 400.

Then let $r = 400$, $e =$ the Number that 400 is too much, which being the *ſecond Caſe*.

Hence	1	$r - e = a$
1 ⊛ 3	2	$rrr - 3rre + 3ree - eee = aaa$
1 ⊛ 2	3	$rr - 2re + ee = aa$

Becauſe in the given Equation the Quantities aaa and a are *affirmative*, therefore add the firſt and ſecond Equations together.

1 + 2	4	$r - e + rrr - 3rre + 3ree - eee = aaa + a$

Becauſe in the given Equation it is $-aa$, therefore *ſubſtract* the third Equation from the fourth, or Sum of the firſt and ſecond Equations. And here the Reader is to obſerve, that if in the given adfected Equation, any Powers of the unknown Quantity have the Sign $-$, the Equation which ariſes from involving $r + e$ to ſuch Powers, is to be *ſubſtracted* inſtead of being added.

4 − 3	5	$r - e + rrr - 3rre + 3ree - eee - rr$ $+ 2re - ee = aaa - aa + a$
But	6	$aaa - aa + a = 46526760$, by the given Equation,
5 . 6	7	$r - e + rrr - 3rre + 3ree - eee - rr$ $+ 2re - ee = 46526760$

Putting this Equation in Numbers, and rejecting all the Powers of *e* above *ee*.

7 in Numbers	8	$400 - e + 64000000 - 480000e + 1200ee - 160000 + 800e - ee = 46526760$
8 contracted	9	$63840400 + 1199ee - 479201e = 46526760$
		Tranſpoſe 46526760 it being leſs than 63840400
9 —	10	$17313640 + 1199ee - 479201e = 0$
		Then tranſpoſe the Quantities that have *e*, to the other Side of the Equation.
10 +	11	$479201e = 17313640 + 1199ee$
11 —	12	$479201e - 1199ee = 17313640$
		Dividing by the Co-efficient of *ee*.
12 ÷	13	$399.66e - ee = 14440.06$
		Now dividing by the Co-efficient of *e*, *minus e*.
13 ÷	14	$e = \frac{14440.06}{399.66 - e}$

In Numbers thus,

399.66) 14440.06 (40.16 = *e*,
— 40.

Diviſor 359.66 143864
— 40.1

53660
Diviſor 319.56 31956
— .16

217040
Diviſor 319.40 191640

25400

Now $r = 400$
$-e = 40.16$

$r - e = 359.84 = a$, and to try if this is the true Root, raiſe it to the ſeveral Powers of *a*, in the given Equation.

$a =$

$$\begin{array}{rr}
a = & 359.84 \\
a = & 359.84 \\
\hline
 & 143936 \\
 & 287872 \\
 & 323856 \\
 & 179920 \\
 & 107952 \\
\hline
aa = & 129484.8256 \\
a = & 359.84 \\
\hline
 & 5179393024 \\
 & 10358786048 \\
 & 11653634304 \\
 & 6474241280 \\
 & 3884544768 \\
\hline
aaa = & 46593819.643904 \\
-aa = & 129484.8256 \\
\hline
\text{Remains} & 46464334.818304 \\
+a = & 359.84 \\
\hline
\end{array}$$

Sum, or $aaa - aa + a = 46464694.658304$ which being leſs than 46526760 the Number in the given Equation, the Root or a muſt be more than 359.84.

Therefore, for a ſecond Operation, ſuppoſe $r = 359.84$ and $e =$ what it wants of the true Root, then it being $r + e = a$, it is now the *firſt Caſe*.

Therefore	1	$r + e = a$
1 ⊛ 3	2	$rrr + 3rre + 3ree + eee = aaa$
1 ⊛ 2	3	$rr + 2re + ee = aa$
		Add the firſt and ſecond Equation together, becauſe in the given Equation it is $a + aaa$.
1 + 2	4	$rrr + 3rre + 3ree + eee + r + e = aaa + a$
		From this Equation, ſubſtract the third Equation, becauſe it is $-aa$ in the given Equation.
4 — 3	5	$rrr + 3rre + 3ree + eee + r + e - rr - 2re - ee = aaa - aa + a$
But	6	$aaa - aa + a = 46526760$ by the given Equation,
5 . 6	7	$rrr + 3rre + 3ree + eee + r + e - rr - 2re - ee = 46526760$

Put

		Put this Equation in Numbers, and reject the Powers of e, above ee.
7 in Numbers	8	$46593819.644 + 388454.4768e + 1079.52ee + 359.84 + e - 129484.8256 - 719.68e - ee = 46526760$
8 contracted	9	$46464694.6584 + 387735.7968e + 1078.52ee = 46526760$
9 —	10	$387735.7968e + 1078.52ee = 62065.3416$
		Dividing by the Co-efficient of ee.
10 ÷	11	$359.5e + ee \doteq 57.547$
		Now dividing by the Co-efficient of e, *plus* e.
12 ÷	12	$e = \frac{57.547}{359.5+e}$

In Numbers thus.

$359.5)\ 57.547\ (.16 \doteq e.$
$+\ \ .1$

Diviſor 359.6 $\quad 3596$
$.16$
$\quad 215870$
Diviſor 359.76 $\quad 215856$
$\quad 14$

The Reader will obſerve that in this Diviſion, I have taken at once two Figures from the Dividend, *viz.* 70, becauſe in adding the .16 to the Diviſor, the Number of Places there is increaſed by one, therefore I take one Figure more from the Dividend than is uſual; which is recommended to the Reader's Attention, as he may again meet with the ſame Caſe.

Now $r = 359.84$
$+ e = \quad .16$

$r + e = 360.00 = a$, which will be found to be the true Root, by involving it to the ſeveral Powers of a in the given Equation, and adding or ſubſtracting them according as thoſe Powers of a are there connected by the Signs + or —.

It may not be improper to inform the Learner, that the nearer the Number is taken to the true Root, the nearer the Operation will come to the Truth, and therefore after he has tried the *firſt Suppoſition*, if he thinks he can make a ſecond Suppoſition nearer the Truth, it may be convenient to do it, which perhaps may bring out the Root ſo near at the firſt, that it may ſave him the Trouble of making a ſecond Operation. Thus,

If $aaa+aa+a=4942070$, to find a.

Suppoſe a to be 100.

Then the Cube of a, or aaa is - -	1000000
And the Square of a, or aa is - - -	10000
And a is - - - - - - - - -	100
	1010100
The Number in the given Equation is - -	4942070
If we ſuppoſe $a=100$, then the Sum of its ſeveral Powers are - - - -	1010100
Difference *wanting* - - - - -	3931970

Now let us make a ſecond Suppoſition, thus,
If $a=200$,

Then the Cube of a, or aaa is - -	8000000
And the Square of a, or aa is - - -	40000
And a is - - - - - - - -	200
Sum of the ſeveral Powers of a, if a is 200 -	8040200
The Number in the given Equation - -	4942070
Difference *over* - - - - - -	3098130

And as the Difference on the *ſecond* Suppoſition is not ſo much as the Difference on the *firſt* Suppoſition, I conclude that 200 is nearer the true Root of the given Equation than 100, therefore it is beſt to proceed upon that Suppoſition, and as the Sum of the Powers of a when a is 200, is greater than the Number in the given Equation, hence 200 is greater than the true Root, therefore proceed as at *Caſe* 2, Page 235; a little Attention will aſſiſt the Learner in making theſe Suppoſitions.

Having explained the Method of reſolving adfected Equations, we proceed to ſuch Queſtions as produce theſe Equations.

The Manner of ſolving Equations, when the unknown Quantity has ſeveral Powers in one Equation, and only to the firſt Power in another Equation.

69. WHEN the unknown Quantities are to the Square and firſt Power in one Equation, and but to the firſt Power in the other Equation; then find the Value of that unknown Quantity, in the Equation where its Terms are the more ſimple, raiſe this Equation, or Value of the unknown Quantity, to the ſeveral Powers the unknown Quantity has in the other Equation; then in that Equation for the ſeveral Powers of the unknown Quantity, write, or put theſe Values, which *exterminates* that unknown Quantity, leaving an Equation with only one unknown Quantity, which may be reſolved by ſome of the Methods already explained.

Queſtion 81. *There are two Numbers, that if the Square of the greater is divided by the leſſer, to this Quotient adding the greater, from which Sum ſubſtracting the Square of the leſſer, the Remainder is* 100:

And the Sum of the two Numbers is 50. *What are the Numbers ſought?*

Let $a =$ the greater, $e =$ the leſſer Number, $m = 100$, $p = 50$.

$$\left.\begin{array}{c|l} 1 & \frac{aa}{e} + a - ee = m \\ 2 & a + e = p \end{array}\right\} \text{By the Queſtion.}$$

In the firſt Equation both the unknown Quantities are to the firſt and ſecond Power, but in the ſecond Equation they are only to the firſt Power; therefore, according to the Directions, find the Value of a or e, in the ſecond Equation.

$$2 - e \mid 3 \mid a = p - e$$

Becauſe a is to the ſecond Power in the firſt Equation, raiſe the third Equation to the ſecond Power.

3 & 2	4	$aa = pp - 2pe + ee$

Now for aa and a in the firſt Equation, write their reſpective Values, or $pp - 2pe + ee$, and $p - e$, found by the third and fourth Equations, then we have,

1 . 3 . 4	5	$\frac{pp-2pe+ee}{e} + p - e - ee = m$, an Equation from which a is exterminated, and contains only the unknown Quantity e.
$5 \times e$	6	$pp - 2pe + ee + pe - ee - eee = me$
That is	7	$pp - pe - eee = me$
$7 + eee$	8	$pp - pe = me + eee$
$8 + pe$	9	$pp = eee + me + pe$
9 in Numbers	10	$eee + 100e + 50e = 2500$
That is	11	$eee + 150e = 2500$

Here the Equation appears to be *adfected*, and to reſolve it, let us ſuppoſe $e = 9$.

Then $eee = 729$
And $150e = 1350$

2079 which being leſs than 2500, therefore e muſt be more than 9.

Then let $r = 9$, and $y =$ what 9 wants of the true Value of e, then by *Caſe* 1, Art. 67, we have,

	1	$r + y = e$
1 & 3	2	$rrr + 3rry + 3ryy = eee$, rejecting the Powers of y above yy.

Becauſe in the given Equation e is multiplied by 150, therefore multiply the firſt Step by 150.

1×150	3	$150r + 150y = 150e$

Now add the ſecond and third Equations together, becauſe the like Powers of e in the *adfected* Equation, are connected by the Sign +.

$2 + 3$	4	$rrr + 3rry + 3ryy + 150r + 150y = eee + 150e$
But	5	$eee + 150e = 2500$, by the given Equation.

4 · 5

$4 \cdot 5$	6	$rrr + 3rry + 3ryy + 150r + 150y = 2500$
6 in Numbers	7	$729 + 243y + 27yy + 1350 + 150y = 2500$
7 contracted	8	$2079 + 393y + 27yy = 2500$
$8 - \overline{2079}$	9	$393y + 27yy = 421$ Dividing by the Co-efficient of *yy*.
$9 \div \overline{27}$	10	$14.55y + yy = 15.59$ Now dividing by the Co-efficient of *y* *plus y*.
$10 \div 14.48 + y$	11	$y = \frac{15.59}{14.55 + y}$

Operation $14.55)\ 15.58\ (1. = y$

$1.$

Divisor 15.55 $\quad 15.55$

3 Remainder neglected.

$r = 9$
$y = 1$

$r + y = 10 = e$, which being involved and tried will be found to be the true Root: Hence 10 is the leſſer Number ſought.

Then by the third Step of the Work to the Queſtion $a = p - e = 40$, the greater Number ſought.

In the Diviſion for finding *y*, the Learner may obſerve, that as the two next Figures in the Quotient will be *Cyphers*, and in the Places of Fractions, and the third Figure in the Place of Fractions being of ſo ſmall a Value, I proceed no further in the Diviſion but leave it as in the Work, and ſo happen to find the true Value of *e*.

Queſtion 82. *Two Men,* A *and* B, *diſcourſing of their Money,* A *who had the moſt ſays to* B, *if my Money was divided by your's, and from that Quotient ſubſtracting three times the Square of your's, and to this Remainder adding the Square of mine, the Sum is* 27 *Pounds:*

But if your Money is ſubſtracted from mine, the Remainder is 5 *Pounds. How many Pounds had each Man?*

Put $a =$ the Money of A, $e =$ the Money of B, $d = 27$, $x = 5$.

1	$\frac{a}{e} - 3ee + aa = d$	By the Queſtion.
2	$a - e = x$	

Here we find a and e are both to the firſt and ſecond Power in the firſt Equation, and to the firſt Power only in the ſecond Equation, therefore by the Directions find the Value of a, or e, in the ſecond Equation, ſuppoſe we find the Value of a.

$2 + e$	3	$a = x + e$

Raiſe this to the ſecond Power, becauſe a is to the ſecond Power in the firſt Equation.

$3 \circledast 2$	4	$aa = xx + 2xe + ee$

Then for a and aa in the firſt Equation, write theſe reſpective Values, or $x + e$, and $xx + 2xe + ee$.

$1 \cdot 3 \cdot 4$	5	$\frac{x+e}{e} - 3ee + xx + 2xe + ee = d$, here a is exterminated, for the Equation contains only the unknown Quantity e.
$5 \times e$	6	$x + e - 3eee + xxe + 2xee + eee = de$
That is	7	$x + e - 2eee + xxe + 2xee = de$
7 in Numbers	8	$5 + e - 2eee + 25e + 10ee = 27e$
$8 + 2eee$	9	$5 + e + 25e + 10ee = 2eee + 27e$
$9 - 10ee$	10	$5 + e + 25e = 2eee - 10ee + 27e$
$10 - e$	11	$5 + 25e = 2eee - 10ee + 26e$
$11 - 25e$	12	$2eee - 10ee + e = 5$

To reſolve this Equation, ſuppoſe $e = 6$.

Then $2eee = 432$
$-10ee = -360$
72
$+e = 6$
78 which being greater than 5, the Number in the given Equation, hence e cannot be ſo much as 6, therefore,

Let $r = 6$, and $y =$ what 6 is too much, then by *Caſe* 2, Article 68.

	1	$r - y = e$
$1 \circledast 3$	2	$rrr - 3rry + 3ryy = eee$ rejecting the Powers of y above yy.

Becauſe in the given Equation eee is multiplied by 2, therefore multiply the laſt Equation by 2.

$2\times\overline{2}$	3	$2rrr - 6rry + 6ryy = 2eee$

Now raiſe $r - y = e$ to the ſecond Power, after which multiply it by 10, becauſe it is $10ee$ in the given adfected Equation.

$1 \odot 2$	4	$rr - 2ry + yy = ee$
$4\times\overline{10}$	5	$10rr - 20ry + 10yy = 10ee$

Then add, or ſubſtract the Equations that are equal to $2eee$, $10ee$ and e, according as thoſe Quantities have the Signs + or —, in the given *adfected* Equation.

$3 - 5 + 1$	6	$2rrr - 6rry + 6ryy - 10rr + 20ry - 10yy + r - y = 2eee - 10ee + e$
But	7	$2eee - 10ee + e = 5$, by the given Equation.
$6 \,.\, 7$	8	$2rrr - 6rry + 6ryy - 10rr + 20ry - 10yy + r - y = 5$
& in Numbers	9	$432 - 216y + 36yy - 360 + 120y - 10yy + 6 - y = 5$
9 contracted	10	$78 - 97y + 26yy = 5$

Tranſpoſe 5 it being leſs than 78.

$10 - \overline{5}$	11	$73 - 97y + 26yy = 0$, for one Side of the Equation ſubſtracted from the other, the Remainder muſt be *nothing*, both Sides of the Equation being equal to one another.
$11 + 97y$	12	$73 + 26yy = 97y$
$12 - 26yy$	13	$97y - 26yy = 73$ Divide by the Co-efficient of yy.
$13 \div \overline{26}$	14	$3.73y - yy = 2.807$ Now divide by the Co-efficient of y *minus* y.
$14 \div 3.73 - y$	15	$y = \dfrac{2.807}{3.73 - y}$

Operation 3.73) 2.807 (1. = y

— 1.

Diviſor 2.73 2.73

7 Remainder neglected.

$r = 6$

$$\begin{array}{l} r = 6 \\ -y = -1 \\ \hline \end{array}$$

$r - y = 5 = e$, which being involved and tried it will be found to be the true Root; hence B had 5 Pounds.

Then by the third Step of the Work to the Queſtion $a = x + e = 10$ Pounds, the Money A had.

70. *Every adfected Equation has as many Roots either* real or imaginary, *as are the higheſt Dimenſions of its unknown Quantity.*

It may not be amiſs to indulge the Curioſity of the Learner, to ſhow him adfected Equations have as many Roots as they have Dimenſions, that is, if the higheſt Power of the unknown Quantity is the Biquadratic, or fourth Power, then there may be four Values of the unknown Quantity; if it is only to the third Power, then there may be three Values of the unknown Quantity and ſo on: But there cannot be more Roots than there are Dimenſions in the Equation.

Theſe Roots are ſometimes *affirmative*, and ſometimes *negative*, and ſome Roots are *impoſſible*. The Reader obſerving how Quadratic Equations were compounded and generated, may better underſtand the Nature of theſe Roots. Thus,

Suppoſe $a = 1$, then $a - 1 = 0$, again ſuppoſe $a = 2$, then $a - 2 = 0$.

Now multiply theſe two together - -

$$\begin{array}{r} a - 1 = 0 \\ a - 2 = 0 \\ \hline aa - a \\ -2a + 2 \\ \hline \end{array}$$

An Equation of two Dimenſions, which has two Roots, *viz.* 1 and 2. - - $aa - 3a + 2 = 0$

Again, let $a = 3$, then - - - $a - 3 = 0$

$$\begin{array}{r} \hline aaa - 3aa + 2a \\ -3aa + 9a - 6 \\ \hline \end{array}$$

From multiplying theſe together, we have an Equation of three Dimenſions, and which has 3 Roots, *viz.* 1 . 2 and 3. - - - - $aaa - 6aa + 11a - 6 = 0$

Laſtly, ſuppoſe $a = -5$, then - - $a + 5 = 0$

$$\begin{array}{r} \hline aaaa - 6aaa + 11aa - 6a \\ +5aaa - 30aa + 55a - 30 \\ \hline \end{array}$$

An Equation of 4 Dimenſions, and which has 4 Roots, *viz.* 1 . 2 . 3 and -5, and ſo of any other Power. $aaaa - aaa - 19aa + 49a - 30 = 0$

By the ſame Method that we found the two Roots in Quadratic Equations, we may find the Roots of theſe Equations. For ſuppoſe we had this Equation $aaaa-aaa-19aa+49a-30=0$ given, which being reſolved by the Method of *Converging Series*, we ſhall find $a=1$, whence 1 is one of the Roots of the given adfected Equation; now tranſpoſe 1 to make it $a-1=0$, take the given Equation and make it equal to *nothing*, and dividing the given Equation by $a-1$, the Quotient muſt be equal to *nothing*, thus,

$$\begin{array}{l}
a-1=0)\ aaaa-aaa-19aa+49a-30=0\ (aaa-19a+30=0 \\
\qquad\qquad aaaa-aaa \\
\hline
\qquad\qquad\qquad -19aa+49a-30 \\
\qquad\qquad\qquad -19aa+19a \\
\hline
\qquad\qquad\qquad\qquad\quad 30a-30 \\
\qquad\qquad\qquad\qquad\quad 30a-30 \\
\hline
\qquad\qquad\qquad\qquad\qquad\quad 0
\end{array}$$

Here we find the Quotient to be $aaa-19a+30=0$, and by the Solution of this Equation we ſhall find $a=3$, for another of the Roots of the given adfected Equation.

$$\begin{array}{l}
\text{Then } a-3=0)\ aaa-19a+30=0\ (aa+3a-10=0 \\
\qquad\qquad\qquad aaa-3aa \\
\hline
\qquad\qquad\qquad\qquad 3aa-19a+30 \\
\qquad\qquad\qquad\qquad 3aa-9a \\
\hline
\qquad\qquad\qquad\qquad\qquad -10a+30 \\
\qquad\qquad\qquad\qquad\qquad -10a+30 \\
\hline
\qquad\qquad\qquad\qquad\qquad\qquad 0
\end{array}$$

Hence we have got this Equation $aa+3a-10=0$, whence $aa+3a=10$, the two Roots of which are 2 and -5, the two remaining Roots of the given adfected Equation; in the ſame Manner all the poſſible Roots of any other Equation are determined.

And to give the Learner an inſtance where ſome of the Roots of an Equation are impoſſible:

Suppoſe $aaa-4aa+4a-16=0$, by tranſpoſing 16 and reſolving the Equation, we ſhall find $a=4$: Then tranſpoſing 4 to make it $a-4=0$, and making the given Equation equal to nothing and dividing thus,

$$\begin{array}{l} a-4=0)\ aaa-4aa+4a-16=0\ (aa+4=0 \\ \quad\quad\quad\ \ \underline{aaa-4aa\quad\quad\quad\quad\quad\quad} \\ \quad\quad\quad\quad\quad\quad\quad\quad 4a-16 \\ \quad\quad\quad\quad\quad\quad\quad\quad \underline{4a-16} \\ \quad\quad\quad\quad\quad\quad\quad\quad\quad 0 \end{array}$$

Becauſe the Dividend and Diviſor are both equal to *nothing*, therefore the Quotient muſt be equal to *nothing*, but if $aa+4=0$, then $aa=-4$ an Equation which has no real or *poſſible* Root in Nature, it being impoſſible to generate or produce a *negative* Square, for *minus* multiplied into *minus*, as well as *plus* multiplied into *plus*, makes the Product *affirmative*, or *plus*.

Queſtion 83. *Three Merchants,* A, B, *and* C, *found that what* A *and* B *had gained was equal to twice what* C *had gained:*

But if A's *Gain was added to twice* B's *Gain, and this Sum added to what* C *gained, it made* 19 *Pounds:*

And the Sum of the Squares of each Perſon's Gain was equal to 77 *Pounds. How much did each Perſon Gain?*

Let $a=$ the Gain of A, $e=$ the Gain of B, $y=$ the Gain of C, $m=19$, $p=77$.

	1	$a+e=2y$ } By the Queſtion.
	2	$a+2e+y=m$ } By the Queſtion.
	3	$aa+ee+yy=p$ } By the Queſtion.
$1-e$	4	$a=2y-e$. Raiſe this Equation to the ſecond Power, it being aa at the third Equation.
4 @ 2	5	$aa=4yy-4ye+ee$

Now for a, and aa in the ſecond and third Equations write their reſpective Values, *viz.* $2y-e$, and $4yy-4ye+ee$.

2 . 4	6	$3y+e=m$
3 . 5	7	$5yy-4ye+2ee=p$

Here the Queſtion is reduced to two Equations and two unknown Quantities, for a is exterminated, therefore in the ſixth Equation, find the Value of e, or y, and raiſe it to the ſecond Power, for thoſe Quantities are to the ſecond Power in the ſeventh Equation.

6 — 3

$6 - 3y$	8	$e = m - 3y$
$8 \odot 2$	9	$ee = mm - 6my + 9yy$ Multiply this Equation by 2, becauſe it is $2ee$ in the ſeventh Equation.
9×2	10	$2ee = 2mm - 12my + 18yy$

Now in the ſeventh Equation for e and $2ee$ write their Values at the eighth and tenth Steps.

7 . 8 . 10	11	$5yy - 4ym + 12yy + 2mm - 12my + 18yy = p$, an Equation with only the unknown Quantity y.
11 contracted	12	$35yy - 16my + 2mm = p$
$12 - 2mm$	13	$35yy - 16my = p - 2mm$, here the Equation appears *quadratic*, and it is likewiſe *ambiguous*, for $2mm$ is greater than p.
$13 \div \overrightarrow{35}$	14	$yy - \frac{16my}{35} = \frac{p - 2mm}{35}$
$14\ c\ \square$	15	$yy - \frac{16my}{35} + \frac{256mm}{4900} = \frac{256mm}{4900} + \frac{p - 2mm}{35}$

The Co-efficient of y is $\frac{16m}{35}$, which being divided by 2, or $\frac{2}{1}$ by the Rule in common Arithmetic for Diviſion of Vulgar Fractions, the Quotient is $\frac{16m}{70}$, the Square of which is $\frac{256mm}{4900}$.

15 ɯ 2	16	$y - \frac{16m}{70} = \sqrt{\frac{256mm}{4900} + \frac{p - 2mm}{35}}$
$16 + \frac{16m}{70}$	17	$y = \frac{16m}{70} \pm \sqrt{\frac{256mm}{4900} + \frac{p - 2mm}{35}}$ $= 4.9999$, or 3.6857 But 4.9999 is the Number, the Anſwer being 5. then if $y = 5$
By the 8th Step	18	$e = m - 3y = 4$
By the fourth Step	19	$a = 2y - e = 6$

Queſtion 84. A, B, *and* C, *having been at the Gaming-Tables, found the Loſs of* A *added to the Loſs of* C *was equal to twice the Loſs of* B:

But the Loſs of A *added to the Loſs of* B, *and this added to twice the Loſs of* C, *the Sum was* 22 *Pounds:*

And the Product of what A *and* B *loſt, being added to three times the Product of what* B *and* C *loſt, the Sum was* 120 *Pounds. How much did each Perſon loſe?*

Let $a =$ the Sum A loſt, $e =$ the Sum B loſt, $y =$ the Sum C loſt, $d = 22$, $n = 120$.

	1	$a + y = 2e$ } By the Queſtion.
	2	$a + e + 2y = d$
	3	$ae + 3ey = n$
$1 - y$	4	$a = 2e - y$
$2 \cdot 4$	5	$3e + y = d$
$3 \cdot 4$	6	$2ee - ey + 3ey = n$

By the fifth and ſixth Steps, the Queſtion is reduced to two Equations, and two unknown Quantities, and becauſe y is only to the firſt Power in both Equations, find the Value of y in each of them.

$5 - 3e$	7	$y = d - 3e$
$6 - 2ee$	8	$2ey = n - 2ee$
$8 \div 2e$	9	$y = \frac{n - 2ee}{2e}$
$7 \cdot 9$	10	$\frac{n - 2ee}{2e} = d - 3e$
$10 \times 2e$	11	$n - 2ee = 2de - 6ee$
$11 + 6ee$	12	$4ee + n = 2de$
$12 - n$	13	$4ee = 2de - n$
$13 - 2de$	14	$4ee - 2de = -n$ Here the Equation is *quadratic* and *ambiguous.*
$14 \div 4$	15	$ee - \frac{de}{2} = -\frac{n}{4}$
15 c □	16	$ee - \frac{de}{2} + \frac{dd}{16} = \frac{dd}{16} - \frac{n}{4}$ for the Co-efficient of e is $\frac{d}{2}$, which being divided by 2 as in the laſt Queſtion, the

		the Quotient is $\frac{d}{4}$, the Square of which is $\frac{dd}{16}$.
16 ω 2	17	$e - \frac{d}{4} = \sqrt{\frac{dd}{16} - \frac{n}{4}}$
$17 + \frac{d}{4}$	18	$e = \frac{d}{4} \pm \sqrt{\frac{dd}{16} - \frac{n}{4}} = 5.5 \pm .5$ (=6, or 4, if $e = 6$,
Then by Step 7th	19	$y = d - 3e = 4$
And by Step 4th	20	$a = 2e - y = 8$

But if $e = 4$, then by the ſeventh Step $y = d - 3e = 10$, and by the fourth Step $a = 2e - y = -2$, which is an Impoſſibility for a an *affirmative* Quantity, to be equal to -2 a *negative* Quantity. Therefore the Loſs of A was 8 Pounds, the Loſs of B was 6 Pounds, and the Loſs of C was 4 Pounds.

Queſtion 85. *There are two Numbers, that the Sum of their Squares added to their Sum, is* 338 :
And their Product is 156. *What are the Numbers?*

Let a and e, be the two Numbers ſought, $b = 338$, $m = 156$.

Then	1	$aa + ee + a + e = b$ } By the Queſtion.
	2	$ae = m$
1 ÷ e	3	$a = \frac{m}{e}$
3 @ 2	4	$aa = \frac{mm}{ee}$
1 . 3 . 4	5	$\frac{mm}{ee} + ee + \frac{m}{e} + e = b$, this Equation has the unknown Quantity e only.
5 × ee	6	$mm + eeee + \frac{eem}{e} + eee = bee$
		But as $\frac{eem}{e}$ is only em, the e being rejected by Art. 20.
Hence	7	$mm + eeee + em + eee = bee$

There being only the known Quantity mm, tranſpoſe the others ſo that mm may be at laſt *affirmative*, and this may be obſerved that in tranſpoſing the Quantities in theſe *adfected* Equations, the Side of the Equation which is known may at laſt be *affirmative*.

$7 - eeee$	8	$mm + em + eee = bee - eeee$
$8 - eee$	9	$mm + em = bee - eeee - eee$
$9 - em$	10	$mm = bee - eeee - eee - me$
Or	11	$-eeee - eee + bee - me = mm$, it being the common Method to place theſe Equations, according to the higheſt Power of the unknown Quantity.
11 in Numbers	12	$-eeee - eee + 338ee - 156e = 24336$.

Now ſuppoſe $e = 10$.

$$\text{Then } -eeee = -10000$$
$$-eee = -\ \ 1000$$
$$-11000$$
$$+338ee = 33800$$
$$22800$$
$$-156e = -\ \ 1560$$

$-eeee - eee + 338ee - 156e = 21240$ which being leſs than 24336 the Number in the given Equation, therefore e muſt be more than 10.

Let $r = 10$, and put $y =$ what it wants of being the true Root.

Then	1	$r + y = e$
1 ⊚ 4	2	$rrrr + 4rrry + 6rryy = eeee$, all the Powers of y above yy being rejected.
1 ⊚ 3	3	$rrr + 3rry + 3ryy = eee$, rejecting all the Powers of y above yy.
1 ⊚ 2	4	$rr + 2ry + yy = ee$

Becauſe in the given Equation it is $338ee$, therefore multiply the fourth Equation by 338.

$4 \times \overline{338}$	5	$338rr + 676ry + 338yy = 338ee$

Becauſe in the given Equation it is $156e$, therefore multiply the firſt Equation by 156.

$$1 \times \overline{156} \mid 6 \mid 156r + 156y = 156e$$

Now the ſecond, third, fifth, and ſixth Equations being equal to the ſeveral Powers of e in the given Equation, connect them by the ſame Signs thoſe Powers have in that Equation.

$-2-3+5-6$	7	$-rrrr-4rrry-6rryy-rrr-3rry$ $-3ryy+338rr+676ry+338yy$ $-156r-156y=-eeee-eee$ $+338ee-156e$
But	8	$-eeee-eee+338ee-156e=24336$ by the given Equation.
$7 \cdot 8$	9	$-rrrr-4rrry-6rryy-rrr-3rry$ $-3ryy+338rr+676ry+338yy$ $-156r-156y=24336$
9 in Numbers	10	$-10000-4000y-600yy-1000$ $-300y-30yy+33800+6760y$ $+338yy-1560-156y=24336$
10 contracted	11	$21240+2304y-292yy=24336$
$11-21240$	12	$2304y-292yy=3096$ Now divide by the Co-efficient of yy.
$12 \div \overline{292}$	13	$7.89y-yy=10.6$ And dividing by the Co-efficient of y *minus* y.
$13 \div 7.89-y$	14	$y=\dfrac{10.6}{7.89-y}$

Operation 7.89) 10.60 (1.7 = y
— 1
———— 6.89
Diviſor 6.89 ————
— 1.7 3710
———— 3633
Diviſor 5.19 ————
77

$r = 10$
$+y = 1.7$

$r+y=11.7$, which being involved and tried, it will be found too little; therefore for a ſecond Operation,

Suppoſe $r = 11.7$ and $y =$ what it wants of the true Root. Then

Then	1	$r+y=e$
1 @ 4	2	$rrrr+4rrry+6rryy=eeee$, the Powers of y, above yy being rejected.
1 @ 3	3	$rrr+3rry+3ryy=eee$, the Powers of y, above yy being rejected.
1 @ 2	4	$rr+2ry+yy=ee$

Becauſe in the given Equation it is $338ee$, therefore multiply the laſt Equation by 338.

$4\times\overline{338}$	5	$338rr+676ry+338yy=338ee$

Becauſe in the given Equation it is $156e$, therefore multiply the firſt Equation by 156.

$1\times\overline{156}$	6	$156r+156y=156e$

Now connect the Equations that are equal to $eeee$, eee, $338ee$ and $156e$, by the ſame Signs as thoſe Quantities have in the given *adfected* Equation.

$-2-3+5-6$	7	$-rrrr-4rrry-6rryy-rrr-3rry-3ryy+338rr+676ry+338yy-156r-156y=-eeee-eee+338ee-156e$
But	8	$-eeee-eee+338ee-156e=24336$, by the given Equation.
7 · 8	9	$-rrrr-4rrry-6rryy-rrr-3rry-3ryy+338rr+676ry+338yy-156r-156y=24336$
9 in Numbers	10	$-18738.8721-6406.452y-821.34yy-1601.613-410.67y-35.1yy+46268.82+7909.2y+338yy-1825.2-156y=24336$
10 contracted	11	$24103.1349+936.078y-518.44yy=24336$
11 —	12	$936.078y-518.44yy=232.8651$ Dividing by the Co-efficient of yy.
12 ÷	13	$1.805y-yy=.4491$ Now dividing by the Co-efficient of y, *minus* y, that is, by $1.805-y$.
$13\div 1.805-y$	14	$y=\dfrac{.4491}{1.805-y}$

Operation $1.805)\ .4491\ (.297 = y.$
$-\ .2$

Diviſor 1.605 3210
$-\ .29$
12810
Diviſor 1.315 11835
$-\ 97$
9750
Diviſor 1.218 8526
1224

$r = 11.7$
$+ y = \ \ .297$

$r + y\ 11.997 = e$, which is ſomething too little, the true Value being 12. but this may inform the Learner of the Nature of ſolving theſe high *adfected* Equations, and that every Operation approaches nearer and nearer to the true Root, from whence it may be found to any aſſignable degree of Exactneſs.

And having found e to be 12, then by the third Step of the Work to the Queſtion, we have $a = \frac{m}{e} = 13$, the other Number ſought.

71. *The Method of reſolving Equations when the unknown Quantity is to ſeveral Powers in both Equations.*

When both the unknown Quantities are to the firſt and ſecond Power in both Equations, then find the Value of the Square of the unknown Quantity in each Equation, and make theſe two Equations equal to one another; which Equation will have the firſt power only of the unknown Quantity, its Square being exterminated by the foregoing Equation.

Then find the Value of the firſt Power of the unknown Quantity in this laſt Equation, which raiſe to the ſecond Power, and in either of the two given Equations in which it may be moſt conveniently done; for this unknown Quantity and its ſeveral Powers write their reſpective Values, which will give an Equation with only one unknown Quantity, and is to be reduced by the Rules already explained.

Queſtion 86. *To find two Numbers that the Sum of their Squares is equal to the leſſer multiplied by 20:*

And the Square of the leſſer being added to their Product, the Sum is 16.

Let $a =$ the greater Number, $e =$ the leſſer Number, $m = 20$; $d = 16$.

	1	$aa + ee = me$ By the Queſtion.
	2	$ee + ae = d$
		Begin to exterminate ee according to the Directions, that is, find the Value of ee in both the given Equations.
$1 - aa$	3	$ee = me - aa$
$2 - ae$	4	$ee = d - ae$
$3 \cdot 4$	5	$me - aa = d - ae$, here ee is exterminated, now find the Value of e.
$5 + ae$	6	$me + ae - aa = d$
$6 + aa$	7	$me + ae = d + aa$
$7 \div m + a$	8	$e = \frac{d + aa}{m + a}$
		Raiſe this Value of e to the ſecond Power.
8 @ 2	9	$ee = \frac{dd + 2daa + aaaa}{mm + 2ma + aa}$

Now in the firſt Equation for ee and e, write their reſpective Values at the eighth and ninth Steps.

$1 \cdot 9 \cdot 8$	10	$aa + \frac{dd + 2daa + aaaa}{mm + 2ma + aa} = \frac{md + maa}{m + a}$ an Equation clear of e, having only the unknown Quantity a.

To clear this Equation of the Fractions, obſerve that $mm + 2ma + aa$ is the Square of $m + a$, the former ariſing from the Involution of the latter by the eighth and ninth Steps, and in the Multiplication of Fractions, it being the ſame to divide the Diviſor, as to multiply the Dividend, to multiply $\frac{dd + 2daa + aaaa}{mm + 2ma + aa}$, by $m + a$, we only change the Diviſor to $m + a$, that being the Quotient of $mm + 2ma + aa$ divided by $m + a$, the reſt of the Multiplication is the ſame as uſual.

10 ×

$10\times m+a$	11	$maa+aaa+\frac{dd+2daa+aaaa}{m+a}=md+maa$
$11\times m+a$	12	$mmaa+maaa+maaa+aaaa+dd+2daa+aaaa=mmd+mmaa+mda+maaa$
$12-maaa$	13	$mmaa+maaa+aaaa+dd+2daa+aaaa=mmd+mmaa+mda$
$14-mmaa$	14	$maaa+aaaa+dd+2daa+aaaa=mmd+mda$
$14-mda$	15	$maaa+2aaaa+dd+2daa-mda=mmd$
$15-dd$	16	$2aaaa+maaa+2daa-mda=mmd-dd$
16 in Numbers	17	$2aaaa+20aaa+32aa-320a=6144$ Becauſe the Co-efficient of $aaaa$, will divide the other Co-efficients without any Remainder divide by it.
$17\div 2$	18	$aaaa+10aaa+16aa-160a=3072$

Which Equation being reſolved by the Method of *Converging Series*, we ſhall find $a=6$, or nearly to it, 6 being the true Root, from whence by the eighth Step $e=2$.

Queſtion 87. *There are two Numbers, that if from the Sum of the greater added to its Square, we ſubſtract the Square of the leſſer, the Remainder is 94:*

But the Square of the leſſer being added to the leſſer, this Sum is equal to twice the greater.

Let $a=$ the greater Number, $e=$ the leſſer Number, $m=94$.

	1 2	$\left.\begin{array}{l}a+aa-ee=m\\ ee+e=2a\end{array}\right\}$ By the Queſtion. Begin with finding the Value of ee in each Equation.
$1+ee$	3	$aa+a=m+ee$
$3-m$	4	$aa+a-m=ee$
$2-e$	5	$ee=2a-e$
4 . 5	6	$aa+a-m=2a-e$, here ee is exterminated, now find the Value of e.
$6+e$	7	$e+aa+a-m=2a$
$7-a$	8	$e+aa-m=a$

$8 + m$	9	$e + aa = a + m$
$9 - aa$	10	$e = a + m - aa$ Raiſe this Value of e to the ſecond Power.
10 Ⓦ 2	11	$ee = aa + 2am + mm - 2aaa - 2maa + aaaa$

Now for ee and e in the ſecond Equation, write their reſpective Values, found at the tenth and eleventh Steps.

2 . 11 . 10	12	$aa + 2am + mm - 2aaa - 2maa + aaaa + a + m - aa = 2a$
12 in Numbers	13	$188a + 8836 - 2aaa - 188aa + aaaa + a + 94 = 2a$
13 contracted	14	$187a + 8930 - 2aaa - 188aa + aaaa = 0$ Tranſpoſe the ſeveral Powers of a, that 8930 the known Part of the Equation may be *affirmative*.
$14 - aaaa$	15	$-aaaa = 187a + 8930 - 2aaa - 188aa$
$15 + 2aaa$	16	$-aaaa + 2aaa = 187a + 8930 - 188aa$
$16 + 188aa$	17	$-aaaa + 2aaa + 188aa = 187a + 8930$
$17 - 187a$	18	$-aaaa + 2aaa + 188aa - 187a = 8930$

Which Equation being reſolved we ſhall find $a = 10$, or nearly to it, 10 being the true Root.

Then by the tenth Step $e = a + m - aa = 4$.

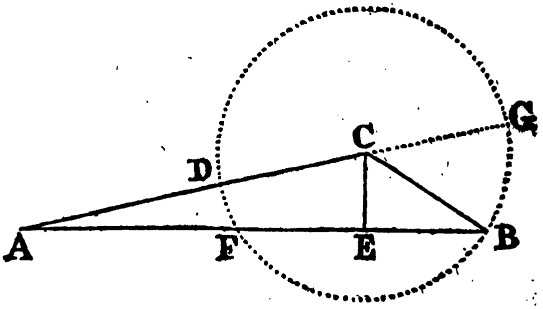
We ſhall now proceed to the Solution of ſeveral Geometrical Problems upon the ſame *general* Principles, and if the Learner is not ſufficiently acquainted with the Elements of Geometry, to diſcover how the Equations are formed from the Properties of the Figure, he may omit theſe Queſtions, and proceed to the others which require no Knowledge in Geometry.

Queſtion 88. *In the oblique Triangle* A B C, *there is given the Difference between the Sides* A C, *and* B C, *or* A D = 8, *and the Difference between the Segments of the Baſe, or* A F = 10, *and the Perpendicular* C E = 16, *let fall from the vertical Angle* C, *upon the Baſe* A B, *to find the Sides* A C, C B, *and Baſe* A B.

Upon C, as a Center with the Radius C B, draw the Circle G B F D, and continue A C to G.

Hence C B = C D, as Radius of the ſame Circle, whence A D is the Difference of the Sides, or the Difference between A C and C B.

And $BE = EF$, as FB is biſſected at E by the Nature of the Perpendicular CE, hence AF is the Difference of the Segments of the Baſe, or the Difference between AE and EB.



Let $AD = d = 8$, and $DC = CB = e$, then $AC = d + e$.

Let $AF = b = 10$, and $FE = EB = a$, hence $AB = b + 2a$.

Let $CE = p = 16$.

Having two unknown Quantities a, and e, and no Equation from the Conditions of the Queſtion, we muſt raiſe two Equations from the Figure.

Now the Lines AG and AB are drawn from within a Circle, and touch without at the Point A, therefore by 37 *e*. 3 $AG \times AD = AB \times AF$, all which Lines are expreſſed in Symbols except AG, but $CG = CD = e$, and $AC = d + e$, therefore $AG = d + 2e$, hence we have in Symbols.

	1	$\overline{d + 2e} \times d = \overline{b + 2a} \times b$ the ſhort Lines over the Quantities, ſignifies that they are both to be multiplied by the Quantity which follows the Sign of Multiplication.
		But the Triangle CEB is Right-angled, therefore by
47 *e*. 1	2	$pp + aa = ee$
From the firſt	3	$dd + 2de = bb + 2ba$
		Now find the Value of either a or e, in the third Equation.
$3 - dd$	4	$2de = bb + 2ba - dd$

But as we ſhall have occaſion to ſquare this Equation, for when the Value of e is found, that Equation muſt be raiſed to the ſecond Power, it being ee in the ſecond Equation; and $bb-dd$ being a known Quantity to avoid Trouble, ſubſtitute $x=bb-dd$.

Then	5	$2de=x+2ba$
$5\div 2d$	6	$e=\frac{x+2ba}{2d}$ Raiſe this to the ſecond Power, becauſe it is ee in the ſecond Equation.
6 ⊙ 2	7	$ee=\frac{xx+4xba+4bbaa}{4dd}$
2, 7	8	$\frac{xx+4xba+4bbaa}{4dd}=pp+aa$, an (Equation clear of e.
$8\times 4dd$	9	$xx+4xba+4bbaa=4ddpp+4ddaa$ Bring all the Powers of a, to one Side of the Equation.
$9-4ddaa$	10	$4bbaa-4ddaa+4xba+xx=4ddpp$
$10-xx$	11	$4bbaa-4ddaa+4xba=4ddpp-xx$

Here the Equation appears *quadratic*, the unknown Quantity being only to the firſt and ſecond Power, but as the Square of the unknown Quantity has Co-efficients, therefore by Article 58. divide the Equation by $4bb-4dd$, the Co-efficients of aa.

$11\div 4bb-4dd$	12	$aa+\frac{4xba}{4bb-4dd}=\frac{4ddpp-xx}{4bb-4dd}$

The Work being now prepared for *compleating the Square*, but the *Co-efficient* of a being a Fraction, to ſave the Trouble of dividing it by 2, and ſquaring the Quotient according to Art. 57. ſubſtitute $y=\frac{4xb}{4bb-4dd}$,

Then	13	$aa+ya=\frac{4ddpp-xx}{4bb-4dd}$
13, c □	14	$aa+ya+\frac{yy}{4}=\frac{4ddpp-xx}{4bb-4dd}+\frac{yy}{4}$
14 ɯ 2	15	$a+\frac{y}{2}=\sqrt{\frac{4ddpp-xx}{4bb-4dd}+\frac{yy}{4}}$
$15-\frac{y}{2}$	16	$a=\sqrt{\frac{4ddpp-xx}{4bb-4dd}+\frac{yy}{4}}:-\frac{y}{2}$ $=16.7$

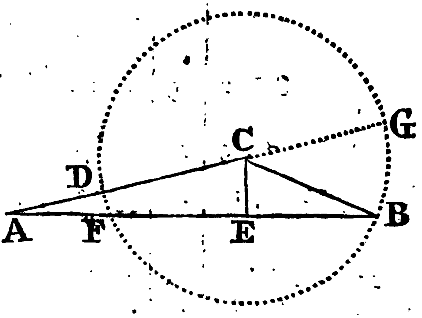
Then by Step 6th | 17 | $e = \frac{x + 2ba}{2d} = 23.12$

Hence | 18 | $AC = d + e = 31.12$

19 | $CB = x = 23.15$

20 | $AB = b + 2a = 43.4$

Queſtion 89. *In the oblique Triangle* A B C, *there is given* 8, *the Sum of the Sides* A C *and* B C, *and* A F = 2, *the Difference of the Segments of the Baſe,* A E *and* B E, *with the Perpendicular* C E = 1, *let fall from the vertical Angle at* C *upon the Baſe* A B. *To find the Sides* A C, *and* B C, *and the Baſe* A B?



Upon C as a Center with the Radius C B, draw the Circle G B F D, and continue A C to G.

Then C G = C B = C D, being all Radii of the ſame Circle, whence A G is the Sum of the Sides, or A C + C B.

And F E = E B, for F B is biſſected at E, by the Nature of the Perpendicular C E, whence A F is the Difference of the Segments of the Baſe, or the Difference between A E and B E.

The Conſtruction of this Figure being the ſame as the laſt, we can raiſe the ſame two Equations from the Figure, but inſtead of A D being given, we have A G given. Let A G, or A C + C B = s = 8, and D C = C G = a, whence D G = 2 a, then $s - 2a$ = A D, the Part of the Line A G without the Circle.

And put A F = d = 2, and F E = E B = e, then A B = $d + 2e$, let C E = p = 1.

Now as in the laſt Queſtion, becauſe the Lines A G and A B, are drawn from the Circumference within the Circle, and touch at the Point A without the Circle, hence by 37 . *e* . 3 A G × A D = A B × A F, that is,

in Symbols | 1 | $s \times \overline{s - 2a} = \overline{d + 2e} \times d$

That is | 2 | $ss - 2sa = dd + 2de$

by 47 . *e* . 1 | 3 | $pp + ee = aa$ the Triangle C E B being right-angled, and D C = C B = a.

Here the Queſtion is expreſſed by two Equations, and two unknown Quantities.

$2 + 2sa$	4	$ss = dd + 2de + 2sa$
$4 - dd$	5	$ss - dd = 2de + 2sa$
$5 - 2de$	6	$2sa = ss - dd - 2de$
		Subſtitute as before $x = ss - dd$, for when the Value of a is found, it muſt be raiſed to the ſecond Power, to compare it with aa in the third Equation.
Then	7	$2sa = x - 2de$
$7 \div 2s$	8	$a = \frac{x - 2de}{2s}$
		Raiſe this Equation to the ſecond Power, becauſe it is aa in the third Equation.
8 ⊙ 2	9	$aa = \frac{xx - 4xde + 4ddee}{4ss}$
$3 \cdot 9$	10	$pp + ee = \frac{xx - 4xde + 4ddee}{4ss}$
$10 \times 4ss$	11	$4sspp + 4ssee = xx - 4xde + 4ddee$
		Now bring all the Powers of e to one Side of the Equation.
$11 - 4ddee$	12	$4ssee - 4ddee + 4sspp = xx - 4xde$
$12 + 4xde$	13	$4ssee - 4ddee + 4xde + 4sspp = xx$
$13 - 4sspp$	14	$4ssee - 4ddee + 4xde = xx - 4sspp$

Here the Equation appears *quadratic*, but is not *ambiguous*, for xx is greater than $4sspp$. Now, by Article 58, divide by the Co-efficients of ee.

$14 \div 4ss - 4dd$	15	$ee + \frac{4xde}{4ss - 4dd} = \frac{xx - 4sspp}{4ss - 4dd}$

The Work being prepared for *compleating the Square*, ſubſtitute $y = \frac{4xd}{4ss - 4dd}$,

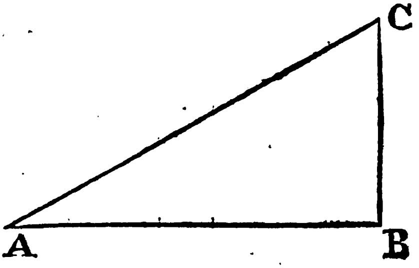
Then	16	$ee + ye = \frac{xx - 4sspp}{4ss - 4dd}$
16 c □	17	$ee + ye + \frac{yy}{4} = \frac{xx - 4sspp}{4ss - 4dd} + \frac{yy}{4}$

17 ω 2

17 ш 2	18	$e + \frac{y}{2} = \sqrt{\frac{xx - 4sspp}{4ss - 4dd} + \frac{yy}{4}}$
18 $- \frac{y}{2}$	19	$e = \sqrt{\frac{xx - 4sspp}{4ss - 4dd} + \frac{yy}{4}} : - \frac{y}{2}$ $= 2.86$
Then by Step 8th	20	$a = \frac{x - 2de}{2s} = 3.03 =$ B C.
Hence	21	A C $= s - a = 4.97$
And	22	B A $= d + 2e = 7.72$

Queſtion 90. *In the right-angled Triangle* A B C, *there is given the Area of the Triangle equal to* 24, *and the Sum of the Hypothenuſe* A C *and Perpendicular* B C *equal to* 16. *To find the Sides of the Triangle?*

Let A C $= y$, A B $= a$, B C $= e$, $s = 24$, $d = 16$.



Here being three unknown Quantities, there muſt be raiſed three Equations from the Queſtion, and the Properties of the Figure.

Now as the Triangle A B C is right-angled, therefore,

By 47 . e 1 .	1	$aa + ee = yy$ } By the Queſtion.
	2	$y + e = d$
	3	$\frac{ae}{2} = s$, from the Rule for finding the Area of the Triangle, for the Product of the Baſe and Perpendicular of any Triangle being divided by 2, the Quotient is the Area of the Triangle.

The firſt Equation has all the three unknown Quantities, but the other two have only two of them. Now if we take that Quantity which is in all the three Equations, and find the Value of it in one of them, and in the Room of that unknown Quantity in the other two, write its Value, the Queſtion will be then reduced to two Equations, and two unknown Quantities, thus in the ſecond Equation find the Value of e.

$2 - y$	4	$e = d - y$
		But as it is *ee* in the firſt Equation, therefore
$4 ⊙ 2$	5	$ee = dd - 2dy + yy$
1 . 5	6	$aa + dd - 2dy + yy = yy$
3 . 4	7	$\frac{ad - ay}{2} = s$ } Two Equations with only two unknown Quantities.
		Find the Value of *y*, in each of theſe Equations.
$6 - yy$	8	$aa + dd - 2dy = 0$, for *yy* being taken away by the Subſtraction, that Side of the Equation is *nothing*.
$9 + 2dy$	9	$2dy = aa + dd$
$9 \div 2d$	10	$y = \frac{aa + dd}{2d}$
7×2	11	$ad - ay = 2s$
$11 + ay$	12	$ad = 2s + ay$
$12 - 2s$	13	$ay = ad - 2s$
$13 \div a$	14	$y = \frac{ad - 2s}{a}$
10 . 14	15	$\frac{aa + dd}{2d} = \frac{ad - 2s}{a}$
$15 \times a$	16	$\frac{aaa + dda}{2d} = ad - 2s$
$16 \times 2d$	17	$aaa + dda = 2dda - 4ds$
17 in Numbers	18	$aaa + 256a = 512a - 1536$
$18 - 512a$	19	$aaa - 256a = -1536$

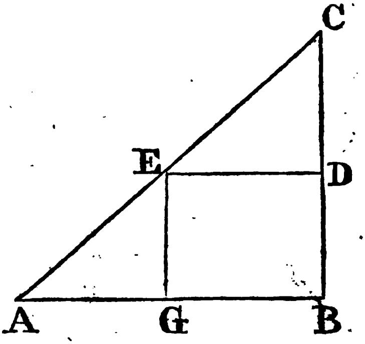
Here the Equation is *adfected*, therefore tranſpoſe the Quantities ſo that the Side of the Equation which is known may have the *affirmative* Sign.

$19 + 256a$	20	$aaa = 256a - 1536$
$20 + 1536$	21	$aaa + 1536 = 256a$
$21 - aaa$	22	$256a - aaa = 1536$

Which Equation being reſolved by the Method of *Converging Series*, we ſhall find $a = 8$ nearly, for 8 is the true Root; from whence the other Sides of the Triangle are eaſily determined.

Queſtion 91. *In the right-angled Triangle* A B C, *there is drawn* G E *parallel to the Perpendicular* B C, *given the Perpendicular* B C = 24, *and the Segment of the Hypothenuſe*

EC = 15, *and the Segment of the Baſe* AG = 20. *To find the Hypothenuſe* AC *and Baſe* AB?



Draw ED parallel to AB. Let BC = c = 24, EC = n = 15, AG = b = 20, GB = ED = a, then AB = $b + a$, AE = e, then AC = $n + e$.

Here being two unknown Quantities, we muſt raiſe two Equations.

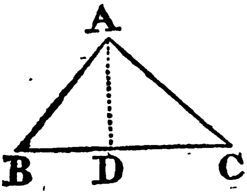
Now the Triangles AGE and EDC are ſimilar, hence,

by 4 . e . 6	1	AG : AE :: ED : EC
in Symbols	2	$b : e :: a : n$
whence	3	$ae = bn$, for Quantities that are in continual Proportion, the Product of the Extreams and Means are equal.
by 47 e . 1 .	4	$bb + 2ba + aa + cc = nn + 2ne + ee$, the Triangle ABC being right-angled, and as theſe two laſt Equations contain the Queſtion, therefore
3 ÷ e	5	$a = \frac{bn}{e}$
5 ⊛ 2	6	$aa = \frac{bbnn}{ee}$
4 . 5 . 6	7	$bb + \frac{2bbn}{e} + \frac{bbnn}{ee} + cc = nn + 2ne + ee$
7 × e	8	$bbe + 2bbn + \frac{bbnn}{e} + cce = nne + 2nee + eee$
8 × e	9	$bbee + 2bbne + bbnn + ccee = nnee + 2neee + eeee$
9 in Numbers	10	$400ee + 12000e + 90000 + 576ee = 225ee + 30eee + eeee$
that is	11	$976ee + 12000e + 90000 = 225ee + 30eee + eeee$
11 − 225ee	12	$751ee + 12000e + 90000 = 30eee + eeee$
12 − 751ee	13	$eeee + 30eee - 751ee = 12000e + 90000$
13 − 12000e	14	$eeee + 30eee - 751ee - 12000e = 90000$

Which Equation being reſolved by the Method of *Converging Series*, we ſhall find $e=25$ nearly, for 25 is the true Root.

Then by the fifth Step $a=12$, whence $AC=n+e=40$, and $AB=b+a=32$.

Queſtion 92. *In the Triangle* ABC, *given the Baſe* $BC=42.5$ *and the Angle at* $B=49°:45'$ *and the Angle at* $C=42°:30'$ *to find the Perpendicular* AD, *let fall from the vertical Angle upon the Baſe* BC.



The Triangle ADB is right-angled, and the Angle ABD being given, all the Angles of the Triangle ADB are known, therefore by plain Trigonometry, we can find the Ratio between the Sides BD, and AD, though we do not know the Length of either of them, for As the Sine of the Angle BAD, is to the Logarithm of any Number aſſumed for the Side BD, ſo is the Sine of the Angle at B to a fourth Number, which is the Logarithm of the proportional Number for the Side AD.

Therefore aſſuming *Unity*, or 1, for the Side BD, we have

As the Sine of the Angle at A -	40° : 15′ -	9.810316
Is to the Log. of the Side BD -	1. - -	0.0
So is the Sine of the Angle at B -	49° : 45′ -	9.882657
		9.882657
		9.810316
To the Log of the Side AD -	1.18 - -	.072341

Hence the Sides AD and BD are to one another, as 1.18 is to 1.

Now let $BC=b=42.5$ $AD=a$, $m=1.18$ and $p=1$, Conſequently,

$$\left|\,1\,\right| m:p::a:\frac{pa}{m}=BD,$$

that is, as the Numbers which expreſs the Proportion of AD and DB, are to one another, ſo is the true Length of AD to the true Length of BD.

By the ſame Reaſoning in the Triangle ADC, becauſe all the Angles are known, therefore the Ratio of the Sides AD and DC are known, and aſſuming AD to be $=1$, and proceeding by Trigonometry as before, we ſhall find the proportional Number for CD to be 1.1

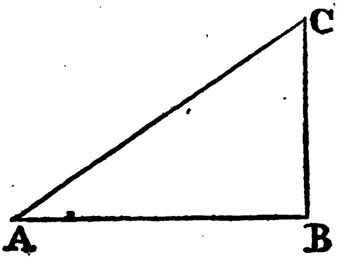
Now putting $d = 1.1$ and $p = 1$, as before, we have

	2	$p : d :: a : \frac{da}{p} = DC$, by the ſame Reaſoning as at the firſt Step.

And as we have now expreſſed in Symbols the two Parts of the Baſe BD and DC.

Hence	3	$\frac{pa}{m} + \frac{da}{p} = b$, that is, $BD + DC = BC$.
$3 \times m$	4	$pa + \frac{mda}{p} = mb$
$4 \times p$	5	$ppa + mda = mbp$
$5 \div pp + mb$	6	$a = \frac{mbp}{pp + md} = 21.82 = AD$.

Queſtion 93. *In the right-angled Triangle* ABC, *there is given the Sum of the Sides equal to* 12, *and the Area equal to* 6. *To find the Hypothenuſe* AC?



Let $s = 12$, $b = 6$, $BC = a$, $AC = y$.

Then by the Property of the Triangle $AB = \sqrt{yy - aa}$.

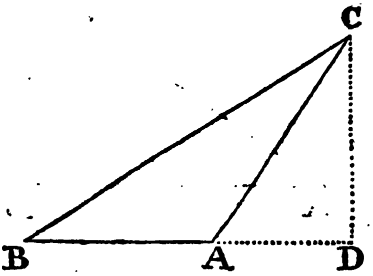
Hence	1	$a + y + \sqrt{yy - aa} = s$
	2	$\frac{a}{2}\sqrt{yy - aa} = b$, from the Rule for (finding the Area of the Triangle.

Now becauſe there is the ſame Surd in both Equations, find what the Surd is equal to in the ſecond Equation, and write it for the Surd in the firſt Equation.

$2 \div \frac{a}{2}$	3	$\sqrt{yy - aa} = \frac{2b}{a}$ for $b = \frac{b}{1}$ and $\frac{b}{1} \div \frac{a}{2} = \frac{2b}{a}$, by the Rule for Diviſion of Fractions in common Arithmetic.

1 · 3

1 . 3 .	4	$a+y+\frac{2b}{a}=s$
$4\times a$	5	$aa+ay+2b=sa$, now the Queſtion is contained in the firſt and fifth Equation.
1 −	6	$\sqrt{yy-aa}=s-a-y$
$6 \odot 2$	7	$yy-aa=ss-2sa-2sy+2ay+aa+yy$
7 ±	8	$2aa=2sa+2sy-2ay-ss$
5 −	9	$aa=sa-ay-2b$
9×2	10	$2aa=2sa-2ay-4b$
8 . 10	11	$2sa-2ay-4b=2sa+2sy-2ay-ss$
$11+2ay$	12	$2sa-4b=2sa+2sy-ss$
$12-2sa$	13	$-4b=2sy-ss$
$13+ss$	14	$2sy=ss-4b$
$13\div 2s$	15	$y=\frac{ss-4b}{2s}=5=AC.$



Queſtion 94. *In the Triangle* A B C, *there is given the Sum of the Sides* BC+BA+AC=85, *and the Area* =200, *and the Angle at* A =124°. *To find the Sides of the Triangle?*

Let $s=85$, $b=200$, $AC=a$, becauſe the Angle BAC =124°, the Angle CAD =56°, and CD being a Perpendicular let fall on BA continued, all the Angles of the Triangle ACD are known, conſequently the Ratio between AC, and CD is known, for aſſuming CD to be *Unity*, or 1, then in the Triangle ACD by Trigonometry,

As the Sine of the Angle CAD -	56° : 00′ -	9.918574
Is to the Log. of the Side CD -	1. -	0.000000
So is Radius - - - -	90° : 00′ -	10.000000
		10.000000
		9.918574
To the Log. of the Side AC -	1.21 -	0.081426

Hence we know the Sides A C and CD are as 1.21 to 1.

Calling $m = 1.21$ $d = 1$, therefore,

	1	$m : d :: a : \frac{da}{m} = CD$, as at the firſt or ſecond Steps, Queſtion 92.

Now B A being conſidered as the Baſe of the Triangle B A C, and C D as its Perpendicular, hence by the Rule for finding the Area of the Triangle $BA \times DC = 2b$, that is in

Symbols	2	$2b = \frac{da}{m} \times BA$.
$2 \div \frac{da}{m}$	3	$\frac{2bm}{da} = BA$, for $2b$ or $\frac{2b}{1} \div \frac{da}{m} = \frac{2bm}{da}$ by the Rule for Diviſion of Vulgar Fractions.
And	4	$AD = \sqrt{aa - \frac{ddaa}{mm}}$ for the Triangle A C D is right-angled, where $a = AC$, and $\frac{da}{m} = CD$.
$3 + 4$	5	$\frac{2bm}{da} + \sqrt{aa - \frac{ddaa}{mm}} = BA + AD = BD$.
$5 \circledast 2$	6	$\frac{4bbmm}{ddaa} + \frac{4bm}{da}\sqrt{aa - \frac{ddaa}{mm}} + aa - \frac{ddaa}{mm} = \overline{BD}^{2}$, or BD ſquared.
$1 \circledast 2$	7	$\frac{ddaa}{mm} = \overline{CD}^{2}$, or C D ſquared
$6 + 7$	8	$\frac{4bbmm}{ddaa} + \frac{4bm}{da}\sqrt{aa - \frac{ddaa}{mm}} + aa = \overline{CB}^{2}$, or C B ſquared.

Having now got an Expreſſion equal to the Square of C B, we muſt endeavour to find another Expreſſion for C B from ſome other *Data*.

Now the Sum of all the Sides is given, that is,

	9	$BC+AC+AB=s$
But	10	$AC=a$, and $AB=\frac{2bm}{da}$ by the third Step.
$9 \cdot 10$	11	$BC+a+\frac{2bm}{da}=s$
$11-\frac{2bm}{da}$	12	$s-\frac{2bm}{da}=BC+a$
$12-a$	13	$s-\frac{2bm}{da}-a=BC$
$13 \odot 2$	14	$ss-\frac{4sbm}{da}-2sa+\frac{4bma}{da}+\frac{4bbmm}{ddaa}+aa=\overset{-2}{BC}$, or BC ſquared
$8 \cdot 14$	15	$ss-\frac{4sbm}{da}-2sa+\frac{4bma}{da}+\frac{4bbmm}{ddaa}+aa=\frac{4bbmm}{ddaa}+\frac{4bm}{da}\sqrt{aa-\frac{ddaa}{mm}}+aa$
$15-\frac{4bbmm}{ddaa}$	16	$ss-\frac{4sbm}{da}-2sa+\frac{4bma}{da}+aa=\frac{4bm}{da}\sqrt{aa-\frac{ddaa}{mm}}:+aa$
$16-aa$	17	$ss-\frac{4sbm}{da}-2sa+\frac{4bma}{da}=\frac{4bm}{da}\sqrt{aa-\frac{ddaa}{mm}}$

Here the Learner may obſerve that the unknown Quantity is under the radical Sign, and therefore as ſuch Equations are generally ſquared to take away the Surds, the ſame is to be done here; but as it is aa in all the Quantities under the radical Sign, we can extract the ſquare Root of aa and join it to the rational Quantity, leaving the remaining Part of the *Surd* under the radical Sign, thus $\frac{4bm}{da}\sqrt{aa-\frac{ddaa}{mm}}=\frac{4bma}{da}\sqrt{1-\frac{dd}{mm}}=\frac{4bm}{d}\sqrt{1-\frac{dd}{mm}}$, whence the ſeventeenth Equation becomes

18 | ss

	18	$ss - \frac{4sbm}{da} - 2sa + \frac{4bma}{da} = \frac{4bm}{d}\sqrt{1 - \frac{dd}{mm}}$, by which means we have ſaved the Trouble of ſquaring the ſeventeenth Step.
$18 \times da$	19	$ssda - 4sbm - 2dsaa + 4bma = 4bma\sqrt{1 - \frac{dd}{mm}}$

The Equation being now cleared of its Fractions, it appears *quadratic*, for the Powers of *a* are only to the firſt and ſecond Power, and the Surd is part of one of the Co-efficients of *a*.

$19 \pm$	20	$2dsaa + 4bma\sqrt{1 - \frac{dd}{mm}} : -4bma - ssaa \; (= -4sbm$
$20 \div 2ds$	21	$aa + \frac{4bma\sqrt{1 - \frac{dd}{mm}} : -4bma - ssda}{2ds} = -\frac{4sbm}{2ds} = -\frac{2bm}{d}$
		Now ſubſtitute $-x = \frac{4bm\sqrt{1 - \frac{dd}{mm}} : -4bm - ssd}{2ds} = -45.$
then	22	$aa - xa = -\frac{2bm}{d}$
$12\,c\,\square$	23	$aa - xa + \frac{xx}{4} = -\frac{2bm}{d} + \frac{xx}{4}$
23 ɯ 2	24	$a - \frac{x}{2} = \sqrt{\frac{xx}{4} - \frac{2bm}{d}}$
$24 + \frac{x}{2}$	25	$a = \frac{x}{2} \pm \sqrt{\frac{xx}{4} - \frac{2bm}{d}}$ the Equation being *ambiguous* by the twentieth Step.

Whence we ſhall find $a = 27.21 = AC$.

And by the third Equation $\frac{2bm}{da} = 17.78 = BA$.

And by the thirteenth Equation $s - \frac{2bm}{da} - a = 40.01 = BC$.

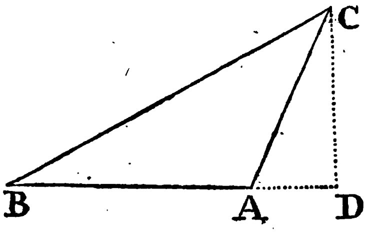
This being the moſt difficult Solution we have yet had, a review or ſummary Account of the Operation may not be uſeleſs to the Learner, as it may tend to give him ſome Idea how to begin and form a Judgment in ſuch Caſes.

Now becauſe the Angles of the Triangle A C D are known, we have the *Ratio* of the Sides given, whence aſſuming C D as known, I find a proportional Number for A C, and from thence I can expreſs C D in Symbols, and C D being conſidered as the Perpendicular to the Triangle B A C, of which the Baſe is B A, then from the Rule for finding the Area of the Triangle, I obtain an Expreſſion for B A; then I expreſs A D in Symbols from knowing A C and C D, and add it to B A, that now I have Expreſſions for B D and D C, each of which being ſquared, their Sum is equal to the Square of B C.

Then from ſome other *Data* I find an Expreſſion for B C, and becauſe the Sum of the Sides is given, and I have Expreſſions for the two Sides B A and A C, therefore it is eaſy to find an Expreſſion for B C as at the thirteenth Step, which being ſquared, is made equal to the former Square of B C, which Equation is reduced as in the Work.

This and ſeveral other Queſtions are taken from Sir ISAAC NEWTON, the perpetual, and everlaſting, Honour, Ornament, and Glory of our Nation; and I have only endeavoured to accommodate his Solutions to the Learner, in explaining them in a more copious Manner.

Queſtion 95. *In the Triangle* A B C, *there is given the Altitude* $CD = 7$, *and the Baſe* $AB = 10$, *and the Sum of the Sides* $BC + AC = 23$. *To find the Sides of the Triangle?*



Becauſe the three Sides of the Triangle B A C will be eaſily expreſſed in Symbols, and the Triangle B C D being right-angled, we ſhall eaſily find to what B D is equal.

Again as the Triangle A C D is right-angled, and the Sides A C and C D are known in Symbols, therefore A D is known in Symbols.

Now if from B D before found in Symbols, we ſubſtract B A, there remains another Value for A D, which being made equal to the former we have an Equation, which is ſufficient, if we uſe but one unknown Quantity.

And as here will be a new Method of expreſſing the Quantities ſought, I would refer the Reader to Queſtion 39, where he will find that in any two Numbers, or Quantities, the leſſer Number is equal to the Difference, between the Sum and Difference of the two Numbers, divided by 2, and at the ſame Queſtion if he exterminates e and finds a, or the greater Number, it will be equal to the Sum and Difference of the two Numbers added together and divided by 2.

Therefore put $x = CD = 7$, $b = BA = 10$, $c =$ half the Sum of the Sides $BC + AC = 11.5$ and $a =$ half their Difference, then the greater Side or $BC = c + a$, and the leſſer Side or $AC = c - a$, now in the Triangle BCD

by 47 e. 1	1	$\sqrt{cc+2ca+aa-xx} = BD$, for $BC = c+a$, and $CD = x$
		And in the Triangle ACD,
by 47 e. 1	2	$\sqrt{cc-2ca+aa-xx} = AD$, for $AC = c-a$, and $CD = x$
but	3	$BA = b$
1 — 3	4	$\sqrt{cc+2ca+aa-xx} : -b = BD - BA = AD$
2 . 4	5	$\sqrt{cc-2ca+aa-xx} = \sqrt{cc+2ca+aa-xx} - b$
5 @ 2	6	$cc-2ca+aa-xx = cc+2ca+aa-xx - 2b\sqrt{cc+2ca+aa-xx} : +bb$
		By tranſpoſing the Quantities which deſtroy one another
we have	7	$-4ca = -2b\sqrt{cc+2ca+aa-xx} : +bb$
7 ±	8	$2b\sqrt{cc+2ca+aa-xx} = bb+4ca$
8 @ 2	9	$4bbcc+8bbca+4bbaa-4bbxx = bbbb+8bbca+16ccaa$
9 — 8bbca	10	$4bbcc+4bbaa-4bbxx = bbbb+16ccaa$
10 — bbbb	11	$16ccaa = 4bbcc+4bbaa-4bbxx-bbbb$
11 — 4bbaa	12	$16ccaa-4bbaa = 4bbcc-4bbxx-bbbb$
12 ÷ 16cc — 4bb	13	$aa = \frac{4bbcc-bbbb-4bbxx}{16cc-4bb} = \frac{bb}{4} - \frac{bbxx}{4cc-bb}$
13 ɯ 2	14	$a = \sqrt{\frac{bb}{4} - \frac{bbxx}{4cc-bb}} = b\sqrt{\frac{1}{4} - \frac{xx}{4cc-bb}} = 3.69$

Whence $c + a = 15.19 = BC$, and $c - a = 7.81 = AC$.

If the Learner ſhould be perplexed to ſee the Contractions at the thirteenth and fourteenth Steps, they may be illuſtrated thus

$$\frac{4bbcc - bbbb - 4bbxx}{16cc - 4bb} = \frac{4bbcc - bbbb}{16cc - 4bb} - \frac{4bbxx}{16cc - 4bb}$$

for it is the ſame thing whether the Quantities that compoſe the Numerator, are placed ſucceſſively one after another like one continued Fraction, or placed ſeparately and diſtinctly, like different Fractions, the Quantities that compoſe the Denominator being placed under each diſtinct Numerator.

$$\begin{array}{rl} \text{But } 16cc - 4bb) & 4bbcc - bbbb \left(\frac{bb}{4}\right. \\ & \underline{4bbcc - bbbb} \\ & \qquad\qquad 0 \end{array}$$

The Quotient Quantity is bb, and as the Co-efficients of the Diviſor are reſpectively four Times more than thoſe of the Dividend, therefore under the Quotient Quantity bb place 4, and $\frac{bb}{4}$ is the Quotient exact.

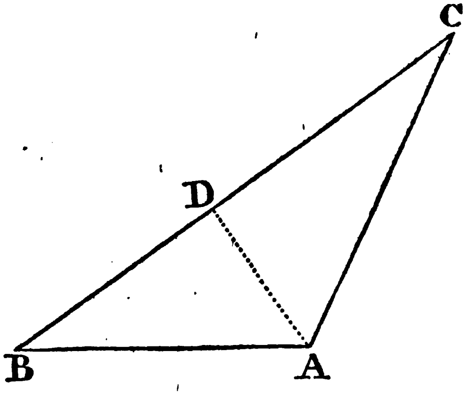
And this Fraction $\frac{4bbxx}{16cc - 4bb} = \frac{bbxx}{4cc - bb}$, for it is only dividing the Co-efficients by 4, therefore the Contractions are as at the thirteenth Step.

The Contractions at the fourteenth Step ariſe from its being bb in all the Terms under the radical Sign, for it is only placing b the ſquare Root of bb without the radical Sign, by which means $\sqrt{\frac{bb}{4}}$, or $\sqrt{\frac{1bb}{4}}$ is $b\sqrt{\frac{1}{4}}$.

Queſtion 96. *In the Triangle* BCA, *there is given the Baſe* $AB = 6$, *and the Sum of the Sides* $AC + BC = 18$, *and the vertical Angle at* $C = 30^\circ : 00'$. *To find the Sides* AC *and* BC.

Let fall the Perpendicular AD, and in the Triangle ACD, becauſe the Angle at C is given, therefore all the Angles of that Triangle are known, and therefore the Ratio of the Sides is known, by which means we can get an Expreſſion for CD.

And becauſe A D is a Perpendicular that falls within the Triangle, and the Angle at C is acute, therefore by 13 *e* 2, B C ſquared added to A C ſquared, is equal to B A ſquared, added to the Product of 2 B C × C D, from whence we ſhall have another Expreſſion for C D, then if we can expreſs the Sides of the Triangle with one unknown Quantity, this Equation between the two Values of C D will be ſufficient.



Now in the Triangle A C D, becauſe the Angle at C is known, and A D being a Perpendicular to C B, all the Angles of the Triangle A C D are known, therefore aſſuming C D = 1, by Trigonometry,

As the Sine of the Angle C A D - 60° : 00′ - 9.937531
Is to the Log. of the Side C D - 1. - - 0.000000
So is the Sine of the Angle C D A 90:00 - 10.000000
10.000000
9.937531
To the Log. of the Side A C - - 1.15 - - 0.062469

Hence we know that as 1.15 is to 1, ſo is A C : C D.

Then let $AB = 6 = x$, half the Sum of the Sides $AC + BC = 9 = b$, and half their Difference $= a$, then as in the laſt Queſtion, the greater Side or $BC = b + a$, and the leſſer Side or $AC = b - a$, $d = 1.15$ $n = 1$.

Becauſe A C is to C D, as 1.15 is to 1.

Therefore

Therefore	1	$d:n::b-a:\frac{nb-na}{d}=CD$
by 13 e. 2	2	$bb+2ba+aa+bb-2ba+aa=xx+\overline{2b+2a}\times CD$
$2-xx$	3	$2bb+2aa-xx=\overline{2b+2a}\times CD$
$3\div 2b+2a$	4	$\frac{2bb+2aa-xx}{\overline{2b+2a}}=CD$

The ſhort Line over the two Quantities $2b+2a$ in the ſecond and third Equations, ſignifies they are both to be multiplied into CD, otherwiſe there would be no Diſtinction whether C D is to be multiplied into $2a$ only, or all the Quantities on that Side of the Equation.

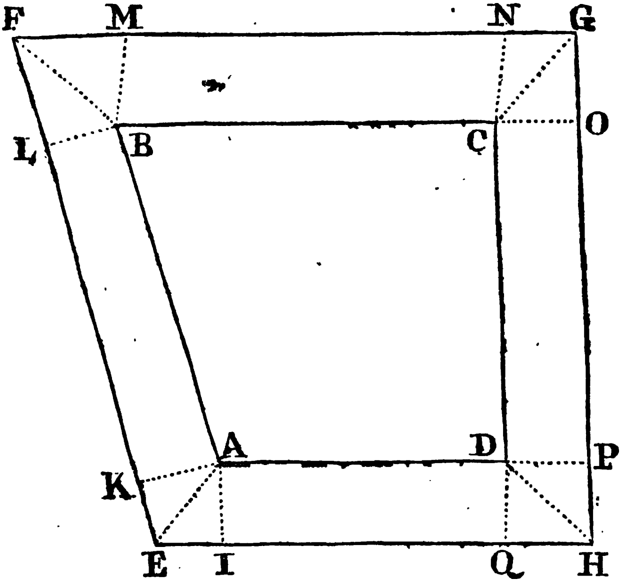
Now make an Equation between the two Values of C D found at the firſt and fourth Equations.

1 . 4	5	$\frac{2bb+2aa-xx}{\overline{2b+2a}}=\frac{nb-na}{d}$
$5\times 2b+2a$	6	$2bb+2aa-xx=\frac{2nbb-2bna+2bna-2naa}{d}$
$6\times d$	7	$2dbb+2daa-dxx=2nbb-2naa$
$7+2naa$	8	$2naa+2dbb+2daa-dxx=2nbb$
$8+dxx$	9	$2naa+2dbb+2daa=2nbb+dxx$
$9-2dbb$	10	$2naa+2daa=2nbb+dxx-2dbb$
$10\div 2n+2d$	11	$aa=\frac{2nbb+dxx-2dbb}{2n+2d}$
11 ɯ 2	12	$a=\sqrt{\frac{2nbb+dxx-2dbb}{2n+2d}}=1.99$

Whence $BC=b+a=16.99$ and $AC=b-a=7.01$

Queſtion 97. *In the Fiſh Pond* A B C D, *there is given the Side* $AD=30$, $DC=35$, $CB=40$, *and* $AB=38$; *the Angle at* $A=113°$, *the Angle at* $B=60°$, *the Angle at* $C=100°$, *and the Angle at* $D=87°$, *and the Fiſh Pond is to be ſurrounded with an Area of* 700, *and every where of the ſame Breadth. To find the Breadth of the Walk?*

Suppoſing the Walk to be drawn round the Pond as in the Figure, let fall the Perpendiculars A K, B L, B M, C N, C O, D P, D Q and A I, by which the Walk is divided into four Parallellograms A K L B, B M N C, C O P D, D Q I A, and into four Trapezia A I E K, B L F M, C N G O and D P H Q, and the Area of theſe four Parallellograms and four Trapezia is equal to the given Area of the Walk.



Let the Breadth of the Walk be a, and the Sum of the Sides $AD + DC + CB + BA = 143 = b$, then the Area of the four Parallellograms will be $= b\,a$. Let $x = 700$.

Draw A E, B F, C G and D H, becauſe the Triangles A I E and A K E are equal, therefore the Angle A E K and A E I are equal, and each of theſe Angles are equal to half the Angle at A which is 113°, hence the Angle A E I is 56° : 30′.

Then in the Triangle A E I all the Angles are known, and conſequently from plain Trigonometry, we can find the Ratio of the Sides E I and I A, for aſſuming E I to be *Unity*, or 1, we have

As the Sine of the Angle E A I -	33° : 30′ -	9.741889
Is to the Log. of the Side E I - -	1.0 - -	0.000000
So is the Sine of the Angle A E I -	56° : 30′ -	9.921107
		9.921107
		9.741889
To the Log. of the Side A I - -	1.51 - -	0.179218

Hence we know that A I is to E I as 1.51 is to 1.
Then let $d = 1.51$ and $e = 1$.

Hence $d : e :: a : \frac{ea}{d} = EI$, which being the Baſe of the Triangle E I A.

Hence	1	$\frac{ea}{d} \times \frac{a}{2} = \frac{eaa}{2d} =$ the Area of the (Triangle E I A, and
1×2	3	$\frac{eaa}{d} =$ the Area of the Trapezium (E I A K.

Now in the Trapezium B L F M, becauſe the Angle B is 60°, for the ſame Reaſons as before, we have the Angle L F B = 30°: whence in the Triangle L F B, if we aſſume B L to be *Unity*, or 1, we ſhall find the proportional Number for F L to be 1.73 hence as 1 is to 1.73 ſo is B L to L F, let $f = 1.73$

Then	4	$e : f :: a : \frac{af}{e} = LF$
$4 \times \frac{a}{2}$	5	$\frac{af}{e} \times \frac{a}{2} = \frac{aaf}{2e} =$ the Area of the (Triangle B L F.
5×2	6	$\frac{aaf}{e} =$ the Area of the Trapezium (B L F M.

Again in the Trapezium C N G O, becauſe the Angle at C is 100°, for the ſame Reaſon as before, the Angle C G N is 50°, and aſſuming *Unity* for N G, we ſhall find 1.19 to be the proportional Number for N C, whence we know that C N and N G are as 1.19 is to 1, let $g = 1,19$

Then	7	$g : e :: a : \frac{ea}{g} = NG$, which being the Baſe of the Triangle C N G,
Hence	8	$\frac{ea}{g} \times \frac{a}{2} = \frac{eaa}{2g} =$ the Area of the (Triangle C N G.
8×2	9	$\frac{eaa}{g} =$ the Area of the Trapezium (C N G O.

Laſtly, in the Trapezium D P H Q, becauſe the Angle D is 87°, for the ſame Reaſon as before, the Angle D H P is 43° : 30′ and aſſuming *Unity* for D P we ſhall find 1.07 to be the proportional Number for P H, hence we know that as 1 is to 1.07 ſo is D P to P H, let $b = 1.07$

Then	10	$e : b :: a : \frac{ab}{e} =$ P H, then as before
$10 \times \frac{a}{2}$	11	$\frac{ab}{e} \times \frac{a}{2} = \frac{aab}{2e}$ the Area of the Triangle D P H, hence
11×2	12	$\frac{aab}{e} =$ the Area of the Trapezium D P H Q,
		But it was before found that
	13	$ba =$ the Area of the four Parallellograms.

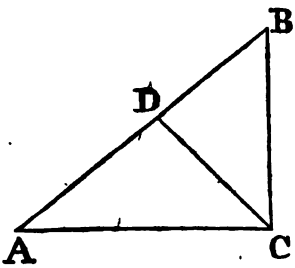
Now collect the Area of the Trapezia and Parallellograms into one Sum, and make them equal to the given Area of the Walk,

$3+6+9+12+13$	14	$\frac{eaa}{d} + \frac{aaf}{e} + \frac{eaa}{g} + \frac{aab}{e} + ba = x = 700$
ſubſtitute	15	$p = \frac{e}{d} + \frac{f}{e} + \frac{e}{g} + \frac{b}{e}$, the Coefficients of $aa = 4.302$
$14 \cdot 15$	16	$paa + ba = x$
$16 \div p$	17	$aa + \frac{ba}{p} = \frac{x}{p}$
$17\ c\ \square$	18	$aa + \frac{ba}{p} + \frac{bb}{4pp} = \frac{bb}{4pp} + \frac{x}{p}$
$18\ \omega\ 2$	19	$a + \frac{b}{2p} = \sqrt{\frac{bb}{4pp} + \frac{x}{p}}$
$19 - \frac{b}{2p}$	20	$a = \sqrt{\frac{bb}{4pp} + \frac{x}{p}} : - \frac{b}{2p} = 4.35$ the Breadth of the Walk.

Becauſe the Angles and Sides of the Fiſh Pond are given, the Figure may be drawn; but for the eaſe of the Numerical Calculation I have choſe ſuch Numbers, as will not exactly agree with a Geometrical Figure.

Queſtion 98. *In the right-angled Triangle* A B C, *given the Perimeter or Sum of the Sides* $AC + CB + AB = 24$, *and the*

Perpendicular CD $=4.75$ *let fall from the right-angle at* C *upon the Hypothenuſe* AB. *To find the Sides of the Triangle?*



Let $CD=b=4.75$ $AB+BC+CA=x=24$, $AB=a$, then the Sum of two of the Sides, or $AC+CB=x-a$, and as at Queſtion 95 let $y=$ the Difference of the ſame two Sides AC and CB. And becauſe the greater Number or Leg is equal to the Sum and Difference of the two Numbers or Legs divided by 2, as in the laſt Queſtion, therefore AC the greater Leg $=\frac{x-a+y}{2}$, and BC the leſſer Leg $=\frac{x-a-y}{2}$.

Having Expreſſions for AB, BC, and AC, the three Sides of the Triangle ABC, in which there are two unknown Quantities, we muſt raiſe two Equations from the Properties of the Figure, and becauſe the Triangles ABC, BCD are ſimilar, therefore by

4 e 6	1	$AB:BC::AC:CD$
in Symbols	2	$a:\frac{x-a-y}{2}::\frac{x-a+y}{2}:b$
whence	3	$ab=\frac{xx-xa+xy-xa+aa-ay-xy+ay-yy}{4}$
that is	4	$ab=\frac{xx-2xa+aa-yy}{4}$
		And becauſe the Triangle ACB is right-angled,
by 47 . e . 1	5	$\frac{xx-2xa-2xy+2ay+aa+yy}{4}+\frac{xx-2xa+2xy-2ay+aa+yy}{4}=aa$
5 contracted	6	$\frac{2xx-4xa+2aa+2yy}{4}=aa$

Hence the two Equations which contain the Queſtion are the fourth and ſixth, and as y is only to the Square in each of them, find in both the Value of yy. But the ſixth Equation becomes

	7	$\frac{xx-2xa+aa+yy}{2}=aa$
7×2	8	$xx-2xa+aa+yy=2aa$
$8\pm$	9	$yy=aa+2xa-xx$
4×4	10	$4ab=xx-2xa+aa-yy$
10 ± 11	11	$yy=xx-2xa+aa-4ab$
9 . 11	12	$aa+2xa-xx=xx-2xa+aa-4ab$
$12-aa$	13	$2xa-xx=xx-2xa-4ab$
$13+4ab$	14	$2xa+4ab-xx=xx-2xa$
$14+2xa$	15	$4xa+4ab-xx=xx$
$15+xx$	16	$4xa+4ab=2xx$
$16\div 4x+4b$	17	$a=\frac{2xx}{4x+4b}=\frac{xx}{2x+2b}=10=AB.$
		Now a being found, therefore
9 ɯ 2	18	$y=\sqrt{aa+2xa-xx}=2$
thence	19	$AC=\frac{x-a+y}{2}=8.$
and	20	$BC=\frac{x-a-y}{2}=6.$

Queſtion 99. *In the right-angled Triangle* A B C, *given the Hypothenuſe* A B $=10$, *and the Sum of the Sides and Perpendicular* C D, *that is,* $AC+CB+CD=18.75$ *To find the Sides* A C *and* B C? *Vide laſt Figure.*

Let $b=10=AB$, $x=18.75$ $CD=a$, then $AC+CB=x-a$; now put $y=$ the Difference between the Legs A C and C B, then as in the two laſt Queſtions A C, or the greater Leg is $\frac{x-a+y}{2}$, and the leſſer Leg, or $CB=\frac{x-a-y}{2}$.

Having expreſſed the Sides of the Triangle in Symbols, in which there are two unknown Quantities, we muſt raiſe two Equations from the Properties of the Figure, and becauſe A B C is a right-angled Triangle, therefore

by 47 e 1	1	$\frac{xx-2xa+2xy-2ay+aa+yy}{4}+\frac{xx-2xa-2xy+2ay+aa+yy}{4}=bb$

Then becauſe the two Triangles A B C and C B D are ſimilar,

by 4 *e* 6	2	AB : CB :: AC : CD that is in Symbols,
	3	$b : \frac{x-a-y}{2} :: \frac{x-a+y}{2} : a$
3 ∵	4	$ba = \frac{xx-xa+xy-xa+aa-ay-xy+ay-yy}{4}$
		Hence the Queſtion is contained in the firſt and fourth Equations.
1 contracted	5	$\frac{xx-2xa+aa+yy}{2} = bb$
4 contracted	6	$ba = \frac{xx-2xa+aa-yy}{4}$

Now in both theſe Equations find the Value of *yy*, there being no other Power of *y*.

$5 \times \overline{2}$	7	$xx-2xa+aa+yy=2bb$
$7 \pm$	8	$yy=2bb-aa+2xa-xx$
$6 \times \overline{4}$	9	$4ba=xx-2xa+aa-yy$
$9 \pm$	10	$yy=xx-2xa+aa-4ba$
8 . 10	11	$xx-2xa+aa-4ba=2bb-aa+2xa-xx$
$11+aa$	12	$xx-2xa+2aa-4ba=2bb+2xa-xx$
$12-xx$	13	$-2xa+2aa-4ba=2bb+2xa-2xx$
$13-2xa$	14	$2aa-4xa-4ba=2bb-2xx$
$14 \div \overline{2}$	15	$aa-2xa-2ba=bb-xx$
		Becauſe *xx* is greater than *bb*, therefore the Equation is *ambiguous.*
ſubſtitute	16	$-z=-2x-2b=-57.5$
then	17	$aa-za=bb-xx$
17 *c* □	18	$aa-za+\frac{zz}{4}=bb-xx+\frac{zz}{4}$
18 ω 2	19	$a-\frac{z}{2}=\sqrt{bb-xx+\frac{zz}{4}}$
$19+\frac{z}{2}$	20	$a=\frac{z}{2}\pm\sqrt{bb-xx+\frac{zz}{4}}=4.78$ (or 52.72
by the 10th Step	21	$y=\sqrt{xx-2xa+aa-4ba}=1.99$

Then AC $=\frac{x-a+y}{2}=7.98$ and CB $=\frac{x-a-y}{2}=6$.

In the above Equation where $a=4.78$ or 52.72 but the Value of a muſt be 4.78 for it cannot be 52.72 as the Sum of the three Quantities is only 18.75

Queſtion 100. *In the right-angled Triangle* ABC, *there is given the Sum of the Sides* AC + BC = 14, *and the Perpendilar* CD = 4.75 *To find the Sides of the Triangle?* See Figure Queſtion 98.

Let $x=14=$ AC + BC, $y=$ AC − BC, or the Difference of the Sides, then as in the preceding Queſtions, the greater Side or AC $=\frac{x+y}{2}$, and the leſſer Side, or BC $=\frac{x-y}{2}$. Put AB $=a$ and DC $=b=4.75$

Having expreſſed all the Sides of the Triangle in Symbols, amongſt which two are unknown, *viz.* a, and y, we muſt raiſe two Equations from the Figure, then becauſe the Triangle ABC is right-angled, therefore

by 47 *e* 1.	1	$\frac{xx+2xy+yy}{4}+\frac{xx-2xy+yy}{4}=aa$
that is	2	$\frac{xx+yy}{2}=aa$
		Becauſe the Triangles ABC, and CBD are ſimilar,
by 4 *e*. 6	3	AB : AC :: CB : CD
in Symbols	4	$a:\frac{x+y}{2}::\frac{x-y}{2}:b$
4 ∵	5	$ba=\frac{xx-yy}{4}$

Here the Queſtion is contained in the ſecond and fifth Equations, and becauſe there are no other Powers of y, but yy in either of thoſe two Equations, find the Value of yy in both Equations.

$2\times\overline{2}$	6	$xx+yy=2aa$
$6-xx$	7	$yy=2aa-xx$
$5\times\overline{4}$	8	$4ba=xx-yy$
$8+yy$	9	$yy+4ba=xx$

$9-4ba$	10	$yy=xx-4ba$
$7 . 10$	11	$2aa-xx=xx-4ba$
$11 \pm$	12	$2aa+4ba=2xx$
$12 \div 2$	13	$aa+2ba=xx$
13 *c* □	14	$aa+2ba+bb=xx+bb$
14 ɯ 2	15	$a+b=\sqrt{xx+bb}$
$15-b$	16	$a=\sqrt{xx+bb}:-b=10.03$ or neglecting the Fraction $AB=10$.
7 ɯ 2	17	$y=\sqrt{2aa-2x}=2$

Then $AC=\frac{x+y}{2}=8$, and $BC=\frac{x-y}{2}=6$.

The same Question done in another Manner.

Let $AC+BC=x=14$, $AC=a$, then $BC=x-a$, $CD=b=4.75$ and becauſe the Triangle ABC is right-angled, therefore $AB=\sqrt{xx-2xa+2aa}$.

Here we have Expreſſions for all the Sides of the Triangle, with only one unknown Quantity, and therefore one Equation will be ſufficient. And as the Triangles ABC and CBD are ſimilar, therefore

by 4 *e*. 6	1	$AB:AC::CB:CD$
in Symbols	2	$\sqrt{xx-2xa+2aa}:a::x-a:b$
2 ∵	3	$b\sqrt{xx-2xa+2aa}=xa-aa$

Square both Sides of the Equation, the unknown Quantity being under the radical Sign.

3 ⊚ 2	4	$bbxx-2bbxa+2bbaa=xxaa-2xaaa+aaaa$ Ranging the Equation according to the Powers of the unknown Quantity.
$4 \pm$	5	$aaac-2xaaa+xxaa-2bbaa+2bbxa=bbxx$

Tho' the Equation here appears as if *adfected*, yet it may be reſolved by *compleating the Square* as in Quadratics.

And to give the Learner a clear Idea how this is done, if he ſquares any three Quantities as $m-n-z$, in the Square he will find ſix Terms, three of them being pure Powers of the

Quantities that are ſquared, and the other three Terms will be double Rectangles, or Products of theſe Quanties, and therefore any Expreſſion that comes under theſe Circumſtances, may have its ſquare Root extracted.

Now $aaaa$ is the Square of - - - - aa
And $xxaa$ is the Square of - - - - xa

And $2xaaa$ is the double Rectangle, or Product of theſe Roots.
And $2bbaa$ is the double Rectangle, or Product of $bb \times aa$.
And $2bbxa$ is the double Rectangle, or Product of $bb \times xa$.

From hence it appears that the above Equation of five Quantities has two of them whoſe ſquare Roots may be taken, and that the other three Quantities are double Rectangles of thoſe two Roots, and a third Quantity bb, therefore multiply this Quantity bb by itſelf, and add the Product $bbbb$ to both Sides of the Equation, which makes it a compleat Square, thus,

	6	$aaaa - 2xaaa + xxaa - 2bbaa + 2bbxa + bbbb = bbbb + bbxx$
6 ɯ 2	7	$aa - xa - bb = \sqrt{bbbb + bbxx} = b\sqrt{bb + xx}$
$7 + bb$	8	$aa - xa = bb + b\sqrt{bb + xx}$
8 c □	9	$aa - xa + \frac{xx}{4} = \frac{xx}{4} + bb + b\sqrt{bb + xx}$
9 ɯ 2	10	$a - \frac{x}{2} = \sqrt{\frac{xx}{4} + bb + b\sqrt{bb + xx}}$
$10 + \frac{x}{2}$	11	$a = \frac{x}{2} + \sqrt{\frac{xx}{4} + bb + b\sqrt{bb + xx}}$ $= 18.9 = AC$, a different Value of what it had before, for then it was only 8.

To explain this to the Learner, if he extracts the ſquare Root of $aa - 2xa + xx$, he will find it to be $a - x$, or $x - a$, the double Rectangle, *viz.* $2ab$ having the Sign $-$ we are ſure either a, or x muſt be *negative*; but in this Caſe we are to determine which is to be *negative* by the Conſequences that follow, for if there follows an impoſſibility in ſuppoſing $a - x$ to be the Root, then the Root muſt be $x - a$.

To apply this to the Square before us, *viz.* $aaaa - 2xaaa + xxaa - 2bbaa + 2bbxa + bbbb$.

Now the ſquare Root of $aaaa$ is - - - aa
And the ſquare Root of $xxaa$ is - - - xa
And the ſquare Root of $bbbb$ is - - - bb

But as $2xaaa$ the double Rectangle, or Product of $aa \times xa$ has the Sign —, therefore it muſt be in the ſquare Root either $aa - xa$, or $xa - aa$, but as an impoſſibility attends putting it $aa - xa$, we now put it $xa - aa$, and to determine what Sign bb muſt have in the Root, now the double Product $2bbaa$ having the Sign —, therefore it muſt be $+bb$ or bb, as $2bb \times -aa$ produces $-2bbaa$, then taking the

ſixth Equation	7	$aaaa - 2xaaa + xxaa - 2bbaa$ $+ 2bbxa + bbbb = bbbb + bbxx$
7 ш 2	8	$xa - aa + bb = \sqrt{bbbb + bbxx}$ $= b\sqrt{bb + xx}$
		Becauſe aa is *negative* tranſpoſe it
8 + aa	9	$aa + b\sqrt{bb + xx} = xa + bb$
9 − xa	10	$aa - xa + b\sqrt{bb + xx} = bb$
10 − $b\sqrt{bb + xx}$	11	$aa - xa = bb - b\sqrt{bb + xx}$

Here the Equation appears *quadratic*, and becauſe $-b\sqrt{bb+xx}$ is greater than bb, it is likewiſe *ambiguous*.

11 c □	12	$aa - xa + \frac{xx}{4} = \frac{xx}{4} + bb - b\sqrt{bb + xx}$
12 ш 2	13	$a - \frac{x}{2} = \sqrt{\frac{xx}{4} + bb - b\sqrt{bb + xx}}$
13 + $\frac{x}{2}$	14	$a = \frac{x}{2} \pm \sqrt{\frac{xx}{4} + bb - b\sqrt{bb + xx}}$ $= 7 \pm 1.11 = 8.11$ or $5.89 = AC.$

Queſtion 101. *In the right-angled Triangle* ABC, *there is given the Sum of the Legs* $AC + BC = 14$, *and the Sum of the Hypothenuſe and Perpendicular* CD, *or* $AB + CD = 14.75$ *To find the Sides of the Triangle? See Figure, Queſtion* 98.

Let $AC + BC = 14 = x$, $AB + CD = 14.75 = b$, $AC = a$, $AB = y$, then $BC = x - a$, and $CD = b - y$.

Having now expreſſed the Sides of the Triangles in Symbols, in which there are two that are unknown, therefore raiſe two Equations from the Properties of the Figure.

And becauſe the Triangle A B C is right-angled, therefore by

47 *e*. 1.	1	$xx-2xa+2aa=yy$

And becauſe the Triangles A B C, and C B D are ſimilar, therefore by

4 *e*. 6	2	$AB:AC::CB:CD$
in Symbols	3	$y:a::x-a:b-y$
3 ∵	4	$by-yy=xa-aa$

Now both the unknown Quantities being to the firſt and ſecond Power, in the fourth Equation, and it being yy only in the firſt Equation, and theſe two Equations containing the Conditions of the Queſtion, find the Value of yy in each Equation.

4 ±	5	$yy=by+aa-xa$
1 . 5	6	$by+aa-xa=xx-2xa+2aa$
6 − *aa*	7	$by-xa=xx-2xa+aa$
7 + *xa*	8	$by=xx-xa+aa$
8 ÷ *b*	9	$y=\frac{xx-xa+aa}{b}$

Raiſe this Equation to the ſecond Power, and make it = to the firſt Equation as there it is only yy, whereas in the fourth Equation it is by and yy, that if we were to exterminate y from the fourth Equation, we muſt uſe the Values of y and yy.

9 @ 2	10	$yy=\frac{xxxx-2axxx+2aaxx-2aaax+aaxx+aaaa}{bb}$
1 . 10	11	$\frac{xxxx-2axxx+2aaxx-2aaax+aaxx+aaaa}{bb}=$ $xx-2xa+2aa$
11 × *bb*	12	$xxxx-2axxx+2aaxx-2aaax+aaxx+aaaa$ $=xxbb-2bbxa+2bbaa$

Tranſpoſing, and ranging the Equation according to the higheſt Dimenſions of the unknown Quantity.

12 ±	13	$aaaa-2xaaa+2aaxx-2bbaa+aaxx$ $-2axxx+2bbxa+xxxx-xxbb=0$

Tho' the Equation now appears to be *adfected*, yet the ſquare Root may be compleated as in the laſt.

To ſhow the Learner how this is to be done, if he ſquares any four Quantities (for the Root of the above Equation will conſiſt of ſo many Quantities) he will find ten Terms in the Square, four of which are pure Powers of the Quantities that were ſquared, and the other ſix will be double Rectangles of thoſe Quantities, of which each particular Root will conſtitute a part of three of the Rectangles.

Now in the above Equation $aaaa$, $aaxx$, $xxxx$,
Are the Squares of - - - aa, ax, xx,
And the Quantities - $2xaaa$, $2aaxx$, $2axxx$ are the double Rectangles of thoſe Parts, or Roots.

And by examining - $2bbaa$, $2bbxa$, $xxbb$, the remaining Terms in the above Equation, the firſt two are double Rectangles of $bb \times aa$ and $bb \times ax$, but the laſt Term is only a ſingle Rectangle of $bb \times xx$, therefore to compleat the Square there wants $-xxbb$, which when added to $-xxbb$, will make that a double Rectangle of $bb \times xx$, and as we have no pure Power of bb, which being ſquared is $bbbb$, hence if we add $-bbxx+bbbb$ to our Equation we ſhall make it a Square, therefore

| 14 | $aaaa-2xaaa+2aaxx-2bbaa+aaxx$
$-2axxx+2bbxa+xxxx-2xxbb$
$+bbbb=bbbb-xxbb$

Before we proceed, perhaps the Learner might have obſerved that $xxbb$ is the Square of xb, and therefore might ſuppoſe that to be one of the Roots, but then he will find xb to make a Part only of two of the Rectangles, whereas, if it had been one of the Roots, it would have made a Part of three of the Rectangles.

14 ω 2 | 15 | $aa-ax+xx-bb=\sqrt{bbbb-xxbb}$
$=b\sqrt{bb-xx}$

The Manner of extracting this Root is, I firſt extract the ſquare Root of $aaaa$ which is aa, then the ſquare Root of $aaxx$ the next pure Power is ax, and to determine whether ax muſt have the Sign + or —, obſerve the Sign of the double Rectangle of theſe two Roots, *viz.* of $2xaaa$, which becauſe it is — I therefore in the Root make it $-ax$.

The next pure Power is $xxxx$ whoſe Root is xx, then obſerve the Sign of the double Rectangle of this, and one of the two former Roots, as of $2aaxx$ which being + and the Root aa being +, therefore in the Root make it $+xx$.

The laſt pure Power is $bbbb$, whoſe Root is bb, then obſerve the Sign of the double Rectangle of this and one of the former Roots, as the laſt Root xx, but the double Rectangle of theſe is $2xxbb$, which being *negative*, and the Sign of xx being $+$, therefore place the Sign — before bb.

Now we go on with our Work.

$15 + bb$	16	$aa-ax+xx=bb+b\sqrt{bb-xx}$
$16 - xx$	17	$aa-ax=bb-xx+b\sqrt{bb-xx}$
17 c □	18	$aa-ax+\frac{xx}{4}=bb-\frac{3xx}{4}+b\sqrt{bb-xx}$, for $\frac{xx}{4}-xx=-\frac{3xx}{4}$
18 ω 2	19	$a-\frac{x}{2}=\sqrt{bb-\frac{3xx}{4}+b\sqrt{bb-xx}}$
$19+\frac{x}{2}$	20	$a=\frac{x}{2}+\sqrt{bb-\frac{3xx}{4}+b\sqrt{bb-xx}}$

$=18.79=AC$ which is impoſſible, for $AC+BC=14$ by the Queſtion, conſequently AC cannot be 18.79

This impoſſible Concluſion is owing to taking the Root of the Equation at the fifteenth Step, for as $-aa\times-aa$ produces $aaaa$, as well as $aa\times aa$, therefore in the Extraction of ſuch Roots, it is doubtful whether the Root is $-aa$, or aa, let us now make a new Extraction, and ſuppoſe it to be $-aa$.

14 ω 2	21	$-aa+ax-xx+bb=\sqrt{bbbb-xxbb}$ $=b\sqrt{bb-xx}$

Having ſuppoſed the Root of $aaaa$ the firſt pure Power to be $-aa$, I go to the next pure Power which is $aaxx$, the Root of which is ax, but to determine its Sign, obſerve the Sign of the double Rectangle of theſe two Roots, *viz.* of $2aaax$, which being —, I therefore make it $+ax$, as — into $+$ produces —.

The next pure Power is $xxxx$, whoſe Root is xx, then obſerve the Sign of the double Rectangle of this, and either of the two former Roots as of ax, now the Sign of $2axxx$ is —, therefore in the Root make it $-xx$, for $+ax\times-xx$ produces $-axxx$.

The laſt pure Power is $bbbb$, whoſe Root is bb, and obſerve the Sign of the double Rectangle of this, and either of the other Roots, as ſuppoſe the laſt, the double Rectangle of theſe two

Roots is $2bbxx$, which being —, therefore make it $+bb$ as $-xx \times bb$ gives $-xxbb$.

Now we go on with the Work, and tranſpoſe aa it being *negative.*

$21 + aa$	22	$aa + b\sqrt{bb - xx} = ax - xx + bb$
$22 - b\sqrt{bb - xx}$	23	$aa = bb + ax - xx - b\sqrt{bb - xx}$
$23 - ax$	24	$aa - ax = bb - xx - b\sqrt{bb - xx}$

Here the Equation is *quadratic*, and becauſe $-xx - b\sqrt{bb - xx}$ is greater than bb, it is therefore *ambiguous.*

24 c □	25	$aa - ax + \frac{xx}{4} = bb - \frac{3xx}{4} - b\sqrt{bb - xx}$
25 ɯ 2	26	$a - \frac{x}{2} = \sqrt{bb - \frac{3xx}{4} - b\sqrt{bb - xx}}$
$26 + \frac{x}{2}$	27	$a = \frac{x}{2} + \sqrt{bb - \frac{3xx}{4} - b\sqrt{bb - xx}}$

$x = 14$
$x = 14$
56
14
$xx = 196$

$b = 14.75$
$b = 14.75$
7375
10325
5900
1475
$217.5625 = bb$
$-196 = -xx$
$21.5625\ (4.64 = \sqrt{bb - xx}$
16
86) 556
516
924) 4025
3796
229

$$14.75 = b$$
$$4.64 = \sqrt{bb - xx}$$

5900
8850
5900

$$68.4400 = b\sqrt{bb - xx}$$
$$147. = \frac{3xx}{4}$$

$$215.44 = b\sqrt{bb - xx} + \frac{3xx}{4}$$

$$xx = 196$$
3

$$4)\ 588 = 3xx$$

$$147 = \frac{3xx}{4}$$

$$217.5625 = bb$$
$$-215.44 = -\frac{3xx}{4} - b\sqrt{bb - xx}$$

$$2.1225\ (1.46 \text{ neareſt} = \sqrt{bb - \frac{3xx}{4} - b\sqrt{bb - xx}}$$

1

24) 112
96

286) 1625

$$7 = \frac{x}{2}$$
$$\pm 1.46 = \sqrt{bb - \frac{3xx}{4} - b\sqrt{bb - xx}}$$

$$8.46 = a = AC.$$
$$\text{or } 5.54 = a = AC.$$

But if $a = 8.46$, then by the ninth Step $y = \frac{xx - xa + aa}{b}$.

$$a = 8.46$$
$$x = 14.$$

3384
846

$$ax = 118.44$$

$$\begin{array}{r}
8.46 = a \\
8.46 = a \\
\hline
5076 \\
3384 \\
6768 \\
\hline
71.5716 = aa \\
196 = xx \\
\hline
267.5716 = aa + xx \\
-118.44 = -ax \\
\hline
b = 14.75)\ 149.1316\ (10.11 = y = AB. \\
1475 \\
\hline
1631 \\
1475 \\
\hline
1566 \\
1475 \\
\hline
91
\end{array}$$

Becauſe $a = AC = 8.46$ therefore $BC = x - a = 5.54$

That theſe are the three Sides of a right-angled Triangle may be tried, by ſquaring and adding them to ſee if they agree with the Property of the Figure.

5.54	10.11	8.46
5.54	10.11	8.46
2216	1011	5076
2770	1011	3384
2770	10110	6768
30.6916	102.2121	71.5716
71.5716		
102.2632		
102.2121		
.0511 Difference which ariſes from the Inaccuracy of the Fractions.		

But if the laſt Proceſs is too perplexing, the ſame Queſtion may be done otherwiſe, thus, Let $AC+BC=14=x$, and $a=$ the Difference between AC and BC, whence as in the former Queſtions the greater Leg or $AC=\frac{x+a}{2}$, and the leſſer Leg $BC=\frac{x-a}{2}$.

Again put $AB+CD=14.75=b$, and the Difference between AB and $CD=y$, then for the Reaſons already mentioned $AB=\frac{b+y}{2}$, and $CD=\frac{b-y}{2}$.

Now becauſe the Triangle ACB is right-angled,

by 47 *e*. 1	1	$\frac{xx+2xa+aa}{4}+\frac{xx-2xa+aa}{4}=\frac{bb+2by+yy}{4}$

Becauſe the Triangles ACB and BCD are ſimilar, therefore

by 4 *e*. 6	2	$AB:AC::BC:CD$
in Symbols	3	$\frac{b+y}{2}:\frac{x+a}{2}::\frac{x-a}{2}:\frac{b-y}{2}$
3 ∵	4	$\frac{bb-yy}{4}=\frac{xx-aa}{4}$ or $bb-yy=xx-aa$
from the firſt	5	$\frac{xx+aa}{2}=\frac{bb+2by+yy}{4}$

The Queſtion being contained in the fourth and fifth Equations, and there being no other Powers of a but aa in both thoſe Equations, exterminate that unknown Quantity.

4 ±	6	$aa=xx+yy-bb$
5 × 4	7	$2xx+2aa=bb+2by+yy$
7 — 2xx	8	$2aa=bb+2by+yy-2xx$
8 ÷ 2	9	$aa=\frac{bb+2by+yy-2xx}{2}$
6 . 9	10	$xx+yy-bb=\frac{bb+2by+yy-2xx}{2}$
10 × 2	11	$2xx+2yy-2bb=bb+2by+yy-2xx$
11 — yy	12	$2xx+yy-2bb=bb+2by-2xx$
12 + 2bb	13	$2xx+yy=3bb+2by-2xx$
13 — 2xx	14	$yy=3bb+2by-4xx$
14 — 2by	15	$yy-2by=3bb-4xx$

Here the Equation is *quadratic*, and ſince $-4xx$ is greater than $3bb$ it is *ambiguous*.

15 c □	16	$yy-2by+bb=bb+3bb-4xx$ $=4bb-4xx$
16 ω 2	17	$y-b=\sqrt{4bb-4xx}=2\sqrt{bb-xx}$
17 + b	18	$y=b\pm2\sqrt{bb-xx}=14.75\pm9.3$ $=24.05$ or 5.45

But y cannot be 24.05 for the Sum of the Legs is only 14.75 therefore $y=5.45$

then by Step 6th | 19 | $a=\sqrt{xx+yy-bb}=2.85$

Then $AB=\frac{b+y}{2}=10.1$ $AC=\frac{x+a}{2}=8.42$ $BC=\frac{x-a}{2}=5.57$ which three Numbers nearly agree with the Property of the right-angled Triangle, but not exactly, becauſe of the Imperfection of the Fractions.

The Reader may obſerve, that in ſeveral of the Geometrical Queſtions, after Letters are put for one or more of the unknown Quantities, we then get Expreſſions for the other Parts of the Figure from its Properties, and therefore avoid uſing a greater Number of unknown Quantities, and in general the ſolution of Queſtions are more neat and elegant, the fewer unknown Quantities are uſed in the Work.

The Method of reſolving Queſtions, which contain four Equations, and four unknown Quantities.

72. BUT if the Queſtion contains four Equations, and there are four unknown Quantities in each Equation: Then find the Value of one of the unknown Quantities in one of the given Equations, and for that unknown Quantity in the other three Equations write this Value of it, which then reduces the Queſtion to three Equations, and three unknown Quantities.

Then find the Value of one of theſe three unknown Quantities in one of theſe three Equations, and for that unknown Quantity in the other two Equations write this Value of it, which reduces the Queſtion to two Equations, and two unknown Quantities.

Then find the Value of one of theſe unknown Quantities in each of theſe Equations, and make theſe two Equations equal to one another, when we ſhall have an Equation with only one unknown Quantity, which, being reduced, will anſwer the Queſtion.

Queſtion 102. *A Father gave* 1000*l. to his four Sons* A, B, C, D. *And if* A's *Share was added to twice* B's *Share, from which ſubſtracting twice* C's *and twice* D's *Share, there remains* 650 *Pounds:*

And if from A's *Share there is ſubſtracted three times* B's *Share, to this Remainder adding twice* C's *Share, from which Sum ſubſtracting five times* D's *Share, there remains* 400 *Pounds:*

But if to A's *Share there is added four times* B's *Share, from which Sum ſubſtracting three times* C's *Share, to this Remainder adding ſix times* D's *Share, the Sum is* 1150 *Pounds. How much had each Son?*

Let $a=$ A's Share, $e=$ B's Share, $y=$ C's Share, $u=$ D's Share, $s=1000$, $m=650$, $n=400$, $b=1150$.

	1	$a+e+y+u=s$	By the Queſtion.
	2	$a+2e-2y-2u=m$	
	3	$a-3e+2y-5u=n$	
	4	$a+4e-3y+6u=b$	
from the firſt	5	$a=s-u-y-e$	
5 · 2	6	$s+e-3y-3u=m$	
5 · 3	7	$s-4e+y-6u=n$	
5 · 4	8	$s+3e-4y+5u=b$	

Here the Queſtion is reduced to three Equations, and three unknown Quantities.

from the ſixth	9	$e=m+3u+3y-s$
9 · 7	10	$s-4m-12u-12y+4s+y-6u=n$
9 · 8	11	$s+3m+9u+9y-3s-4y+5u=b$

Here the Queſtion is reduced to two Equations, and two unknown Quantities.

10 contracted	12	$5s - 4m - 18u - 11y = n$
11 contracted	13	$-2s + 3m + 14u + 5y = b$
from the twelfth	14	$y = \frac{5s - 4m - 18u - n}{11}$
from the thirteenth	15	$y = \frac{b + 2s - 3m - 14u}{5}$
14 . 15	16	$\frac{b+2s-3m-14u}{5} = \frac{5s-4m-18u-n}{11}$
16 ×	17	$11b+22s-33m-154u=25s-20m-90u-5n$
17 ±	18	$64u = 11b - 3s - 13m + 5n$
18 ÷ 64	19	$u = \frac{11b-3s-13m+5n}{64} = 50$, the Share of D.
then by Step 15th	20	$y = \frac{b+2s-3m-14u}{5} = 100$, the Share of C.
and by Step 9th	21	$e=m+3u+3y-s=100$, the Share of B.
and by Step 5th	22	$a=s-u-y-e=750$, the Share of A.

And in the ſame Manner may any other Queſtion in the like Circumſtances be anſwered.

We ſhall now add a few Queſtions of a different Nature, tho' they are ſuch as are generally propoſed to Learners, and which require a little more Sagacity to expreſs their Conditions, and have therefore hitherto been avoided, imagining the Learner is more perplexed to expreſs, or find out the Equations reſulting from theſe Queſtions than to reſolve thoſe Equations; and therefore I thought it not proper to perplex the Learner with them in the beginning of this Work.

Queſtion 103. *A Perſon bought two Horſes* A *and* B, *which with the Trappings coſt* 100 *Pounds:*

Now if the Trappings were laid on the Horſe A, *both Horſes will be of equal Value.*

But if the Trappings be laid on the Horſe B, *he will be double the Value of the Horſe* A. *How much did the Horſes coſt?*

Let $b = 100$, $a =$ the Value of the Horſe B and Trappings,
then $b - a =$ the Value of the Horſe A.

Now becauſe the Horſe B and Trappings are double the Value of the Horſe A,

hence

hence	1	$a = 2b - 2a$ by the Queſtion
$1 + 2a$	2	$3a = 2b$
$2 \div 3$	3	$a = \frac{2b}{3} = \frac{200}{3} = 66\frac{2}{3}$ Pounds the

Price of the Horſe B and Trappings. Conſequently $100 - 66\frac{2}{3} = 33\frac{1}{3}$ Pounds, the Price of the Horſe A.

But to find what the Trappings coſt, and by that means to find the Price of the Horſe B, let $y =$ the Price of the Trappings.

Then becauſe if the Trappings were taken from the Horſe B, and laid upon the Horſe A, both Horſes would be of equal Value.

therefore	1	$33\frac{1}{3} + y = 66\frac{2}{3} - y$
$1 + y$	2	$33\frac{1}{3} + 2y = 66\frac{2}{3}$
$2 - 33\frac{1}{3}$	3	$2y = 33\frac{1}{3}$
$3 \div 2$	4	$y = 16\frac{2}{3}$ Pounds, the Price of the Trappings.

Conſequently $33\frac{1}{3} + 16\frac{2}{3} = 50$ Pounds, the Price of the Horſe B.

Queſtion 104. *A Labourer in 40 Weeks Labour ſaved 28 Crowns — the Pay of three Weeks, and found he had ſpent 36 Crowns + the Pay of eleven Weeks. How much did he receive a Week?*

Let $a =$ his weekly Pay.

Then he had ſaved Crowns - - - -	$28 - 3a$
And ſpent Crowns - - - - -	$36 + 11a$
	$64 + 8a$

And as the Sum of theſe two muſt be equal to what he received for his forty Weeks Labour,

therefore	1	$40a = 64 + 8a$
$1 - 8a$	2	$32a = 64$
$2 \div 32$	3	$a = 2$ Crowns, his weekly Pay.

Queſtion 105. *A Servant was hired for* 12 *Months, for which he was to have* 24 *Pounds with a Cloak, when he had ſerved* 8 *Months he has leave to go away, and inſtead of his Wages receives a Cloak and* 13 *Pounds. How much did the Cloak coſt?*

Let $a =$ the Price of the Cloak, $b = 12$, $d = 24$, $m = 8$, $x = 13$.

Now $d + a$ is what the Servant was to receive for ſerving twelve Months.

But $x + a$ is what he did receive for ſerving eighth Months.

And as the Pay for eighth Months was proportional to what he was to receive for the twelve Months, therefore,

	1	$d + a : b :: x + a : m$; becauſe when any four Quantities, or Numbers, are in Geometrical Proportion, as the Product of the Extreams and Means are equal,
therefore	2	$md + ma = bx + ba$
$2 \pm$	3	$ba - ma = md - bx$
$3 \div b - m$	4	$a = \dfrac{md - bx}{b - m} = 9$ Pounds, the Price (of the Cloak.

Queſtion 106. *There is a Footman* A, *who goes* 6 *Miles a Day, and* 8 *Days after* B *follows him and goes* 10 *Miles a Day. In how many Days will* B *overtake* A?

Let $b = 6$, $d = 8$, $m = 10$, $a =$ the Number of Days B travels to overtake A, then as A began to walk eight Days before B,

Hence the Number of Days that A travels, is - $d + a$
And the Number of Miles A travels, is - $bd + ba$
And the Number of Miles B travels, is - - ma

But when B overtakes A, they muſt have travelled an equal Number of Miles.

Therefore	1	$ma = bd + ba$
$1 - ba$	2	$ma - ba = bd$
$2 \div m - b$	3	$a = \dfrac{bd}{m - b} = 12$, the Number of Days required, or the Time in which B will overtake A.

Queſtion 107. *If a Scribe can in 8 Days write 15 Sheets. How many ſuch Scribes can write 405 Sheets in 9 Days?*

Let $a =$ the Number of Scribes, $b = 8$, $d = 15$, $m = 405$, $n = 9$.

Then 1 $b : d :: n : \frac{dn}{b}$ the Number of Sheets the Scribe can write in nine Days.

and 2 $\frac{dn}{b} : 1 :: m : \frac{bm}{dn} =$ the Number of Scribes to write the 405 Sheets in nine Days;

hence 3 $a = \frac{bm}{dn} = \frac{3240}{135} = 24$, the Number (of Scribes required.

Queſtion 108. A *can do a piece of Work once in* 3 *Weeks,* B *can do it three Times in* 8 *Weeks, and* C *can do it five Times in* 12 *Weeks. In how long Time can they do it jointly?*

Let $a =$ the Time required, $b = 1$, $d = 3$, $g = 8$, $n = 5$, $m = 12$, the Number 3 occuring twice, I put only d for it.

Then 1 $d : b :: a : \frac{ba}{d}$, the Part of the Work that can be done by A in the Time ſought.

and 2 $g : d :: a : \frac{da}{g}$, the Part of the Work that can be done by B in the Time ſought.

and 3 $m : n :: a : \frac{na}{m}$, the Part of the Work that can be done by C in the Time ſought.

And as theſe three Parts are to be equal to 1, or one Work, therefore,

4 $\frac{ba}{d} + \frac{da}{g} + \frac{na}{m} = 1.$

whence 5 $a = \frac{1}{\frac{b}{d} + \frac{d}{g} + \frac{n}{m}} = \frac{1}{\frac{1}{3} + \frac{3}{8} + \frac{5}{12}}$

by reducing the Fractions $\frac{1}{3}+\frac{3}{8}+\frac{5}{12}$ to a common Denominator, and adding and abbreviating them we ſhall find $\frac{1}{3}+\frac{3}{8}+\frac{5}{12}=\frac{9}{8}$.

Whence $a=\frac{1}{\frac{9}{8}}=\frac{8}{9}$ of a Week, by the Rule for Diviſion (of Vulgar Fractions:

If the Week conſiſts of 6 Days
8
9) 48 (5 Days
45
3
And the Day conſiſts of 12 Hours
9) 36 (4 Hours, that is, they will perform the Work in five Days four Hours.
36

Or the Equation $\frac{ba}{d}+\frac{da}{g}+\frac{na}{m}=1$, may be reduced thus;

$4\times d$	6	$ba+\frac{dda}{g}+\frac{dna}{m}=d$
$6\times g$	7	$gba+dda+\frac{gdna}{m}=gd$
$7\times m$	8	$mgba+mdda+gdna=mgd$
$8\div$	9	$a=\frac{mgd}{mgb+mdd+gdn}=\frac{288}{324}$
		$=\frac{8}{9}$ of a Week as above.

73. Having in this eaſy familiar Manner, and by theſe general and univerſal Rules explained to the Learner the Elements of this celebrated Science, it may not be improper to raiſe his Curioſity, and animate him to exerciſe his Judgment in the Choice of Quantities for the Solution of the ſame Queſtion, to give an Inſtance how much the Solution of Queſtions becomes more neat and elegant, by a judicious Choice of repreſenting the unknown Quantities. The Queſtion and its Solution is from the ingenious Mr JOHN WARD's *Young Mathematician's Guide.*

Queſtion 109. *A Man playing at Hazard, or Dice, won the firſt Throw juſt ſo much Money as he had in his Pocket; the ſecond Throw he won the ſquare Root of what he then had, and five Shillings more; the third Throw he won the ſquare of all he then had; after which his whole Sum was* 112 *l.* 16 *s.* 0 *d. What Money had he when he began to play?*

Suppoſe	1	$a =$ his firſt Sum.
then	2	$2a =$ his Sum after the firſt Throw.
and	3	$\surd 2a:+5 =$ his Winnings at the ſecond Throw.
$2+3$	4	$\surd 2a:+2a+5 =$ his Sum after the ſecond Throw.
$4 \odot 2$	5	$2a+4a\surd 2a+10\surd 2a+4aa+20a+25 =$ his Winnings at the third Throw.
$4+5$	6	$24a+4a\surd 2a+11\surd 2a+4aa+30 = 2256$ Shillings.

Now to avoid theſe *ſurd Quantities*, let us make a ſecond Suppoſition; for

Let	1	$2aa =$ his firſt Sum.
then	2	$4aa =$ his Sum after the firſt Throw.
and	3	$2a+5 =$ his Winnings at the ſecond Throw.
$2+3$	4	$4aa+2a+5 =$ his Sum after the ſecond Throw.
$4 \odot 2$	5	$16aaaa+16aaa+40aa+20a+4aa+25 =$ his Winnings at the third Throw.
$4+5$	6	$16aaaa+16aaa+48aa+22a+30 = 2256$

But to avoid theſe high Equations, let us make a third Suppoſition, for

Let	1	$\frac{aa}{2} =$ his firſt Sum
then	2	$aa =$ his Sum after the firſt Throw.
and	3	$a+5 =$ his Winnings at the ſecond Throw.
$2+3$	4	$aa+a+5$ his Sum after the ſecond Throw.

But as it was the Square of $aa+a+5$ he won at the third Throw, to avoid the Trouble of ſquaring it,

ſubſtitute	5	$e = aa + a + 5$
then	6	$ee =$ his Winnings at the third Throw, conſequently
$5 + 6$	7	$ee + e = 2256$ Shillings.
$7\,c\,\square$	8	$ee + e + 0.25 = 2256.25$
$8\,\omega\,2$	9	$e + 0.5 = 47.5$
$9 - 0.5$	10	$e = 47$. Becauſe at the fifth Step e was ſubſtituted for $aa + a + 5$
$5 \cdot 10$	11	$aa + a + 5 = 47$
$11 - \overline{5}$	12	$aa + a = 42$
$12\,c\,\square$	13	$aa + a + 0.25 = 42.25$
$13\,\omega\,2$	14	$a + 0.5 = 6.5$
$14 - 0.5$	15	$a = 6$
whence	16	$\frac{aa}{2} = 18$ Shillings, the Money he had (when he firſt began to play.

The Learner will eaſily obſerve, that the third Solution is more neat and elegant than either of the other two; tho' I know of no *general* Rule that is given for the Choice of the Quantities to ſtate the Queſtion, but it is left to the Judgment and Sagacity of the Reader, and as ſuch Methods muſt be attended with particular Difficulties to a Learner, I have avoided the perplexing him with them; tho' as he has now a general Method of managing Equations, or ſolving of Queſtions, he may exerciſe his Judgment at his own Diſcretion, in the Choice of different Quantities to repreſent the ſame Queſtion.

The Method of expreſſing the Power of any Quantity, by placing a Figure over it.

74. IT may be further obſerved to the Learner, that there is a more compendious Method of expreſſing the high Powers of any Quantity, by placing a Figure over the Quantity thus, $\overset{4}{a}$ is $aaaa$, and $\overset{3}{a}$ is aaa, and $\overset{1}{a}$ is a, and $\overset{2}{a}\,\overset{3}{b}$ is $aabbb$, that is, the Figure that ſtands over the Letter ſhows to what

Power that Letter, or Quantity, is involved, which Method of Notation is generally uſed when the Powers riſe high. The Figures placed over the Quantity are called *Exponents*, the Mind being a little accuſtomed to this Method of Notation, will as eaſily manage an *Algebraic* proceſs, when the Powers are expreſſed by *Exponents*, as if they were repeated at length; and for the further eaſe of the Learner, in this Method of Notation, we will reſume the Solution of Queſtion 87, expreſſing the Powers by *Exponents*, that the Learner may compare both the Operations together.

	1	$a^2+a-e^2=m$	to find a and e.
	2	$e^2+e=2a$	
$1+e^2$	3	$a^2+a=m+e^2$	
$3-m$	4	$a^2+a-m=e^2$	
$2-e$	5	$e^2=2a-e$	
$4 \cdot 5$	6	$a^2+a-m=2a-e$	
$6+e$	7	$a^2+a-m+e=2a$	
$7-a$	8	$a^2-m+e=a$	
$8+m$	9	$a^2+e=a+m$	
$9-a^2$	10	$e=a+m-a^2$	
$10 \circledast 2$	11	$e^2=a^2+2am+m^2-2a^3-2ma^2+a^4$	
$2 \,\vdots\, 11 \,.\, 10$	12	$a^2+2am+m^2-2a^3-2ma^2+a^4+a+m-a^2=2a$	
12 in Numbers	13	$188a+8836-2a^3-188a^2+a^4+a+94=2a$	
13 contracted	14	$187a+8930-2a^3-188a^2+a^4=0$	
$14-a^4$	15	$-a^4=187a+8930-2a^3-188a^2$	
$15+2a^3$	16	$-a^4+2a^3=187a+8930-188a^2$	
$16+188a^2$	17	$-a^4+2a^3+188a^2=187a+8930$	
$17-187a$	18	$-a^4+2a^3+188a^2-187a=8930$	

In the ſame Manner the Learner may attempt the Solution of any of the other Queſtions, expreſſing the Powers by *Exponents*: One thing is to be carefully obſerved, that the *Exponent* belongs only to the Letter which ſtands under it, and when it is only *Unity*, or 1, it is never ſet down, like the Co-efficient when it is *Unity* only, which is generally neglected in the Expreſſion.

The Method of knowing if a Queſtion is limited, or admits but of one Anſwer; or if it is indetermined, that is, admits of ſeveral Anſwers.

75. IT may be further obſerved, that after the Queſtion is ſtated, that is, all the Equations are expreſſed which are neceſſary for the Solution of the Queſtion, then if there are *more* unknown Quantities than there are *Equations*, that then the Queſtion admits of a *variety* of Anſwers, and is therefore unlimited or indetermined, *ex. gr.*

$$\left.\begin{array}{r} \text{Suppoſe } a+e=40 \\ \text{And } e+y=20 \end{array}\right\} \text{ to find } a,\ e, \text{ and } y.$$

Here there are three unknown Quantities, and only two Equations.

Now e being in both the given Equations, you may ſuppoſe it any Number under 20, the leaſt of the two given Numbers, as for Example ſuppoſe $e=16$.

Then the firſt Equation is $a+16=40$.
And the ſecond Equation is $16+y=20$.

From whence it will be eaſy to find a and y, but if e is ſuppoſed any other Number under 20, then there will be found different Numbers for a and y, and the like of any other Queſtion where the Number of unknown Quantities are more than the Equations which ariſe from the Queſtion.

But when the Number of given Equations are *juſt as many* as the *unknown Quantities* required to be found, then the Queſtion, generally admits but of *one* Anſwer, for then each of the Quantities ſought hath generally but one ſingle Value, thus as at Queſtion 77, where we have

$$\left|\begin{array}{c|l} 1 & a+e+y=b=18 \\ 2 & a+3e-2y=m=9 \\ 3 & a+4y-2e=p=21 \end{array}\right.$$

Where $a=5$, $e=6$, and $y=7$.

But when the Number of the given Equations *exceeds* the Number of the *Quantities ſought*, they not only limit the Queſtion, but often render it *impoſſible*, as one of the Equations may be inconſiſtent with another; as for Example,

$$\left|\begin{array}{c|l} 1 & a+e=16 \\ 2 & ae=48 \\ 2 & a-e=20 \end{array}\right\}\text{to find } a \text{ and } e.$$

Now here are three Equations, and but two unknown Quantities, and the firſt and ſecond Equations include a poſſible Caſe, and it may be found what the Numbers are.

And if we take the ſecond and third Equations, they likewiſe include a poſſible Caſe, for it may be determined what thoſe Numbers are.

But all three Equations together render the Caſe impoſſible, the firſt Equation being incompatible with the third, as the Sum of two Numbers cannot be leſs than their Difference.

To raiſe or invent a Method to extract the Cube Root.

76. THIS is no more than the Method of *Converging Series* applied to the Solution of an Equation, one Side of which is the unknown Quantity, and is a pure Cube, or raiſed to the third Power only, *ex. gr.*

Suppoſe $aaa=9261$, where 9261 is a Cube Number, that to find what a, or the Number is that being cubed will produce 9261, is to extract the Cube Root of 9261.

By the common Method of diſtinguiſhing, of how many Places the Root will conſiſt, by placing a Point over the Place of Units, and another over every third Figure, the Root will conſiſt of two Places, therefore ſuppoſe the Cube Root to be - - - -

$$\begin{array}{r} 20 \\ 20 \\ \hline 400 \\ 20 \\ \hline 8000 \end{array}$$

which being leſs than 9261 the given Number, the Cube Root of 9261 muſt be more than 20.

Now put $r=20$, and e for what 20 wants of the true Root, then is $r+e=a$, or the Cube Root of 9261, and proceed as in the Method of *Converging Series*, Caſe 1. Page 230.

If	1	$r+e=a$,

Raiſe this Equation to the third Power, becauſe it is the Cube Root, which is to be extracted.

1 ⊛ 3	2	$rrr+3rre+3ree+eee=aaa$
but	3	$aaa=9261$ by the Example,
2 . 3	4	$rrr+3rre+3ree+eee=9261$

Put this Equation into Numbers, and reject all the Powers of e above ee, as in the Method of *Converging Series.*

4 in Numbers	5	$8000+1200e+60ee=9261$
		Becauſe 8000 is leſs than 9261, tranſpoſe 8000
$5-8000$	6	$1200e+60ee=1261$
		Dividing by the Co-efficient of ee.
$6\div 60$	7	$20e+ee=21.01$
		Dividing by $20+e$, that is, by the Co-efficient of e *plus* e, as in the Method of *Converging Series*,
$7\div 20+e$	8	$e=\dfrac{21.01}{20+e}$

Operation in Numbers,

$$\begin{array}{lll} & 20)\ 21.01\ (1=e & \\ +e=\underline{\ 1\ } & & \\ \text{Diviſor } 21 & \underline{21\qquad} & \\ & \quad 01 & \text{Remainder rejected.} \end{array}$$

$r=20$

$+e=\underline{\ 1\ }$

$r+e=21=a$, which being tried will be found to be the Cube Root of 9261. And by the ſame Method may the Cube Root of any other Number be extracted.

But to ſave the Trouble of repeating this Operation, when any Cube Root is to be extracted, the above Proceſs may be made more general, by not turning the Equation at the fourth Step into Numbers, and putting any Letter for the given Number, whoſe Cube Root is to be extracted.

Suppoſe as before $aaa = 9261$, let $b = 9261$.

Then $aaa = b$, to find a, or to extract the Cube Root.

Now make a Suppoſition that 20 is the Root, which being tried as before, it will be found too little. Then put $20 = r$, and becauſe 20 is too little, r in this Caſe is uſually called *leſs than juſt*; and for what r wants of the true Root put e, whence $r + e$ will be the true Root, or equal to a.

Hence	1	$r + e = a$ Raiſe this Equation to the third Power as before.
1 @ 3	2	$rrr + 3rre + 3ree + eee = aaa$
but	3	$aaa = b$ as above
2 . 3	4	$rrr + 3rre + 3ree + eee = b$

As we know rrr to be leſs than b, by finding the Cube of 20 was leſs than the given Number, therefore tranſpoſe rrr, and reject the Powers of e above ee.

$4 - rrr$	5	$3rre + 3ree = b - rrr$ Dividing by the Co-efficient of ee.
$5 \div 3r$	6	$re + ee = \frac{b - rrr}{3r}$

As there will be another Diviſion before the Operation is finiſhed, to keep the Fraction as ſimple as may be, ſubſtitute $D = \frac{b - rrr}{3r}$.

Then	7	$re + ee = D$

Now dividing by $r + e$, that is, the Co-efficient of e *plus* e.

$7 \div r + e$	8	$e = \frac{D}{r + e}$, *THEOREM* I.

Operation in Numbers,

$$\begin{array}{rl} b = & 9261 \\ -rrr = & -8000 \\ \hline 3r = 60) & 1261\ (21.01 = D. \\ & 120 \\ \hline & 61 \\ & 60 \\ \hline & 100 \\ & 60 \\ \hline & 40 \end{array}$$

$r = 20)\ 21.01 = D\ (1 = e.$
$+e = \ 1$
Diviſor 21 $\ 21$
$\qquad 01$ Remainder neglected.

$r = 20$
$+e = \ 1$
$r + e = 21 = a$, the Cube Root required as before.

Now ſuppoſe it was required to extract the Cube Root of 132651.

Here according to the Method of pointing, the Root will conſiſt of two Places, and to make a tolerable near Suppoſition at the firſt Trial, the firſt Period being 132, I conſider what whole Number cubed will be the neareſt to 132, and I find it to be 5, then as the Root conſiſts of two Places, I ſupply the next Place with a *Cypher*, and ſuppoſe the Root to be 50, which I know is leſs than the true Root, as the Cube of 5 is leſs than 132.

Hence as before we are to determine what the Number is, that 50 wants of the true Root of 132651.

That putting $r = 50$, and e what it wants of the true Root, and $b = 132651$, we have juſt the ſame ſubſtituted Letters as in the laſt Example; and as the Operation being repeated will be exactly the ſame, it is therefore needleſs to repeat the Work, but only obſerving the Equation, or *Theorem* to find e, which is $e = \frac{D}{r+e}$, and by Subſtitution we have $D = \frac{b - rrr}{3r}$.

$$\begin{array}{rl}
\text{Now } b = & 132651 \\
-rrr = - & 125000 \\
\hline
3r = 150) & 7651\ (51.006 = D. \\
 & 750 \\
\hline
 & 151 \\
 & 150 \\
\hline
 & 1000 \\
 & 900 \\
\hline
 & 100
\end{array}$$

$r = 50$

$r = 50)\ 51.006 = D\ (1 = e.$
$+e = 1$
Diviſor 51 $\quad$ 51
$\qquad\qquad$ 006 Remainder neglected.

Now it was $r = 50$
We have found $e = 1$

$r + e = 51 =$ the Cube Root of 132651, which being tried will be found to be true.

And in the ſame Manner the Cube Root of any other Number may be extracted without repeating the *Algebraic* Work, when the Number aſſumed for the Root is leſs than the *true* Root: But when the Number aſſumed for the Root is too much, or more than the true Root, then we proceed as in the following Example, in the ſame Manner as at the ſecond *Caſe* of *Converging Series*, Page 235.

Required to extract the Cube Root of 24389, or $aaa = 24389$.

By the uſual Method the Root will conſiſt of two Figures, the firſt Period of the given Number is 24, and the Cube of 3 being the neareſt of whole Numbers to 24, and ſupplying the other Place of the Root with a *Cypher*, I ſuppoſe 30 to be the Cube Root of 24389, but the Cube of 30 is 27000, which being more than the given Number, the Cube Root cannot be ſo much as 30.

Therefore let $r = 30$, which is now too great or *more than juſt*, and what 30 is too much call e, then will $r - e = a$, or the true Cube Root required, and calling the given Number $24389 = b$,

we have	1	$r - e = a$ Raiſe this Equation to the third Power as before.
1 ⊙ 3	2	$rrr - 3rre + 3ree + eee = aaa$
but	3	$aaa = 24389 = b$, as b is put for the given Number.
2 . 3	4	$rrr - 3rre + 3ree + eee = b$

Becauſe b is leſs than rrr, tranſpoſe b and reject the Powers of e, above ee.

$4 - b$	5	$rrr - b - 3rre + 3ree = 0$, for one Side of the Equation ſubſtracted from the other Side muſt leave 0, or *nothing*.

Then tranſpoſe all the Powers of e, to the other Side of the Equation.

$5 + 3rre$	6	$3rre = rrr - b + 3ree$
$6 - 3ree$	7	$3rre - 3ree = rrr - b$
		Dividing by the Co-efficient of ee.
$7 \div 3r$	8	$re - ee = \frac{rrr - b}{3r}$

As there will be another Diviſion before the Operation is finiſhed, to keep the Fraction as ſimple as may be, ſubſtitute $G = \frac{rrr - b}{3r}$.

Then | 9 | $re - ee = G$

Now dividing by $r - e$, that is, by the Co-efficient of e *minus* e,

$9 \div r - e$ | 10 | $e = \frac{G}{r - e}$, *THEOREM* II.

Operation,

$$rrr = 27000$$
$$-b = -24389$$

$3r = 90)\ 2619\ (29.01 = G$.
180
811
810
100
90
10

$r = 30)\ 29.01 = G\ (1 = e$
$-\ 1$
Diviſor 29 29
01 Remainder neglected.

$$r = 30$$
$$-e = -1$$

$r - e = 29 = a$, which being cubed will be found to be the true Cube Root of 24389.

In this Caſe the Quotient Figure is ſubſtracted from the Diviſor as it is found, the Diviſor at the tenth Step being $r-e$, whereas in *Theorem* 1 it was at the eighth Step $r+e$.

Now as the firſt ſuppoſed Root muſt be too great or too little, unleſs it happens to be taken exact at the firſt Time, therefore theſe two *Theorems* will extract the Cube Root of any Number, as in the following Example.

Now let it be required to extract the Cube Root of 14526.784

From pointing the whole Numbers according to the uſual Method in Vulgar Arithmetic, the Root will conſiſt of two Places of Integers, the firſt Period of the given Number being 14, and the Cube of the whole Number which is neareſt to 14 is 2; and ſupplying the other Place of the Root with a *Cypher*, I ſuppoſe the Root of the given Number to be 20, which is too little, or *leſs than juſt*, the Cube of 2 the firſt Figure in the Root being leſs than 14, the firſt Period in the given Number.

Then putting $b=14526.784$ $r=20$, and e what 20 wants of the true Root, we proceed as at *Theorem* 1, where $e=\frac{D}{r+e}$, and by Subſtitution $D=\frac{b-rrr}{3r}$.

$$\begin{array}{rl}
b = & 14526.784 \\
-rrr = & -8000. \\
3r = 60) & 6526.784\ (108.78 \text{ neareſt} = D. \\
 & 60 \\
 & 526 \\
 & 480 \\
 & 467 \\
 & 420 \\
 & 478
\end{array}$$

$$\begin{array}{rl}
r = 20) & 108.78 = D\ (4.44 = e. \\
+e = 4 & \\
\text{Diviſor } 24 & 96 \\
4.4 & 1278 \\
\text{Diviſor } 28.4 & 1136 \\
.44 & 14200 \\
\text{Diviſor } 28.44 & 11376 \\
 & 2824
\end{array}$$

The Reader will obſerve that the Quotient Figure is added to the Diviſor to compleat it, in the ſame Manner as at the Method of *Converging Series*, Page 230.

Now $r = 20$

$+e = 4.44$

$r + e = 24.44$ and to try whether this is the true Root of the given Number cube it.

$$\begin{array}{r}
24.44\\
24.44\\
\hline
9776\\
9776\\
9776\\
4888\\
\hline
597.3136\\
24.44\\
\hline
23892544\\
23892544\\
23892544\\
11946272\\
\hline
14598.344384
\end{array}$$

which being greater than the given Number, the Root cannot be ſo much as 24.44

To approach ſtill nearer to the true Root, make a ſecond Operation ſuppoſing the Number laſt found, *viz.* 24.44 to be r, and put e for what that Number is too much, then $r - e$ will be the true Root, and putting the given Number 14526.784 $= b$, we proceed as at *Theorem* 2, where $e = \frac{G}{r - e}$, and by Subſtitution $G = \frac{rrr - b}{3r}$.

$$\begin{array}{rl}
rrr = & 14598.344384\\
-b = & -14526.784\\
\hline
3r = 73.32) & 71.560384\ (.976 = G.\\
 & 65988\\
\hline
 & 55723\\
 & 51324\\
\hline
 & 43998\\
 & 43992\\
\hline
 & 6
\end{array}$$

$r = 24.$

$r = 24.44$) .9760 $= G$ (.04 $= e$.
$-e = -$ .04

Diviſor 24.40 9760

0

Now $r = 24.44$
$-e = -$ .04

$r - e = 24.4$ which being raiſed to the Cube, will be found the true Root of 14526.784

Therefore by the ſecond Operation the true Root is found.

For a further variety, let it be again required to extract the Cube Root of the ſame Number 14526.784

But let us ſuppoſe the Cube Root to be 30, the Cube of which being 27000, the Root cannot be ſo much as 30, then putting $r = 30$, we ſhall have r too great or *more than juſt*, and putting e what it is too much, then $r - e$ will be the true Root, and calling the given Number $14526.784 = b$, we proceed as at *Theorem* 2, where $e = \frac{G}{r - e}$, and by Subſtitution $G = \frac{rrr - b}{3r}$.

$rrr = 27000$.
$-b = -$ 14526.784

$3r = 90$) 12473.216 (138.591 $= G$.
90
347
270
773
720
532
450
821
810
116
90
26

$r = 30)\ \ 138.591 = G\ (5.7$

$-e = -\ 5$

Diviſor 25 $\ \ 125$

$-\ 5.7\ \ 1359$

Diviſor 193 $\ \ 1351$

8

$r = 30.$

$-e = -\ 5.7$

$r - e = 24.3$ to try whether this is the Cube Root of 14526.784 cube 24.3

24.3

24.3

729

972

486

590.49

24.3

177147

236196

118098

14348.907 which being leſs than the given Number 14526.784 the Cube Root muſt be more than 24.3

Now for a ſecond Operation, and let $r = 24.3$ and what it wants of the true Root call e, then will $r + e$ be the true Root, and ſtill calling the given Number $14526.784 = b$, we now proceed as at *Theorem* 1, where $e = \frac{D}{r+e}$, and by Subſtitution $D = \frac{b - rrr}{3r}$.

$b = 14526.784$
$-rrr = -14348.907$
$3r = 72.9)\ 177.877\ (2.44 = D.$
1458
3207
2916
2917
2916
1

$r = 24.3)\ 1.440\ (.1 = e.$
$+\ .1$
Diviſor 24.4 $\quad 244$
0

$r = 24.3$
$+e = .1$
$r + e = 24.4$ the true Root as before.

In the ſame Manner may the Cube Root of any other Number be extracted, and tho' the true Root cannot always be exactly had, yet by repeating the Operation you may approach to it within any aſſignable Degree of Exactneſs, and if a ſmall miſtake happens in the firſt it will be corrected at the ſecond Operation.

To extract the Biquadrate, or fourth Root.

THIS Operation proceeds in the ſame Manner, only raiſing the $r + e$ or $r - e$ to the fourth Power, thus,

Required the *Biquadrate* or fourth Root of 194481, or of $aaaa = 194481$.

By placing a Point over the Place of *Unity*, and another over every fourth Figure, we ſhall find the Root will conſiſt of two Figures: And the firſt Period of the given Number being 19, now the Biquadrate or fourth Power of 2 being 16, and that being the neareſt in Integers, therefore ſupplying the other Place of the Root with a *Cypher*, we ſuppoſe 20 to be the Biquadrate Root of 194481; but the Biquadrate of 20 being only 160000, the Root muſt be more than 20. Now let $r = 20$, and putting e for what 20 wants of the true Root, then will $r + e = a$ be the true Root required; and calling the given Number $194481 = b$, then $aaaa = b$.

Now	1	$r+e=a$

Raiſe this Equation to the fourth Power, becauſe it is the *Biquadrate* Root that is to be extracted.

1 ⓦ 4	2	$rrrr+4rrre+6rree=aaaa$, all the Powers of e above ee being rejected.
But	3	$aaaa=b$, b being put equal to the given Number.
2 . 3	4	$rrrr+4rrre+6rree=b$. Becauſe $rrrr$ is leſs than b, tranſpoſe $rrrr$.
$4-rrrr$	5	$4rrre+6rree=b-rrrr$ Dividing by the Co-efficient of ee.
$5\div 6rr$	6	$\frac{2re}{3}+ee=\frac{b-rrrr}{6rr}$.

As there will be another Diviſion before the Operation is finiſhed, therefore as before, ſubſtitute $D=\frac{b-rrrr}{6rr}$.

Then	7	$\frac{2re}{3}+ee=D$.

Now dividing by $\frac{2r}{3}+e$, that is, the Co-efficient of e *plus* e,

$8\div\frac{2r}{3}+e$	8	$e=\frac{D}{\frac{2r}{3}+e}$ *THEOREM* 1.

Operation $b=194481$

$-rrrr=-160000$

$6rr=2400)\ 34481\ (14.367=D$.

2400

10481

9600

8810

7200

16100

14400

17000

$\frac{2r}{3} = 13.33$) 14.367 $= D$ (1. $= e$.
$+$ 1.

14.33 1433

37 Remainder neglected.

$r = 20$
$+e = 1$

$r + e = 21 = a$, which being raiſed to the fourth Power, it will be found to be the *Biquadrate* Root of the given Number.

And if we here take the firſt Root too great, or more *than the Truth*, the Operation is the ſame as raiſing the ſecond *Theorem*, for the Cube Root.

Suppoſe $aaaa = 456976$, to find the *Biquadrate* Root.

The Root being found to conſiſt of two Figures as before, and the firſt Period in the given Number being 45, and the *Biquadrate* of 3 being 81, ſupplying the other Place of the Root with a *Cypher*, let us ſuppoſe 30 to be the Root, but the *Biquadrate* of 30 is 810000, which being more than 456976, the Root cannot be ſo much as 30.

Then putting $r = 30$, and e what 30 is too much, we have $r - e = a$ the Root required; and putting $b = 456976$, we then have $aaaa = b$.

Now	1	$r - e = a$ Raiſing this Equation to the fourth Power as before, and neglecting the Powers of e above ee.
1 ⊙ 4	2	$rrrr - 4rrre + 6rree = aaaa$
But	3	$aaaa = b$
2 . 3	4	$rrrr - 4rrre + 6rree = b$

Becauſe b is leſs than $rrrr$ therefore tranſpoſe b.

$4 - b$	5	$rrrr - b - 4rrre + 6rree = 0$, one Side of the Equation being ſubſtracted from the other, muſt leave *nothing*, now tranſpoſe the ſeveral Powers of e.
$5 + 4rrre$	6	$4rrre = rrrr - b + 6rree$

$6-6rree$	7	$4rrre-6rree=rrrr-b$ Divide by the Co-efficient of ee.
$7\div 6rr$	8	$\frac{2re}{3}-ee=\frac{rrrr-b}{6rr}$

For the ſame Reaſon as in the laſt Operation, ſubſtitute $G=\frac{rrrr-b}{6rr}$.

Then	9	$\frac{2re}{3}-ee=G$. Now divide by $\frac{2r}{3}-e$, that is, by the Co-efficient of e *leſs* e.
$9\div\frac{2r}{3}-e$	10	$e=\frac{G}{\frac{2r}{3}-e}$ *THEOREM* 2.

Operation,

$rrrr=810000$
$-b=-456976$

$6rr=5400)$ 353024 (65.374 $=G$.
32400
29024
27000
20240
16200
40400
37800
26000
21600
4400

$\frac{2r}{3}=20.)$ 65.374 $=G$ (4.08 $=e$.
$-$ 4
Diviſor 16. 64
$-$ .08 13740
Diviſor 15.92 12736
1004

$r=30$

$$\begin{array}{r} r = 30 \\ -e = -\ 4.08 \\ \hline \end{array}$$

$r - e = 25.92 = a$, and to try if this is the true Root, raiſe 25.92 to the fourth Power.

$$\begin{array}{r}
25.92 \\
25.92 \\
\hline
5184 \\
23328 \\
12960 \\
5184 \\
\hline
671.8464 \\
671.8464 \\
\hline
26873856 \\
40310784 \\
26873856 \\
53747712 \\
6718464 \\
47029248 \\
40310784 \\
\hline
451377.58519296
\end{array}$$

which being leſs than the given Number 456976, the true Root muſt be more than 25.92

Then for a ſecond Operation, let $r = 25.92$ and for what it wants of the true Root put e, that now $r + e = a$, and ſtill calling the given Number $456976 = b$, this is exactly the ſame Caſe as when we raiſed the firſt *Theorem* for the *Biquadrate* Root, whence we have no occaſion to repeat the *Algebraic* Work, but to uſe the firſt *Theorem* where $e = \dfrac{D}{\dfrac{2r}{3} + e}$, and by Subſtitution $D = \dfrac{b - rrrr}{6rr}$.

$$\begin{array}{r}
b = 456976. \\
-rrrr = -451377.5852 \\
\hline
6rr = 4031.0784)\ 5598.4148\ (1.3888 = D. \\
40310784 \\
\hline
156733640 \\
120932352 \\
\hline
358012880 \\
322486272 \\
\hline
355266080 \\
322486272 \\
\hline
32779808
\end{array}$$

$$\begin{array}{rrl}
\frac{2r}{3} = 17.28)\ 1.3888 = D & & (.08 = e. \\
+\ \ 08 & & \\
\hline
\text{Diviſor } 1736 & 13888 & \\
\hline
 & 0 &
\end{array}$$

$$\begin{array}{r}
r = 25.92 \\
+e = \ \ .08 \\
\hline
\end{array}$$

$r + e = 26. = a$, which being involved to the fourth Power will be found to be the true *Biquadrate* Root of 456976.

The Reader will eaſily obſerve that theſe two *Theorems* will extract the *Biquadrate* Root of any given Number, in the ſame Manner as the two *Theorems* did for the Cube Root.

In the ſame Method may *Theorems* be raiſed to extract any Root, it being no more than to ſuppoſe a Number to be the required Root, and try whether it is too great or too little; then calling it $r + e$, or $r - e = a$, or the true Root as the occaſion requires, and raiſe this Equation as high as the Root requires, after which the Manner of the Operation is the ſame as before.

To turn Equations into Analogies.

77. SUPPOSE there was given this Proportion $a:b::c:d$; then multiplying Extreams and Means we have this Equation $ad=bc$, that as we get an Equation from Quantities in continual Proportion, by multiplying the Extreams and Means and making one Product equal to the other. Hence to turn an Equation into an Analogy, is only the reverſe, by taking the Quantities that compoſe one Side of the Equation, and making them the two Extreams, and the Quantities that compoſe the other Side of the Equation, and making them the two Means in the Proportion.

To turn the Equation $md=za$ into an Analogy.

One Side of the Equation is compoſed of the Quantities m and d.

And the other Side of the Equation is compoſed of the Quantities z and a.

That placing theſe Quantities according to the Direction, we have $m:z::a:d$

Or $d:z::a:m$
Or $z:d::m:a$
Or $z:m::d:a$, &c.

For multiplying the Extreams and Means of either of theſe Proportions, we ſhall ſtill have the given Equation $dm=za$.

Again, ſuppoſe the Equation $an=bdx$, and it is required to find the Ratio of a to b.

Now one Side of the Equation is compoſed of the Quantities a and n.

And the other Side of the Equation is compoſed of the Quantities b and dx.

But in ranging theſe Quantities, make the given Quantities a and b the firſt and ſecond Terms in the Proportion, and Place the other two Quantities that if the Extreams and Means were to be multiplied they would produce the given Equation, and then we ſhall find $a:b::dx:n$.

From the Equation $dny=bxz$, to find the Proportion of d to b.

By the Directions we ſhall find $d:b::xz:ny$.

From $\frac{an}{m} = pxd$, to find the Proportion of a to d, which is $a : d :: px : \frac{n}{m}$ for multiplying Extreams and Means $\frac{an}{m} = dpx$.

To find the Proportion of a to z from $\frac{an}{y} = \frac{bz}{x}$, here $a : z :: \frac{b}{x} : \frac{n}{y}$.

To find the Proportion of ab to d, from ab or $1ab = dny$. Here $ab : d :: ny : 1$, for multiplying Extreams and Means we have $ab = dny$.

78. I ſhall now, for a Concluſion, endeavour to ſhow the Learner the Certainty of the Rules on which this Science is founded; this I have purpoſely omitted in the beginning of the Work, imagining it in general to be an unreaſonable Expectation for a Learner to ſee the Force of a Demonſtration in this Science, before he is acquainted with its Characters and Language, and in

SUBSTRACTION.

I ſay to ſubſtract a *negative* Quantity from a *poſitive*, is only to change the Sign of the *negative* Quantity and *add* it to the *poſitive* Quantity, and this Sum will be the *Remainder* required. That is,

If $x - y$ is ſubſtracted from $x + y$, I ſay the Remainder is $2y$, for

$$\begin{array}{r|c|l} \text{Suppoſe} & 1 & x + y = m \\ \text{And} & 2 & x - y = n \end{array}$$

Now if it can be proved that $2y$ is equal to the Difference between m and n, it follows that to ſubſtract a *negative* Quantity is to change its Sign and add it.

$$\begin{array}{r|c|l} 2 + y & 3 & x = n + y \end{array}$$

Now in the firſt Equation for x, write $n + y$ for that is equal to x.

Then

Then	4	$n + 2y = m$
$4 - n$	5	$2y = m - n$. Q. E. D.

I ſay further, that to ſubſtract a *negative* Quantity from a *negative* Quantity, is done by *changing* the Sign of the Quantity which is to be ſubſtracted, and then *adding* them by the Rules in Addition, which will be the *Difference* required.

Suppoſe	1	$x - 2y = m$
And	2	$x - y = n$

Now at the ſecond Equation ſuppoſing the $-y$ to become $+y$, then $-2y + y = -y$; and if it can be proved that $-y = m - n$, then to ſubſtract a *negative* Quantity from a *negative* Quantity, is only to change its Sign and add it.

$2 + y$	3	$x = n + y$
$1 \cdot 3$	4	$n + y - 2y = m$
$5 - n$	5	$y - 2y = m - n$
That is	6	$-y = m - n$. Q. E. D.

And that $m - n$ is a *negative* Quantity is evident, for $x - 2y$ cannot be ſo great as $x - y$ they being ſuppoſed poſitive Quantities, and therefore m cannot be ſo great as n; conſequently $m - n$ is a *negative* Quantity, and therefore may be equal to $-y$. And in

MULTIPLICATION.

I ſay unlike Signs being multiplied gives $-$ in the Product; that is, $-a \times a = -aa$.

To prove which, I take for granted the following

LEMMA.

That no Quantities connected by the Sign $+$ only, or by the Sign $-$ only, can be equal to *nothing*. That is, it cannot be $-a - b = 0$, or $a + b = 0$, though it may be $a - b = 0$, or $b - a = 0$.

Now if poſſible, let $-a \times a$ produce aa where the Sign of the Product is *affirmative*.

Let | 1 | $m - a = 0$
| 2 | $a = a$, that is, every Quantity is equal to itſelf.
1 × 2 | 3 | $ma + aa = 0a$, that is, $ma + aa$ is equal to *nothing*, which is againſt the *Lemma*, therefore $- a \times a$ cannot produce aa.

But, I ſay $- a \times a = - aa$.

Let | 1 | $m - a = 0$
| 2 | $+ a = + a$, for any Quantity is equal to itſelf.
1 × 2 | 3 | $ma - aa = 0a$, that is, $ma - aa$ is equal to *nothing*, whence $ma = aa$. Now that $ma = aa$ is evident, for $m - a = 0$, therefore $m = a$, and multiplying by a we have $ma = aa$, conſequently $- a \times a = - aa$. Q. E. D.

I ſay further, that like Signs tho' — being multiplied, produce + in the Product. That is, $- a \times - a$ produces aa and not $- aa$, for

Let | 1 | $m - a = 0$
| 2 | $- a = - a$, every *negative* Quantity being equal to itſelf.

Now, if poſſible, let $- a \times - a$ produce $- aa$. Then

1 × 2 | 3 | $- ma - aa = - 0a$, that is, $- ma - aa$ is equal to *nothing*, which is againſt the *Lemma*, therefore $- a \times - a$ cannot produce $- aa$, but the Sign muſt be + or *affirmative*, which may be further proved thus,

Let | 1 | $m - a = 0$
| 2 | $- a = - a$
1 × 2 | 3 | $- ma + aa = 0a$, that is, $- ma + aa$ is equal to *nothing*, from whence $ma = aa$. And that $ma = aa$ is evident, for $m - a = 0$, therefore $m = a$, and multiplying by a, we have $ma = aa$. Hence $- a \times - a = aa$. Q. E. D.

DIVISION.

As unlike Signs in Multiplication produce — in the Product, I ſay that,

In Diviſion, unlike Signs being divided gives — in the Quotient, that is, if $ab - bb = 0$, and both Sides of the Equation be divided by b, I ſay the Quotient will be $a - b$ and not $a + b$. Suppoſing unlike Signs to give + in the Quotient.

If 1 $ab - bb = 0$
Let 2 $b = b$
1 ÷ 2 3 $a + b = \frac{0}{b}$, that is, $a + b$ is equal to *nothing*, which is againſt the *Lemma*, therefore an Abſurdity follows the Suppoſition, that unlike Signs give + in the Quotient; but I ſay unlike Signs give — in the Quotient.

Let 1 $ab - bb = 0$
2 $b = b$
1 ÷ 2 3 $a - b = \frac{0}{b}$, that is, $a - b$ is equal to *nothing*, whence $a = b$, and that $a = b$ is thus proved.

1 + bb 4 $ab = bb$
4 ÷ b 5 $a = b$. Q. E. D.

I ſay further, that like Signs being divided, though they are *negative*, give + in the Quotient, that is, $ab - bb$ divided by $-b$, the Quotient is $-a + b$ and not $-a - b$.

If like Signs though — give — in the Quotient; then

Let 1 $ab - bb = 0$
2 $-b = -b$
1 ÷ 2 3 $-a - b = \frac{0}{b}$; that is, $-a - b$ is equal to *nothing*, which is againſt the *Lemma*, therefore an Abſurdity follows the Suppoſition, that like Signs though — give — in the Quotient.

But, I ſay $ab - bb$ divided by $-b$, the Quotient is $-a + b$, that is, like Signs though — give + in the Quotient. For

Let 1 $ab - bb = 0$
2 $-b = -b$
1 ÷ 2 3 $-a + b = \frac{0}{b}$, that is, $-a + b$ is equal to *nothing*, whence $b = a$, and that $b = a$ is evident, thus,

1 + bb 4 $ab = bb$
4 ÷ b 5 $a = b$. Q. E. D.

By